Shape analysis of human face

Identification and statistical analysis of anatomical features from high-resolution facial images

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Face3D Supported by wellcome trust

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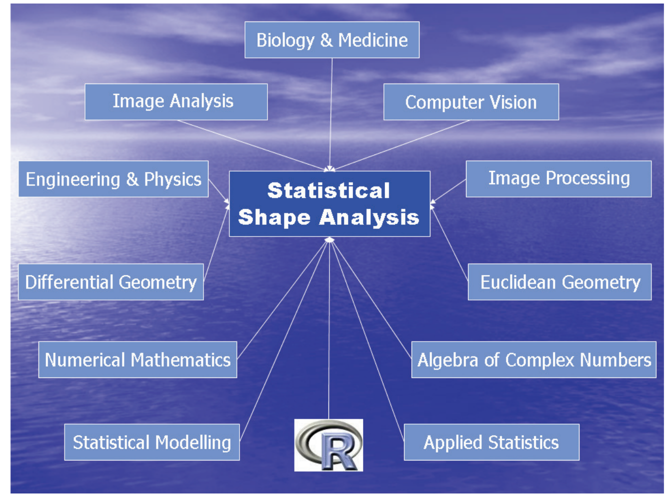
Shape Analysis—Interdisciplinary View



Landmarks to curves to surfaces
Triangle meshes

A **triangle mesh** $\mathcal{M}_i, i = 1, 2, \ldots n$, consists of

- a set of **vertices** $\mathcal{V}_i = \{v_{i1}, v_{i2}, \ldots, v_{iV}\}$
- a set of **triangular faces** connecting them $\mathcal{F}_i = \{f_{i1}, f_{i2}, \ldots, f_{iF}\}, f_j \in \mathcal{V}_i \times \mathcal{V}_i \times \mathcal{V}_i$
- a set of **edges** $\mathcal{E}_i = \{e_{i1}, e_{i2}, \ldots, e_{iE}\}, e_j \in \mathcal{V}_i \times \mathcal{V}_i$
- **a 3D position** $\mathbf{p}_j$ to each vertex $v_j \in \mathcal{V}$

  $$\mathcal{P} = \{\mathbf{p}_{i1}, \mathbf{p}_{i2}, \ldots, \mathbf{p}_{iV}\}, \mathbf{p}_j = \mathbf{p}(v_j) = \begin{pmatrix} x(v_j) \\ y(v_j) \\ z(v_j) \end{pmatrix} \in \mathbb{R}^3, \text{ such}$$

  that each face $f \in \mathcal{F}_i$ corresponds to a triangle in 3D space specified by its three vertex position

Landmarks to curves to surfaces
Configuration matrices and (semi)landmarks

A **configuration matrix**

$$\mathbf{X}_i = \left(\mathbf{x}_i^{(1)}, \mathbf{x}_i^{(2)}, \mathbf{x}_i^{(3)}\right), \mathbf{x}_i^{(\cdot)} = (x_{i1}^{(\cdot)}, x_{i2}^{(\cdot)}, \ldots, x_{ik}^{(\cdot)})^T, k \ll V$$

$$\mathbf{X}_i = (\mathbf{x}_{i1}, \mathbf{x}_{i2}, \ldots, \mathbf{x}_{ik})^T, \mathbf{x}_{ij} = (x_{ij}^{(1)}, x_{ij}^{(2)}, x_{ij}^{(3)})^T, j = 1, 2, \ldots, k$$

The **(semi)landmarks** $\mathbf{x}_{ij}$

- k_1 **landmarks**
- k_2 **semilandmarks on curve** $(k_{21}, k_{22}, \ldots, k_{2C})$
- k_3 **semilandmarks on surface** $(k_{31}, k_{32}, \ldots, k_{3S})$

Landmarks to curves to surfaces
Capturing human face in 3D

A 3D image of the human face can be captured by:

- **cephalometry**
- **CT scanning**
- **laser scanning**
- **stereo-photogrammetry**

Subsequent smoothing is often needed because of ***imperfections of surface representation***

3D imaging of the face
Summary (Hajeer et all 2004)

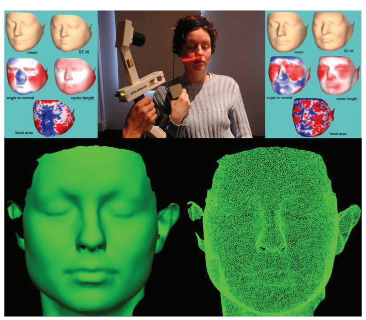
Laser scanning provides a **less invasive method** of capturing the face for planning or evaluating outcome of orthodontic or orthodontic-orthognathic surgical treatment.

