Lectures take place at the Institute of Mathematics and Statistics, No.8 building within the Faculty of Science, Kotlarska 2, Brno

Wednesday, November 4, 2015, at 4:00 p.m. in M1

David Kraus (Praha, Bern)
Functional data analysis: statistics for data that are not just a few numbers

Abstract:

Contemporary statistics is often faced with data sets consisting of data units that are complex objects, such as trajectories, curves, surfaces or images that can be seen as realizations of stochastic processes. Functional data analysis is a branch of statistics that deals with collections of such random variables in function spaces. I will present two areas of my recent research experience in this field. Both problems originate from applied settings and lead to the development of novel statistical methods involving ill-posed inverse problems and the asymptotic analysis of their regularized solutions. The first one is motivated by a problem from molecular biology, namely the study of the mechanical properties of DNA molecules. I will present procedures for comparing covariance operators using spectrally truncated approximations of the Hilbert--Schmidt distance of the empirical covariance operators. The second part comes from a public health study of heart activity patterns. A complete inferential framework is developed for a new type of data, partially observed functional data (fragments of temporal heart rate profiles in this example), with a focus on prediction of principal components and function completion via regularized conditional expectation.

Friday, November 6, 2015, at 2:00 p.m. in M1

Vítězslav Kala (Purdue, Göttingen)
Universal quadratic forms and continued fractions

Abstract:

Quadratic forms have long played a central role in number theory - for example, the

investigation of which primes are of the form (x^2+ny^2) led to the development of a number of crucial tools in algebraic number theory. In the talk, we will be mostly interested in universal forms, i.e., positive definite quadratic forms which represent all natural numbers - a classical example is the sum of four squares ($x^2 + y^2 + z^2 + w^2$). After giving an overview of the theory over the integers, I shall focus on the situation over number fields and discuss some recent results concerning the number of variables required by a universal form over a real quadratic field. In particular, for a given positive integer n, one can use continued fractions to construct in finitely many such fields that admit no (n)-ary universal forms.

Thursday, November 12, 2015, at 2:00 p.m. in M5

Vitaly Vougalter (Atlanta, Toronto)
Existence of stationary solutions for some integro-differential equations with anomalous diffusion

Abstract:

The work deals with the existence of solutions of an integro-differential equation arising in population dynamics in the case of anomalous diffusion involving the negative Laplace operator raised to a certain ractional power. The proof of existence of solutions is based on a fixed point technique. Solvability conditions for non-Fredholm elliptic operators in unbounded domains along with the Sobolev inequality for a fractional Laplacian are being used.

Wednesday, November 25, 2015, at 5:00 p.m. in M1

Ilya Kossovskiy (Moscow, Vienna)
Dynamical Approach in Cauchy-Riemann Geometry

Abstract:

The subject of Cauchy-Riemann Geometry (shortly: CR-geometry), founded in the research of Henri Poincare, is remarkable in that it lies on the border of several mathematical disciplines, among which we emphasize Complex Analysis and Geometry, Differential Geometry, and Partial Differential Equations. Very recently, in my research, I have discovered a new face of CR-geometry. This is a novel approach of interpreting objects arising in CR-geometry (called CR-manifolds) as certain Dynamical Systems, and vice versa. It turns out that geometric

properties of CR-manifolds are in one-to-one correspondence with that of the associated dynamical systems. In this way, we obtain a certain vocabulary between the two theories. The latter approach has enabled us recently to solve a number of long-standing problems in CR-geometry related to mappings of CR-manifolds with degeneracies of the CR-structure. We call this method the CR (Cauchy-Riemann manifolds) - DS (Dynamical Systems) technique. In this talk, I will outline the CR - DS technique, and describe its recent applications to Complex Geometry and Dynamics.

Friday, November 27, 2015, at 2:00 p.m. in M1

Marek Krčál (Praha, Vienna)
Computational and applied homotopy theory

Abstract:

In this talk we survey, first, the recent development in computational homotopy theory and, second, the consequent applications in computer science and computational geometry. The basic and important example of the former is the computational complexity of the *topological extension problem*
. Here we ask, given a continuous map (fcolon Ato Y) where (Y) is ((d-1))-connected, (d>1) (say, a (d) sphere), if there is a continuous extension of (f) to a superspace (X) of (A). The input (A, X, Y) and (f) is represented as simplicial complexes and a simplicial map. We proved that the problem is polynomial-time solvable for fixed (dim Xle 2d-1) and is undecidable otherwise. Among the applications of computational homotopy theory we will particularly cover the new algorithms for
nonlinear systems of equations with uncertainty
. Here we are given a system (f(x)=0) where the continuous map (fcolon Xto mathbf R^n) is known only up to an error (r>0) in the (ell_infty) norm. The question is what can be told about the set of solution (for instance, is it surely nonempty)? We will show that all the answers can be found by the means of homotopy theory, concretely, they are encoded in the homotopy class of the restriction (f|_Acolon Ato mathbf R^nsetminus{0}) where (A={xin Xcolon |f(x)|ge r}). (for instance, in its extendability to (X), respectively).

Wednesday, December 2, 2015, at 5:00 p.m. in M1

Phan Thanh Nam (Copenhagen, Vienna)
Mathematics of many-body quantum systems: an ongoing challenge

Abstract:

Quantum mechanics is a fundamental theory to investigate the structure of matter, from the small scale of atoms to the large scale of stars. In principle, the full quantum theory is linear, but it is very hard to compute explicitly when the number of particles becomes large. Therefore, physical properties of large systems are often addressed using mathematical tools from functional analysis, spectral theory and partial differential equations. I will discuss several mathematical problems in many-body quantum mechanics, from the structure of atoms in the periodic table to the Bose-Einstein condensation and superfluidity of cold gases.

Friday, December 4, 2015, at 2:00 p.m. in M1

Arman Taghavi-Chabert (Oxford, Brno)
Geometric aspects of light rays

Abstract:

According to Einstein's theory of general relativity, the model for our universe is a four-dimensional spacetime, whose points represent events. Distances between two events are measured by means of a metric, and there are non-zero vectors, said to be null, with zero length. In particular, light rays in our universe are geometrically described by null geodesics. How particles move through spacetime is dictated by Einstein's field equations, a system of non-linear partial differential equations that are notoriously difficult to solve in general. Remarkably, many interesting particular solutions, such as those modelling black holes, admit very special families of light rays: these rays do not shear. A deeper geometric property of such light rays is however revealed only once the spacetime is viewed as being complex rather than real. Their existence is also determined by how curved the spacetime is. These facts lead to drastic simplifications of Einstein's field equations, and would have profound consequences in many branches of mathematics thanks to Penrose's brainchild, Twistor Theory. Motivated by the emergence of new theories of physics that posit the existence of extra dimensions, I have worked on a generalisation of these notions of light rays, or rather their complex analogue, to spacetimes of dimension greater than four. In fact, such a generalisation turns out to be rather versatile in its range of geometric applications. I will touch upon some of the open questions connected to the geometric aspects of null geodesics.