Title: **The Scott adjunction**

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During his doctoral study, the author has mostly dealt with a categorical model theory. The first results were devoted to an understanding of weak Fraiisse limits (see [Kubis]) from the point of view of category theory and were published in cite{zbMATH07106179}. Later, the author studied codensity monads playing an important role in categorical universal algebra (see, e.g., [Leinster]) and prepared the paper [Lib19]. Both the papers should be considered as a part of this thesis.

Since then, the author has concentrated his efforts to the development of the Scott adjunction, relating accessible categories with directed colimits to topoi. This adjunction has already appeared in the literature in collaboration with Simon Henry [Hen19]. The resulting theory form the most important part of the thesis and will we submitted for a publication in a near future. After establishing basic properties of the Scott adjunction, we study both its applications to model theory and its geometric interpretation. From the geometric point of view, we introduce the categorfied Isbell duality, relating bounded (possibly large) ionads to topoi. The categorified Isbell duality interacts with the Scott adjunction offering a categorfication of the Scott topology over a poset (hence the name). We show that the categorfied Isbell duality is idempotent, similarly to its uncategorified version. From the logical point of view, we use this machinery to provide candidate (geometric) axiomatizations of accessible categories with directed colimits. In particular, we study the $2$-category of accessible categories with directed colimits using the tools and the mind-setting of formal category theory, this formal approach is part of a very general research-motive of the author and is prominently evident in cite{liberti2019codensity}. We discuss the connection between the above-mentioned adjunctions and the theory of classifying topoi. We relate our framework to the more classical theory of abstract elementary classes. We discuss the relation between atomic topoi and categoricity, providing a more conceptual understanding of our previous contributions to the topic [Di 19], and continuing the research line of [Ros97]. From a more categorical perspective, we show that the $2$-category of topoi is enriched over accessible categories with directed colimits and we relate this result to the Scott adjunction.