1. the slowness of the method, making distortion of the scanned image likely
2. safety issues related to exposing the eyes to the laser beam, especially in growing children
3. inability to capture the soft tissue surface texture, which results in difficulties in identification of landmarks that are dependent on surface color

Even with the new **white-light laser approaches** that capture surface texture color, the shortcomings persist.

3D laser-scan capture
3D facial shape—VCFS data, differences between cases and controls (paired data)

42 pairs of laser-scanned faces
$\approx$ 60000 mesh-points triangulated with 120000 faces



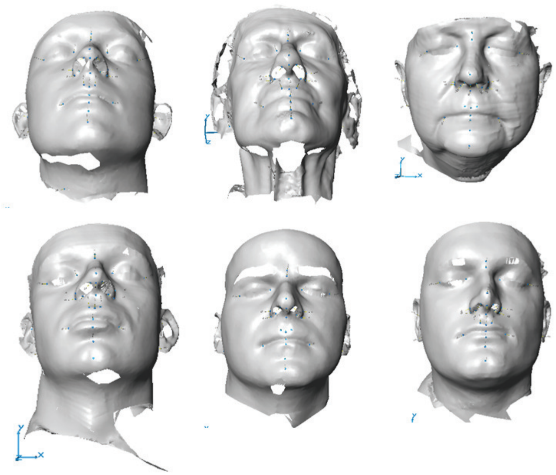
Royal College of Surgeons in Ireland, Dublin; **Face 3D data**
FastSCAN^{TM} Polhemus handheld 3D laser scanner

3D laser-scan capture
VCFS data, differences between cases and controls (matched-pair data)

Why we study the VCFS faces?

- **multiple abnormalities**; extensive and variable phenotype that includes psychiatric disorders and craniofacial dysmorphology
- **increased risk for psychotic illness** [$\approx$ 25-fold]; second only to having an affected monozygotic co-twin $\approx$ 45-fold]
- **schizophrenia** characterised by subtle craniofacial dysmorphology that reflects underlying disturbance in early brain development
- To what extent is craniofacial dysmorphology in VCFS similar to or different from that evident in schizophrenia?
- OVER EARLY FETAL LIFE THE BRAIN AND FACE DEVELOP IN EXQUISITE EMBRYOLOGICAL INTIMACY

3D laser-scan capture
VCFS data, differences between cases and controls (matched-pair data)

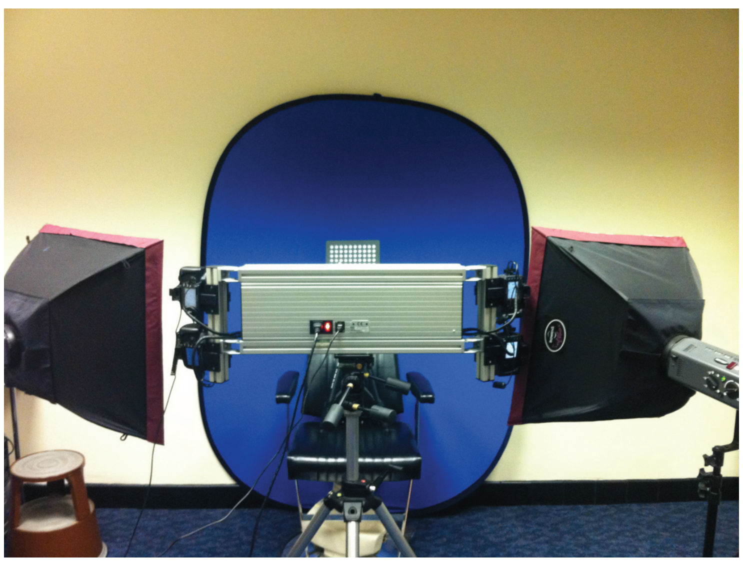


3D stereo-camera capture
Summary (Hajeer et all 2004)

Stereo-photogrammetry uses one or more converging pairs of views to build up a 3D model that can be viewed from any perspective and measured from any direction.

1. four cameras, configured as a two stereopairs, are used to recover 3D distances of features on the surface of the face by means of triangulation
2. Di3D system is based on the use of stereo digital cameras and special textured illumination, with a capture time of 50 milliseconds
3. Di3D system captures the natural surface appearance of the facial skin and "drapes" this skin texture over the captured 3D model of the face

3D stereo-camera capture
System of 3D cameras—School of Maths & Stats, The University of Glasgow, UK



Di3D camera system

3D stereo-camera capture
3D facial shape—control data

$\approx$ 300 stereo-photogrammetric images
$\approx$ 150000 mesh-points triangulated with 300000 faces

