# MARLUK Project – Marcionate and Lukan Christianities

## • Mathematical Models •

Zdeněk Pospíšil

Department of Mathematics and Statistics, Faculty of Science
Masaryk University, Brno

Network Theory and Computer Modeling in the Study of Religion
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# Inspiration

# Peter Turchin

Пётр Валентинович Турчин

Born 1957, Obninsk, USSR; since 1977 in exile


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Cliodynamics – description of historical dynamics by means of models developed in mathematical ecology and evolutionary biology

# Metapopulation dynamics

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\begin{aligned}
x_i(t+1) &= f_i\big(x_i(t), y_i(t)\big) + \sum_{j=1}^{k} c_{ij}\big(x_j(t), y_j(t)\big)\Big(f_j\big(x_j(t), y_j(t)\big) - f_i\big(x_i(t), y_i(t)\big)\Big), \\
y_i(t+1) &= g_i\big(x_i(t), y_i(t)\big) + \sum_{j=1}^{k} d_{ij}\big(x_j(t), y_j(t)\big)\Big(g_j\big(x_j(t), y_j(t)\big) - g_i\big(x_i(t), y_i(t)\big)\Big),
\end{aligned}
\qquad i = 1, 2, \ldots, k.
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Two interacting populations (species) reside in $k$ habitats and they may move from one to another site.

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Two interacting populations (species) reside in $k$ habitats and they may move from one to another site.

**Populations:** two types of Christianity (Jewish $\times$ non-Jewish, Lukan $\times$ Marcionate)
**State variables:** $x$, $y$ – intensities of them (proportion of respective types)
**Space:** simple network
  nodes of two kinds – with $\times$ without Jewish networking activity (synagogues, etc.)
  edges – connect the nearest nodes
**Time:** divided into several intervals (framed by breaking historical events)
**Interactions:** $f$, $g$ various "ecological" or "game-theoretic" interplays (within a node)
**Diffusivities:** $c$, $d$ differ in dependence of the kind of connected nodes

# Model I

# Model description

$$
\begin{aligned}
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\end{aligned}
\qquad i = 1, 2, \ldots, k.
$$

**Space:**

$k$ = number of nodes = approx. 1 800
$K$ = number of "Jewish" nodes = approx. 200

# Model description

$$x_i(t+1) = f_i\big(x_i(t), y_i(t)\big) + \sum_{j=1}^{k} c_{ij}\big(x_j(t), y_j(t)\big)\Big(f_j\big(x_j(t), y_j(t)\big) - f_i\big(x_i(t), y_i(t)\big)\Big),$$
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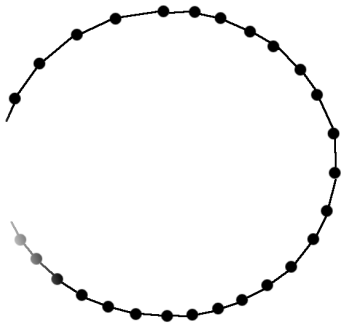
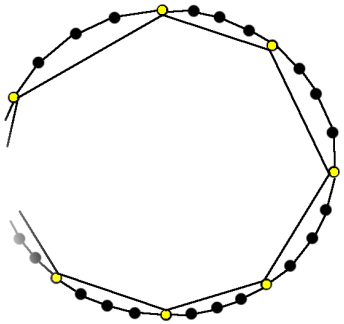
$K$ = number of "Jewish" nodes = approx. 200

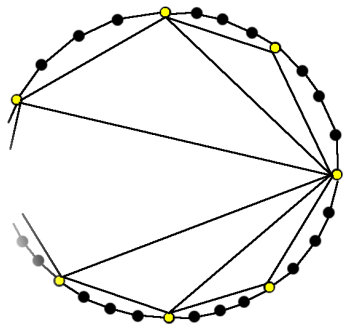
Kinds of network: (i) homogeneous, $K = 0$





(ii) with "special" nodes, $K \approx \frac{1}{5}k$

(iii) one of the "special" nodes is "central"



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\end{aligned}
\qquad i = 1, 2, \ldots, k.
$$

**Time:**

Time unit – one year

Periods of time (breaking historical events):

| | | |
|---|---|---|
| (I) | 30–70 C.E. | (defeat of the Jewish revolt, the fall of Jerusalem) |
| (II) | 71–117 C.E. | (defeat of Jewish rebellions during the Parthian wars) |
| (III) | 118–135 C.E. | (defeat of the Bar Kokhba revolt) |
| (IV) | 136–165 C.E. | |

# Model description

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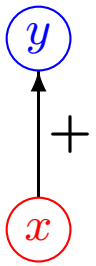
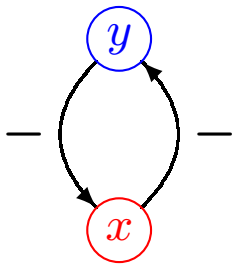
**Interactions within nodes**

(Jewish and non-Jewish networking activities of Christians):

