0Kpisemce

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16. 5. 2018

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```
%latex
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```

Příkazy k písemce

Základy

```
reset() #Celá nová
#Promenna
var('x,y')
```

(x, y)

```
r=[1..54, step=9];r #Definice seznamu
```

[1, 10, 19, 28, 37, 46]

```
type(r) #Napíše typ!
```

<type 'list'>

```
((3*x^2+5*x*y-2*y^2)*(x-y)^3)^2
expand(_).show()
#žPodrtítko odkazuje na řšpedelí výstup
#Expand roznásobuje závorky
#Show ukazuje hezký výstup
```

(3*x^2 + 5*x*y - 2*y^2)^2*(x - y)^6

$$9\,x^{10} - 24\,x^{9}y - 32\,x^{8}y^{2} + 172\,x^{7}y^{3} - 146\,x^{6}y^{4} - 188\,x^{5}y^{5} + 484\,x^{4}y^{6} - 428\,x^{3}y^{7} + 193\,x^{2}y^{8} - 44\,xy^{9} + 4\,y^{10}$$

```
factor(_).show() #Factor dává závorky zpátky do kupy
```

$$(3\,x - y)^{2}(x + 2\,y)^{2}(x - y)^{6}$$

```
((x^2-1)/(3*x^3-11*x^2+13*x-5)).partial_fraction().show() #.\
    partial_fraction ěrozdlí výraz na parciální zlomky
```

$$\frac{4}{3x-5} - \frac{1}{x-1}$$

```
%typeset_mode True #Zapíná hezké vvyýpisy i bez řpíkazu show()
factor(456465465464) #Rozkládá číslo na čsouin čprvoísel
```

$2^3 \cdot 37 \cdot 929 \cdot 1659971$

```
n((e^((1/3)*pi*sqrt(163))),digits=10) #Argument digits=x v řpíkazu n\
    () čuruje čpoet číslic
```

640320.0000

```
n((pi^(pi^pi)),digits=100);
```

1.340164183006357435297449129640131415099374974573499237787927516586034092619094068148269472
10^{18}

```
bool(1==1) #Funkce bool vrací hodnotu True nebo False
```

True

```
bool(2.divides(168)) # x.divides(y) žjestlie x ědlí y
```

True

```
2*I+8 #I je imaginátní jednotka
```

$2i + 8$

```
i=2; x=3; sum(x^i,'j',0,4); #sum() funguje jako suma. Druhý argument\
    je "indexová ěpromnná"
```

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```
var('i'); x=3; sum(x^i,'i',0,5); #Nyní je i ěpromnná a zářrove \
    indexová ěpromnná, to znamená, že se bude ěmnit mocnina u x od 0 \
    do 5 a 6-krát se to čsete díky ěsum
```

i
364

```
var('i,x'); sum(x^i,'i',0,5);
```

(i, x)
$x^5 + x^4 + x^3 + x^2 + x + 1$

```
%latex
\subsection*{Úpravy ůvýraz}
```

Úpravy výrazů

```
reset()
%typeset_mode True
```

```
#Tohle ůvbec nechápu!!!
```

```
p1=(x^4 + x^3 - 4*x^2 - 4*x)/(x^4 + x^3 - x^2 - x);p1
```

$$\frac{x^4 + x^3 - 4\,x^2 - 4\,x}{x^4 + x^3 - x^2 - x}$$

```
p1.numerator(normalize=True)/p1.denominator(normalize=True)
p1.numerator(normalize=False)/p1.denominator(normalize=False)
p1.numerator(normalize=True)/p1.denominator(normalize=False)
p1.numerator(normalize=False)/p1.denominator(normalize=True)
```

$$\frac{x^2 - 4}{x^2 - 1}$$
$$\frac{x^4 + x^3 - 4\,x^2 - 4\,x}{x^4 + x^3 - x^2 - x}$$
$$\frac{x^2 - 4}{x^4 + x^3 - x^2 - x}$$
$$\frac{x^4 + x^3 - 4\,x^2 - 4\,x}{x^2 - 1}$$

```
factor(p1.numerator(normalize=False))/p1.denominator(normalize=False\
    )
p1.numerator(normalize=False)/factor(p1.denominator(normalize=False)\
    )
factor(p1.numerator(normalize=False))/factor(p1.denominator(\
    normalize=False))
```

$$\frac{(x+2)(x+1)(x-2)x}{x^4 + x^3 - x^2 - x}$$
$$\frac{x^4 + x^3 - 4\,x^2 - 4\,x}{(x+1)^2(x-1)x}$$
$$\frac{(x+2)(x-2)}{(x+1)(x-1)}$$