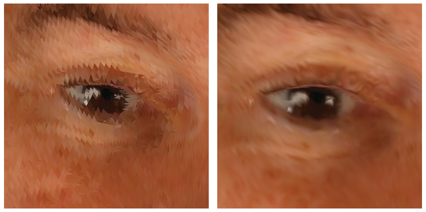
School of Maths & Stats, The University of Glasgow, UK; **Face 3D data**

Di3D camera system

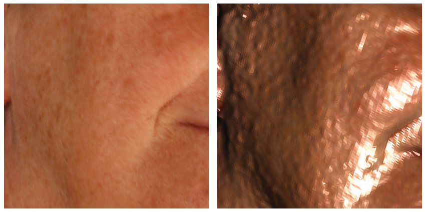
3D stereo-camera capture
3D facial shape—image quality

3D stereo-camera capture
3D facial shape—image quality

3D stereo-camera capture
3D facial shape—image quality

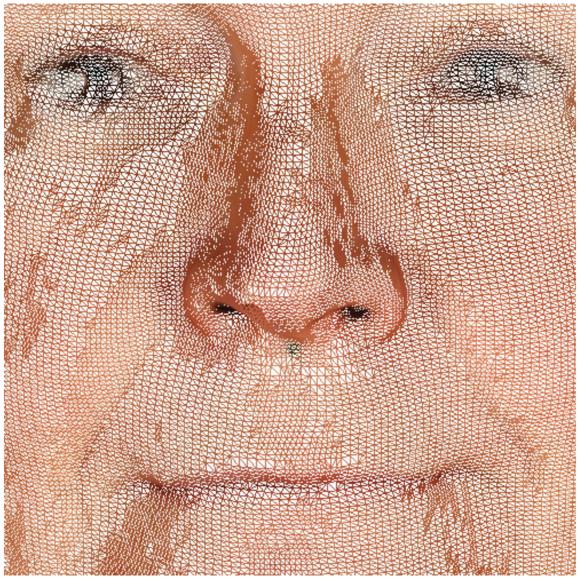


the effect of flat and smooth shading



the effect of lighting calculation in geometry

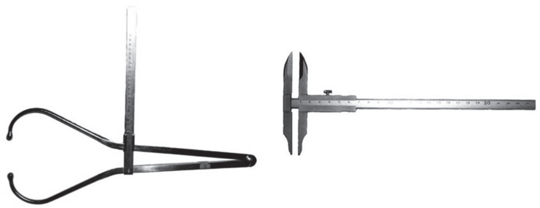
3D stereo-camera capture
3D facial shape—image quality



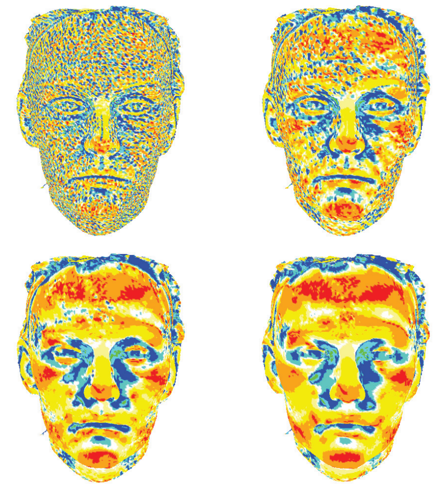
3D face—laser-scan and stereo-camera capture
Data acquisition and pre-processing

Data acquisition and pre-processing:

1. **laser-scan or stereo-camera capture**—data capture protocol (questionnaire, equipment calibration, participants, and timing)
2. **extraction of 3D coordinates, surface normals, faces (the mixture of triangles and quadrangles), and color intensity in RGB space**—from `.obj`, `.ply`, `.wrl`, and `.jpeg` files to `.dmp` files readable in R; and valid `.ply` files (with rescaled intensity) readable in Landmark software [IDAV, University of California, Davis, US]
3. **image capture validation (reliability) study**—selected distances measured with calipers, reconstruction of the coordinates, image landmarking



Shape Index
Smoothing vs surface inhomogeneities



$$s = \frac{2}{\pi}\arctan\left(\frac{\kappa_2+\kappa_1}{\kappa_2-\kappa_1}\right) = \frac{2}{\pi}\arctan\left(\frac{-H}{\sqrt{H^2-K}}\right), \kappa_1 \geq \kappa_2$$

3D face—anatomical curve identification
Summary

Outline:

1. the identification of anatomical curves, with the aim of providing a **much richer characterisation of surface shape than landmarks** and as a potential intermediate step to a suitable characterisation of the full anatomical surface
2. curves often define **the boundaries of particular anatomical features of interest**, allowing the position of these to be identified and, if appropriate, extracted from the larger object for separate analysis
3. types of curves:
   - **valley curve**—the curve following deepest path in the valley
   - **ridge curve**—the curve following the ridge
   - **geodesic**—shortest path between two (semi)landmarks
4. **smooth curves** across the surface with "orange peel" effect—disregarding these locally noisy areas