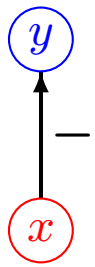
(A) $\dfrac{\partial f}{\partial y} = 0,\ \dfrac{\partial g}{\partial x} > 0$ commensalism: $x$ represents "substrate" for $y$




(B) $\dfrac{\partial f}{\partial y} < 0,\ \dfrac{\partial g}{\partial x} < 0$ competition




(C) $\dfrac{\partial f}{\partial y} = 0,\ \dfrac{\partial g}{\partial x} < 0$ amensalism: $x$ harms $y$



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\end{aligned}
\qquad i = 1, 2, \ldots, k.
$$

**Presuppositions:**

- The "Jewish" sub-net was of some relevance for the early spreading of Christianity.
- The relevance of the "Jewish" sub-net did not increase during time.

**Questions:**

- Was the "Jewish" sub-net necessary for spreading Christianity all over the considered time?
- Did the centre play a role in this process?
- If not, when did importance of them fade away?

# Model specification

**Warnings:**

- There are over 100 of qualitatively different particular models.
- Two different models may produce similar behaviour; one model can exhibit qualitatively different kinds of behaviour!

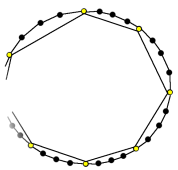
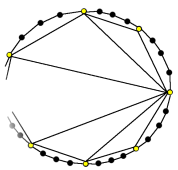
# Model specification

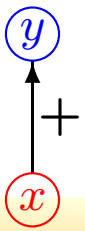
| Period | | Variant of the model |
|---|---|---|
| (I)<br>30–70 | networks:<br>diffusivities:<br>interactions: | (iii)<br>$c > d$<br>(A) |
| (II)<br>71–117 | networks:<br>diffusivities:<br>interactions: | (ii)<br>$c \leq d$<br>(A) |
| (III)<br>118–135 | networks:<br>diffusivities:<br>interactions: | (ii)<br>$c < d$<br>(B) |
| (IV)<br>136–165 | networks:<br>diffusivities:<br>interactions: | (i)<br>$c < d$<br>(C) |

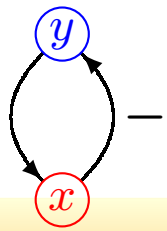
networks: (i) homogeneous
(ii) with "Jewish" nodes
(iii) with "central" node

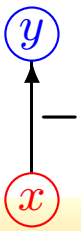
diffusivities: $c$ "Jewish" spread rate
$d$ "non-Jewish" spread rate

interactions: (A) commensalism
(B) competition
(C) amensalism

(i) 
(ii) 
(iii) 

(A) 

(B) 

(C) 

# Model specification

| Period | Model | | Historical context |
|---|---|---|---|
| | formal description | meaning of symbols | |
| (I)<br>30–70 | network: (iii)<br>diffusivities: $c > d$<br>interaction: (A) | "Jewish" with "centre"<br>"Jewish" is faster<br>commensalism | central role of Jerusalem (pilgrimage festivals, temple tax, place of final resurrection);<br>missionary dissemination from Jerusalem, importance of synagogues;<br>fall of Jerusalem (70) reflected significantly in Christian sources; |
| (II)<br>71–117 | network: (ii)<br>diffusivities: $c \leq d$<br>interaction: (A) | "Jewish" without "centre"<br>"Jewish" is not faster<br>commensalism | the role of Jerusalem counterbalanced with diaspora network;<br>Roman restrictions against the Jews (*fiscus Iudaicus*);<br>during the Parthian wars (115–117) Marcion started his carrier (supplying grain to Roman armies), Jewish rebellions broke out in the same time; |
| (III)<br>118–135 | network: (ii)<br>diffusivities: $c < d$<br>interaction: (B) | "Jewish" without "centre"<br>"non-Jewish" is faster<br>competition | Marcionate churches established;<br>sharpening Roman restrictions, Jews forbidden from entering Jerusalem after the Bar Kokhba revolt (132–135);<br>"defining struggle" between Marcion and Luke, Roman authors distinguished between Judaism and Christianity; |
| (IV)<br>136–165 | network: (i)<br>diffusivities: $c < d$<br>interaction: (C) | homogeneous<br>"non-Jewish" is faster<br>amensalism | Jerusalem replaced by Aelia Capitolina;<br>Christians withdrew from Jewish diaspora network;<br>replacement theology, *adversus Iudaeos* literature; |

# Results

Visualization of solution:

- The specified model
- The model without the central role of Jerusalem
- The model with the central role of Jerusalem just on the beginning (30 CE)

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- The traditional conception of Christianity spreading can be reconstructed "in silico".

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- The phenomenon can't spread on homogeneous space (network); even if the "Jewish" sub-net is considered.

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- The existence of a religious center represents a sufficient condition for spreading.