```
factor(p1.numerator(normalize=True))/factor(p1.denominator(normalize\
   =True))
factor(p1.numerator(normalize=True))/factor(p1.denominator(normalize\
   =False))
factor(p1.numerator(normalize=False))/factor(p1.denominator(\
    normalize=True))
```

$$\frac{(x+2)(x-2)}{(x+1)(x-1)}$$
$$\frac{(x+2)(x-2)}{(x+1)^2(x-1)x}$$
$$\frac{(x+2)(x-2)x}{x-1}$$

```
p1.simplify_full() #Maximální šzjednoduení
```

$$\frac{x^2 - 4}{x^2 - 1}$$

```
p1.partial_fraction() #Parciální zlomky
```

$$\frac{3}{2\,(x+1)} - \frac{3}{2\,(x-1)} + 1$$

```
var('y,z')
f = (x+y)^2+1/(x+y);f
p = f.subs({(x+y): z});p #substituce x+y=z
```

$(y,\, z)$

$$(x+y)^2 + \frac{1}{x+y}$$

$$z^2 + \frac{1}{z}$$

```
s=4*x^4-x^2;s
b=s.subs({x^2:z});b
d=s.subs({x^2:z}).subs({x^4:z^2});d #Neumí to substituovat šmení \
    mocninu ve ěšvtí ěmocnin!
```

$4\,x^4 - x^2$

$4\,x^4 - z$

$4\,z^2 - z$

```
%latex
\subsection*{Rovnice}
```

Rovnice

```
reset();
%typeset_mode True
var('y,a');
```

$(y,\, a)$

```
r=x^3-(a-1)*x^2+a^2*x-a^3;r
s=solve(r,x);s #I complexní řšeení
```

$$-a^3 + a^2x - (a-1)x^2 + x^3$$

$$[x = -\frac{\left((a-1)^2 - 3\,a^2\right)\left(-i\,\sqrt{3}+1\right)}{18\left(\frac{1}{27}\,(a-1)^3 - \frac{1}{6}\,(a-1)a^2 + \frac{1}{2}\,a^3 + \frac{1}{6}\,\sqrt{\frac{1}{3}}\sqrt{(16\,a^3 + 8\,a^2 + 11\,a - 4)a}a\right)^{\frac{1}{3}}} - \frac{1}{2}\left(\frac{1}{27}\,(a-1)^3 - \frac{1}{6}\,(a$$

$$\frac{1}{3}\,a - \frac{1}{3},\ x = -\frac{\left((a-1)^2 - 3\,a^2\right)\left(i\,\sqrt{3}+1\right)}{18\left(\frac{1}{27}\,(a-1)^3 - \frac{1}{6}\,(a-1)a^2 + \frac{1}{2}\,a^3 + \frac{1}{6}\,\sqrt{\frac{1}{3}}\sqrt{(16\,a^3 + 8\,a^2 + 11\,a - 4)a}a\right)^{\frac{1}{3}}} -$$

$$\frac{1}{2}\left(\frac{1}{27}\,(a-1)^3 - \frac{1}{6}\,(a-1)a^2 + \frac{1}{2}\,a^3 + \frac{1}{6}\,\sqrt{\frac{1}{3}}\sqrt{(16\,a^3 + 8\,a^2 + 11\,a - 4)a}a\right)^{\frac{1}{3}}\left(-i\,\sqrt{3}+1\right) + \frac{1}{3}\,a -$$

$$\frac{1}{3},\ x = \frac{1}{3}\,a + \frac{(a-1)^2 - 3\,a^2}{9\left(\frac{1}{27}\,(a-1)^3 - \frac{1}{6}\,(a-1)a^2 + \frac{1}{2}\,a^3 + \frac{1}{6}\,\sqrt{\frac{1}{3}}\sqrt{(16\,a^3 + 8\,a^2 + 11\,a - 4)a}a\right)^{\frac{1}{3}}} + \left(\frac{1}{27}\,(a-1)^3 - \frac{1}{6}\,(a$$

$\frac{1}{3}]$

```
s=solve((x^2+y^2==5,x*y==y^2-2),(x,y));s
```

$[[x = -\frac{3}{2}\sqrt{2}, y = \frac{1}{2}\sqrt{2}], [x = \frac{3}{2}\sqrt{2}, y = -\frac{1}{2}\sqrt{2}], [x = (-1), y = (-2)], [x = 1, y = 2]]$

```
k=s[0];k #Hranatá závorka u seznamu vybere prvek toho seznamu v \
    hranatách závorkách
```

$[x = -\frac{3}{2}\sqrt{2}, y = \frac{1}{2}\sqrt{2}]$

```
l=k[0];l
```

$x = -\frac{3}{2}\sqrt{2}$

```
l.rhs() #rhs() - right hand side
```

$-\frac{3}{2}\sqrt{2}$

```
l.lhs()
```

x

```
s=solve(x<(x-3)^2,x);s #Nerovnice se řšeí analogicky jako rovnice.
```

$[[x < -\frac{1}{2}\sqrt{13} + \frac{7}{2}], [x > \frac{1}{2}\sqrt{13} + \frac{7}{2}]]$