3D face—anatomical curve identification
Summary

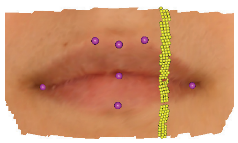
Surface navigation

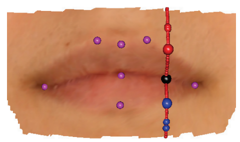
- each anatomical surface of interest—a **two-dimensional manifold** in three-dimensional space (a suitably oriented **local surface patch**)
- while moving around this manifold it is necessary to **remain on the surface**
- a **co-ordinate system** which indexes locations on this manifold, but does not index locations off the manifold, is required
- construct local co-ordinate systems through **planar transects** of the surface, which create **one-dimensional planar curves**
- this reduces the dimensionality of the problem, while allowing the information derived from these curves to be collated across the surface at a later stage

3D face—anatomical curve identification
Summary

Identification of boundary points

- for each one-dimensional curve derived from the **planar transects,** the **point of intersection with the boundary curve of interest** can be identified
- these intersection points are often defined by the **locations of maximum or minimum curvature**
- on some occasions it is necessary to assess the evidence for whether any intersection point exists or whether there is more than one intersection point (points of interest)





- **the collection of candidate boundary points** provides the key information from which a **boundary curve** can then be constructed

3D face—anatomical curve identification
Geodesic curvature along the curve (Koenderink 1990)

How quickly the curve bends within the surface?
geodesic curvature along a curve $\kappa(s)$ at the point $\{\widehat{x}(s), \widehat{y}(s), \widehat{z}(s)\}$ is defined as

$$\frac{\sqrt{\{\widehat{x}''(s)\widehat{y}'(s)-\widehat{y}''(s)\widehat{x}'(s)\}^2+\{\widehat{x}''(s)\widehat{z}'(s)-\widehat{z}''(s)\widehat{x}'(s)\}^2+\{\widehat{y}''(s)\widehat{z}'(s)-\widehat{z}''(s)\widehat{y}'(s)\}^2}}{(\widehat{x}'(s)^2+\widehat{y}'(s)^2+\widehat{z}'(s)^2)^{3/2}}$$

3D face—anatomical curve identification
P-spline—illustration in the case of *x*

Smoothing spline idea leads to the popular **penalized least square regression** with the familiar spline penalty on the integral of the squared second derivative (Fan & Gijbels 1996)

$$\widehat{m}_\lambda(x) = \arg \min_{\forall \lambda \in \mathbb{R}^+} \sum_{j=1}^{k_c} \{x_j - m(s_j)\}^2 + \lambda \int \{m''(s)\}^2 dx$$

***P*-spline** idea leads to the popular **penalized least square regression** with a difference penalty on coefficients of adjacent *B*-splines (Eiler & Marx 1996)

$$\widehat{m}_\lambda(x) = \arg \min_{\forall \lambda \in \mathbb{R}^+} \sum_{j=1}^{k_c} \left\{ x_j - \sum_{i=1}^{m} \alpha_i B_i(s_j) \right\}^2 + \lambda \sum_{i=d+1}^{m} (\Delta^d \alpha_i)^2$$

3D face—anatomical curve identification
P-spline with linear constraint—illustration in the case of *x*

A **p-spline curve** takes the form of a **linear regression**, $x = B\beta$, where

- the columns of the design matrix B evaluate a set of local, **B-spline basis functions** at the values of the observed covariate
- the regression coefficients $\hat{\beta}$ minimise **the penalized sum-of-squares** $SS(\beta) = (x - B\beta)^T(x - B\beta) + \lambda\beta^T D_2^T D_2 \beta$, where the matrix D_2 creates the *second differences* of the elements of the β vector

Linear constraint (Seber 1977)

- to force the solution to pass through particular landmarks
- constraint $A\beta = c$, where the columns of the matrix A evaluate the basis functions **at the constraint locations** and the vector c contains **the constrained response values**
- A has two rows which evaluate the basis functions at s_l and s_r, the arc length values at which the left and right hand corner points are located
- c is the vector (x_l, x_r)
- **the constrained coefficients** $\hat{\beta}_c$ are given by

$$\hat{\beta}_c = \hat{\beta} + (B^T B + D_2^T D_2)^{-1} A^T \left[A(B^T B + D_2^T D_2)^{-1} A^T \right]^{-1} (c - A\hat{\beta})$$

