‘SHAPE SPACES’ AND THEIR APPLICATIONS: A STROLL THROUGH INFINITE DIMENSIONS

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Abstract. On the infinite dimensional space Emb of all smooth embeddings (more general: immersions) from a compact manifold M into a Riemannian manifold $(N, \bar{g})$ acts the Lie group of all diffeomorphisms of M from the right, and various groups of diffeomorphisms of N from the left. Quotiening out the right action leads to a prime example of “shape space”, also called the “differentiable Chow variety” or the “nonlinear Grassmannian”, consisting of all submanifolds of N of type M. Invariant Riemannian metrics on Emb lead to Riemannian metrics on shape space, whose geodesic distances can be used to differentiate between shapes, and whose curvatures affect statistics of shapes.

Among the corresponding geodesic equations, in particular on $\mathrm{Diff}(S^1)$, one finds many famous PDEs: Burgers, KdV, Camassa-Holm, etc.

The left action, via right invariant Riemannian metrics on the diffeomorphism groups of N, also induces various metrics on shape spaces which have found many applications from paleontology to computational anatomy, among others under the name LDDMM.

In this overview talk I will illustrate many aspects of this circle of ideas; the results were started in collaboration with David Mumford and later with Martin Bauer, Martins Bruveris, and Philipp Harms. I will always use the space of differentiable immersed plane curves as the most basic example.