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- The traditional conception of Christianity spreading can be reconstructed "in silico".
- The phenomenon can't spread on homogeneous space (network); even if the "Jewish" sub-net is considered.
- The existence of a religious center represents a sufficient condition for spreading.

The model does not imply that an religious center would be necessary!

# Model II

# Model description

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$t$ ... time
$x = x_i(t)$ ... intensity of the modelled phenomenon
$c = c_{ij}(x)$ ... diffusivity
$f = f_i(x)$ ... change of the intensity

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**Time:** discrete, time unit – one year

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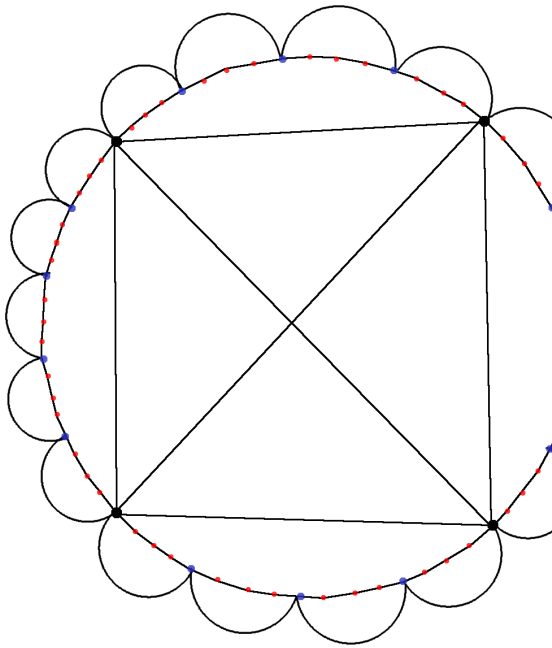
$x = x_i(t)$ ... intensity of the modelled phenomenon at time $t$ in node $i$
$c = c_{ij}(x)$ ... diffusivity (from the node $j$ to the node $i$)
$f = f_i(x)$ ... change of the intensity in a node

**Time:** discrete, time unit – one year

**Space:** Network (dynamic oriented graph) consisting of $k$ nodes; $k = 1\,800$
Among nodes, there are $K$ "Jewish" ones; $K = 200$.
Among them, there are $p$ "priviledged" ones, big cities; $p = 20$.

There is an edge from the node $j$ to the node $i$ at the time $t$ if:
- $i$ is one of the two nodes nearest to $j$
- $j$ is a "Jewish" node and $i$ is one of the two nearest "Jewish" nodes
- $j$ is a "priviledged" node and $i$ is any different "priviledged" node
- $x_j(t) > \theta_D \geq 0$



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**Diffusivity:**
$D = D(t)$ ... maximal degree of a node in the net on time $t$,

$$c_{ij} = c_{ij}\big(x_i(t)\big) = \begin{cases} C\dfrac{1}{D(t)}, & \text{there is an edge from } j \text{ to } i, \\ 0, & \text{otherwise,} \end{cases}$$

$0 < C \leq \frac{1}{2}$.

# Model description

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**Reaction term $f$:** Evolution of the phenomenon within one node

$$f = f(x) = r^{\alpha}(x)x$$

$r$ ... intrinsic growth rate (population growth and missionary activities)
$\alpha$ ... intra-node competition and/or cooperation

$$\alpha(x) = \begin{cases} -0.05, & x \leq \theta_R, \\ \dfrac{1-x}{1-\theta_R}, & x > \theta_R. \end{cases}$$

# Analysis of the model

$$x_i(t+1) = f_i\big(x_i(t)\big) + \sum_{j=1}^{k} c_{ij}\big(x_j(t)\big)\Big(f_j\big(x_j(t)\big) - f_i\big(x_i(t)\big)\Big), \quad i = 1, 2, \ldots, k.$$

**Asymptotic dynamics:**

There are two spatially homogeneous equilibria: $x \equiv 0$, $x \equiv 1$

$r < 1 \quad \Rightarrow \quad x \equiv 0$ is stable, $x \equiv 1$ is unstable,

$1 < r < r_{\text{krit}} \quad \Rightarrow \quad x \equiv 0$ is unstable, $x \equiv 1$ is stable,

$r > r_{\text{krit}} \quad \Rightarrow$ both equilibria are unstable.

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**Transient dynamics:**

The model with the "Jewish sub-net"

The model without the "Jewish sub-net"

The model with the "Jewish sub-net" abandoned in the first "Christian generation"

The model with the "Jewish sub-net" abandoned in the second "Christian generation"

The model with the "Jewish sub-net" abandoned in the third "Christian generation"

# Results

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- A network of connected great cities is sufficient for spreading of Christianity (Christianities); a religious centre is not necessary.
- The Christianity can't spread without a Jewish network.
- The Christians could renounce the Jewish network during the third Christian generation without significant loss of the christianisation level.

# Future prospects

- To create a more realistic model network, i.e. to identify the nodes with particular cities and/or places from available databases.
- To compare the (expected) results and historical records.