3D face—anatomical curve identification

P-spline with linear constraint—illustration in the case of x

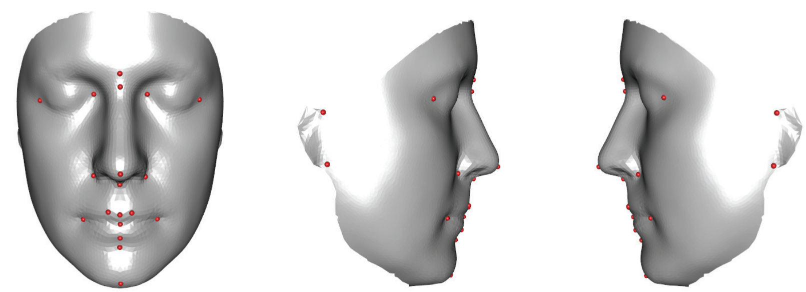
Shape constraints (Bollaerts, Eilers and van Mechelen 2006) through further use of penalty terms [in our case to adopt the anatomy of upper and lower lip to the model]

- **the penalty for monotonicity** is $\kappa\beta^T D_1^T V_1 D_1 \beta$, where the matrix D_1 constructs *the first differences* of the elements of β and the matrix V_1 is diagonal with elements which are 1 when the required monotonicity constraint is violated and 0 otherwise
- **the penalty for the second derivatives** is $\kappa\beta^T D_2^T V_2 D_2 \beta$, where the matrix V_2 is diagonal with elements which are 1 when the change in *the second differences* of the elements of β has a sign which is inconsistent with the increasing/decreasing criterion for the second derivative
- **the penalized sum-of-squares function** is now

$$SS(\beta) = (x - B\beta)^T (x - B\beta) + \lambda\beta^T D_2^T D_2 \beta + \kappa\beta^T D_1^T V_1 D_1 \beta + \kappa\beta^T D_2^T V_2 D_2 \beta$$

