

Weighted Approximations and Their Applications

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Weighted approximations to uniform empirical and quantile processes by a Brownian bridge have proved to be extremely useful tools to establish the asymptotic distribution of complicated functionals of the empirical distribution. They are especially helpful in situations when classical weak convergence techniques fail or are difficult to apply. Since these approximations first appeared in 1986, they have been shown to hold for more general dependent processes such as certain continuous time martingales and exchangeable processes. A survey of some of the uses of weighted approximations will be given and then an improved version of the weighted approximation to the empirical and quantile processes will be discussed, along with a description of some its applications.

These lectures will be largely based on the following papers:

M. Csörgő, S. Csörgő, L. Horváth and D. M. Mason (1986). Weighted empirical and quantile processes. *Annals of Probability* **14**, 31–85.

U. Einmahl and D. M. Mason (1992). Approximations to permutation and exchangeable processes. *Journal of Theoretical Probability* **5**, 101–126.

E. Haeusler and D. M. Mason (1999). A weighted approximation to continuous time martingales with applications, *Scandinavian Journal of Statistics* **26**, 281-297 (1999).

E. Haeusler, D, M. Mason and T. Turova (2000). A study of serial ranks via random graphs, *Bernoulli* **6**, 541-570.

D. M. Mason (1991). A note on weighted approximations to the uniform empirical and quantile processes. In: *Sums, Trimmed Sums and Extremes*, (M.G. Hahn, D.M. Mason and D.C. Weiner, eds.), pp. 269–283. Birkhäuser, Boston, 1991.

D. M. Mason (2001). An exponential inequality for a weighted approximation to the uniform empirical process with applications. In: *State of the Art in Statistics and Probability Theory, Festschrift for Willem R. Van Zwet*. (M. de Gunst, C. Klassen, A. van der Vaart, eds.), I.M.S. Lect. Notes in Math. Statist. **36**, pp. 477-498, 2001.

D. M. Mason (2001). Notes on the KMT Brownian bridge approximation to the uniform empirical process. In: *Asymptotic Methods in Probability and Mathematical Statistics with Applications*, (N. Balakrishnan, I. A. Ibragimov and V.B. Nevzorov, eds.) Birkhäuser, Boston,. pp. 351-369, 2001.

D. M. Mason and W. R. van Zwet (1987). A refinement of the KMT–inequality for the uniform empirical process. *Annals of Probability* **15**, 871–884.