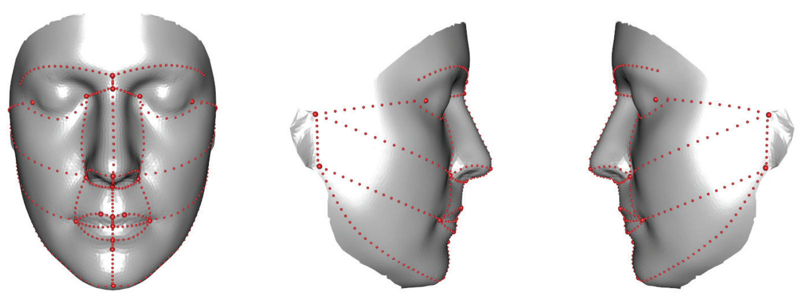
Symmetric Template

Symmetrically cut symmetric mesh with landmarks



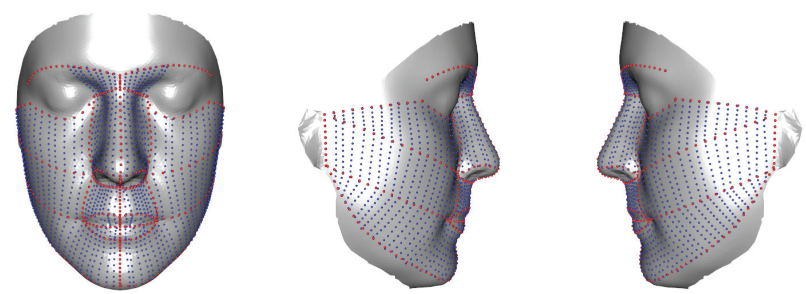
Symmetric Template

Symmetrically cut symmetric mesh with landmarks and curves

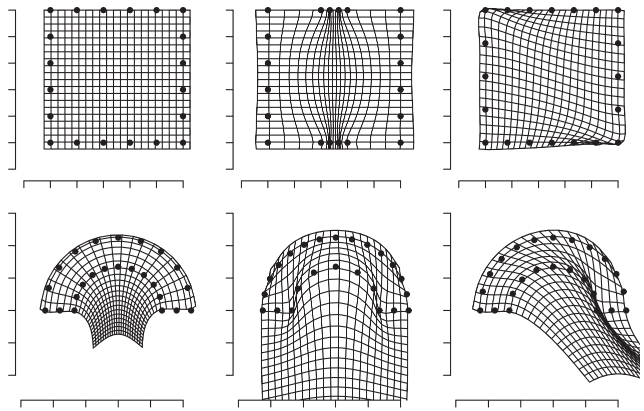


Semilandmarks on surface

Full set of anatomical curves and geodesics

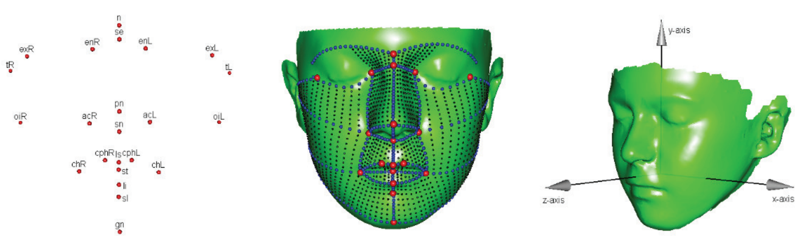


Semilandmarks on curves
Sliding – minimising bending energy



Semilandmarks on surface
Full set of anatomical curves and geodesics

Symmetric Template
Symmetrically cut symmetric mesh

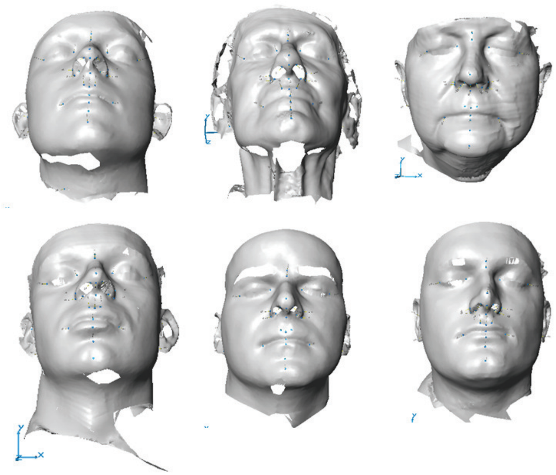


Hierarchical representation of a human face
Summary

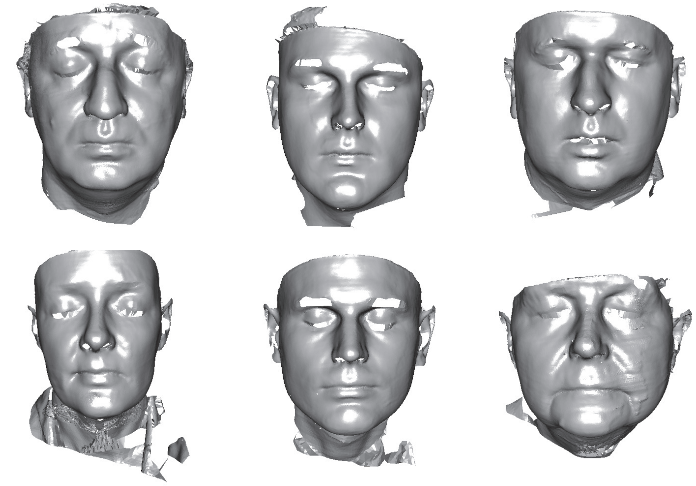
1. **landmarks**, **curves** (ridges, valleys, geodesics) [semilandmarks on curves], **surfaces** [semilandmarks on surfaces]

   the jaw line, the boundary between the lower and upper lip and surrounding skin, the philtrum valley, the nasal profile, the boundary between the nose and surrounding skin, the nasal ridge, the boundary between the lower eyelid and the surrounding skin, the brow ridges, and some geodesics on the nose and cheeks (*the areas without valleys or ridges*) between two carefully chosen anatomical landmarks
2. **automatically identified** by curvature in particular local surface patches, detection of slope discontinuities in local principal curves or optimised surface cuts
3. **a full standardised surface representation** is then available by interpolation across the relatively flat surface patches between identified curves
4. **a high resolution template** can be fitted to the semi-landmark surface by warping

3D laser-scan capture
3D facial shape–StJG data



3D laser-scan capture
3D facial shape–StJG data



3D laser-scan capture
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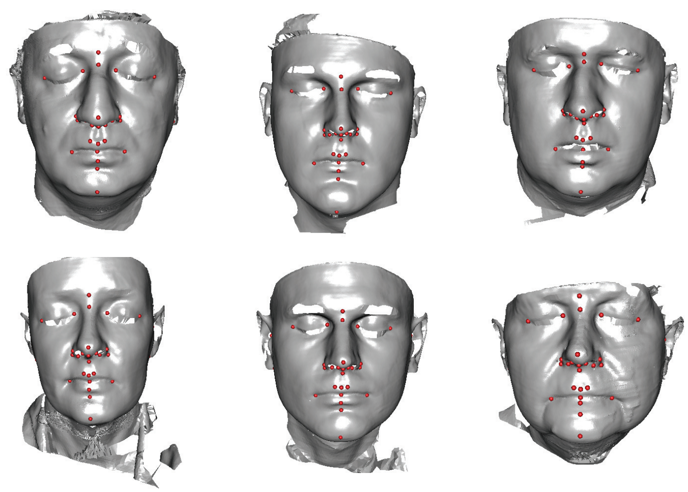

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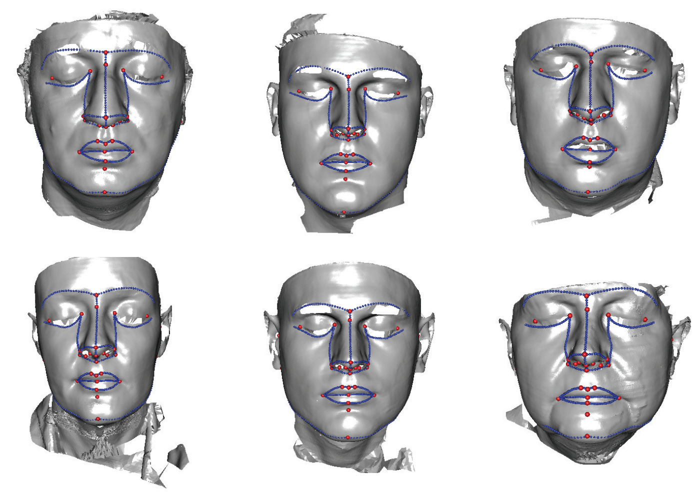

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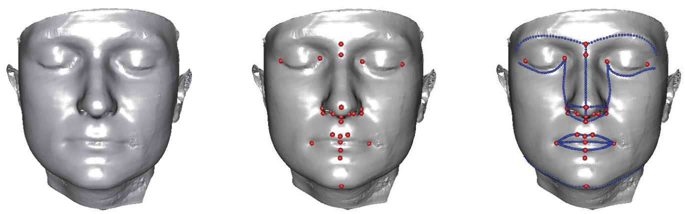
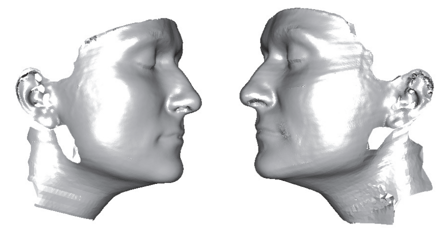
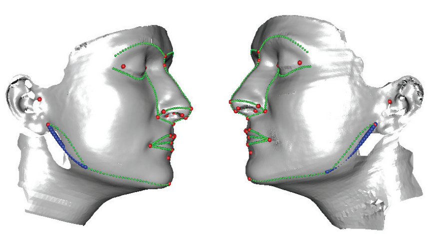


3D laser-scan capture
3D facial shape–StJG data



3D laser-scan capture
3D facial shape–StJG data



Geometric Morphometrics
Generalized Procrustes Analysis—Procrustes k-point registration

Definition (Generalized Procrustes Analysis, GPA)

Procrustes shape coordinates $\mathbf{x}_{P,ij} = c_i \mathbf{\Gamma}_i (\mathbf{x}_{ij} - \mathbf{t}_i)$, where c_i is *scale*, $\mathbf{\Gamma}_i$ is *rotation matrix* and $\mathbf{t}_i$ is *translation*, $\mathbf{x}_{P,ij}$ are rows of $\mathbf{X}_{P,i}$, $i = 1, ..., n$. Then we say that $\mathbf{X}_i, i = 1, 2, ..., n$ are in *optimal position* or have ***the best Procrustes fit*** in the sense of 'shape' if

$$\arg\inf \sum_{1\le i<j\le n} \| \mathbf{X}_{P,i} - \mathbf{X}_{P,j} \|^2 =$$

$$\underbrace{\arg\inf}_{\substack{\mathbf{\Gamma}_1,\dots\mathbf{\Gamma}_n \in SO(2)\\ \mathbf{t}_1,\dots\mathbf{t}_n\in\mathbb{R}^d, c_1, c_2,\dots c_n\in\mathbb{R}^+}} \left\{ \sum_{1\le i<j\le n} \left\| c_i\mathbf{\Gamma}_i \left(\mathbf{X}_i - \mathbf{1}_k \mathbf{t}_i^T\right)^T - c_j \mathbf{\Gamma}_j \left(\mathbf{X}_j - \mathbf{1}_k \mathbf{t}_j^T\right)^T \right\|^2 \right\}$$

Relative Warp Analysis

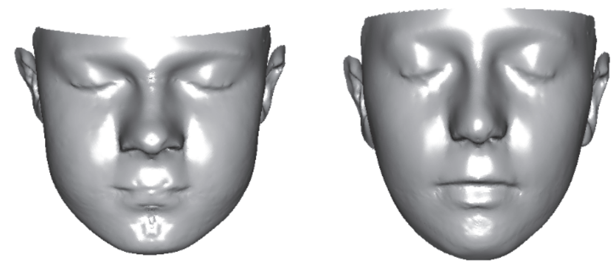
Generalized PCA—from shape space to affine and non-affine subspaces

Definition (Relative Warp Analysis (RWA))

1. ***Affine* contribution** to the variability by performing affine subspace *PCA* on the covariance matrix $\mathbf{S}_A$ of $n \times dk$ matrix $\mathbf{X}_A$ with the rows $Vec\,(\mathbf{X}_{A,i})$, $i = 1, 2, ...n$ (which is equivalent to the *RWA* with $\alpha = 0$)
2. ***Non-affine* contribution** to the variability by performing non-affine subspace *PCA* on the covariance matrix $\mathbf{S}_{NA}$ of $n \times dk$ matrix $\mathbf{X}_{NA}$ with the rows $Vec\,(\mathbf{X}_{NA,i})$, $i = 1, 2, ...n$
3. ***Contribution of (a)symmetry*** by augmenting relabeled and reflected Procrustes configurations to vectorized matrix of Procrustes shape coordinates and performing SVD of $\mathbf{S}_{AS}$
4. ***Size contribution*** by augmenting vectorized matrix of Procrustes shape coordinates by column of **centroid sizes** $\mathbf{x}_{size} = (\ln(CS_1), ..., \ln(CS_n))^T$, where $CS_i = \sqrt{(\sum_{j=1}^{k} \|\mathbf{x}_{ij} - \bar{\mathbf{x}}_i\|_2^2)} = \|\mathbf{X}_i\| = tr(\mathbf{X}_i\mathbf{X}_i^T)$, then $n \times (dk + 1)$ matrix of **vectorized form coordinates** $\mathbf{X}_F = (\mathbf{X}_S \vdots \mathbf{x}_{size})$, and finally performing SVD of $\mathbf{S}_F$

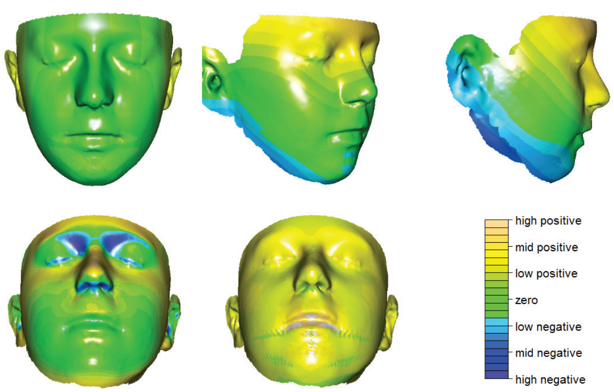
PCA for matched-pair shape data

Estimated extrems in PC2—control and case shape



PCA for matched-pair shape data

Different types of visualisation



R-library Face 3D

Documentation for package `Face3D (Version 0.1)`

Tools for the analysis of three-dimensional surface images

Documentation for package 'Face3D' version 0.1

- DESCRIPTION file.

Help Pages

Face3D-package	Tools for the analysis of three-dimensional surface images
asymmetry.face3d	Construct asymmetry scores for landmark configurations
closest.face3d	Find the closest point in a face3d object
connected.face3d	Identify the connected parts of a shape object
crossproduct	Construct the crossproduct of pairs of vectors
curves.face3d	Find a planar path between anatomicaly pre-specified set of landmarks pairs through the mesh of a face3d object
display.face3d	Display the shape in an rgl window
distances.face3d	Comparing two shapes
face	A face shape
Face3D	Tools for the analysis of three-dimensional surface images
gpa.face3d	Generalized Procrustes registration of (semi)landmarks
index.face3d	Construct shape indices for a face3d object
interpolate.face3d	Thin-plate spline interpolation on the manifold, i.e. from R3 to R1
orient.face3d	Orient a face into a frontal view
planepath.face3d	Find a planar path through the mesh of a face3d object
plot.face3d	Display the shape in an rgl window
read.face3d	Read obj, pts, dilm, tps, landmarkAscii, jpg, and tif files
resamplecurves.face3d	Resample the points of a curves to pre-specified curve length
resizejpg.face3d	Resize the jpg file associated with a camera capture
rotate.face3d	Rotate the matrix of landmarks or face3d object to pre-specified planes based on carefully chosen landmarks.
rp.shapeindex	An interactive demonstration of shape indices
subset.face3d	Create a subset of a face3d object
summary.face3d	Provide a simple summary of a face3d object
warp.face3d	Thin-plate spline interpolation from R3 to R3
write.face3d	Write ply, dmp, jpg, and tif files