```
%latex
\subsection*{Algebra}
```

Algebra

```
reset();
%typeset_mode True
```

```
p=primes_first_n(10);p; type(p) #Definuje seznam (list) a \
    primes_firts definuje prvních n čprvoísel.
```

[2, 3, 5, 7, 11, 13, 17, 19, 23, 29]
<type 'list'>

```
k=[2^i-1 for i in range (1,21)];k; type(k) #Definice pomocí řpíkazu \
    for
```

[1, 3, 7, 15, 31, 63, 127, 255, 511, 1023, 2047, 4095, 8191, 16383, 32767, 65535, 131071, 262143, 524287, 1048575]
<type 'list'>

```
U=set(p);U; type(U) #Defiunuje žmnoinu U
```

set([2, 3, 5, 7, 11, 13, 17, 19, 23, 29])
<type 'set'>

```
V=set(k);V #řVytvoení žmnoiny ze seznamu
```

set([131071, 1, 3, 1023, 2047, 7, 127, 255, 15, 32767, 65535, 16383, 262143, 4095, 511, 8191, 31, 524287, 1048575, 63])

```
V.intersection(U)       #ůPrniuk
V.union(U)              #Sjednocení
```

set([3, 7])
set([1, 2, 3, 17, 5, 7, 8191, 13, 15, 11, 65535, 16383, 262143, 524287, 29, 31, 32767, 4095, 2047, 63, 127, 1048575, 255, 19, 511, 23, 1023, 131071])

```
randint(55,155); #Vygeneruje random hodnotu od 55 do 155
H=[randint(-10,10) for i in range(1,101)];H; type(H)
```

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$[1, -4, -3, 9, 2, -6, 2, -8, -3, 3, -5, 6, -6, -4, 8, -4, 1, 6, 8, 0, -3, -9, 8, 1, 6, -4, -4, 10, -6, -8, -1, -5, -1, 7, -6, 3, -2, 9, -6, 3, -5, -9, -5, -3, -8, 2, 9, 0, 8, -3, 3, 10, -10, 4, 0, 3, -9, 5, -6, -3, 5, 9, -10, 6, -7, 4, 9, -9, 5, 7, -10, -1, -6, 1, -7, -10, -4, 9, 1, 4, 5, 0, -9, 8, 4, -9, -1, 4, 6, -9, 4, 9, 5, 2, 8, -4, -6, -2, -1, -9]$

<type 'list'>

```
uniq(H) #Vypíše ůrzné hodnoty seznamu.
```

$[-10, -9, -8, -7, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10]$

```
[j for j in uniq(H) if j>5] #Vypíše švechny j ze seznamu uniq(H) ěšvt\
   í žne 5
```

[6, 7, 8, 9, 10]

```
#Definice matice
A=Matrix([[1,0,2],[2,-1,3],[4,1,8]]);A
```

$$\begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ 4 & 1 & 8 \end{pmatrix}$$

```
B=A.inverse();B           #Inverzní matice
A*(A.transpose());        #Transponovaná matice
A.det();                  # determinant matice
A*B                       # Maticový čsouin
```

$$\begin{pmatrix} -11 & 2 & 2 \\ -4 & 0 & 1 \\ 6 & -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 8 & 20 \\ 8 & 14 & 31 \\ 20 & 31 & 81 \end{pmatrix}$$

1

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

```
%latex
```

```
\subsection*{Funkce a grafika}
```

Funkce a grafika

```
reset();
%typeset_mode True
```

```
f(y)=x^2;f #Funkce f: y=x^2
```

$y \mapsto x^2$

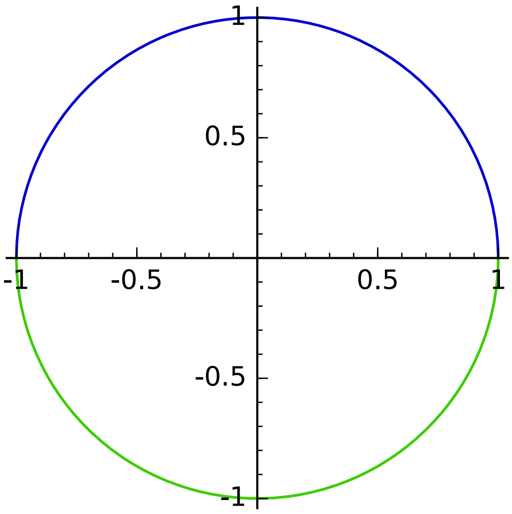
```
g=piecewise([[(-infinity,-1),0,],[[-1,1],1],[(1,infinity),0]],var=x)\
    ;g #Pomocí picewise lze definovat čěrun ůrzné hdonoty na ůrzném \
    intervalu.
```

piecewise(x|-->0 on (-oo, -1), x|-->1 on [-1, 1], x|-->0 on (1, +oo); x)

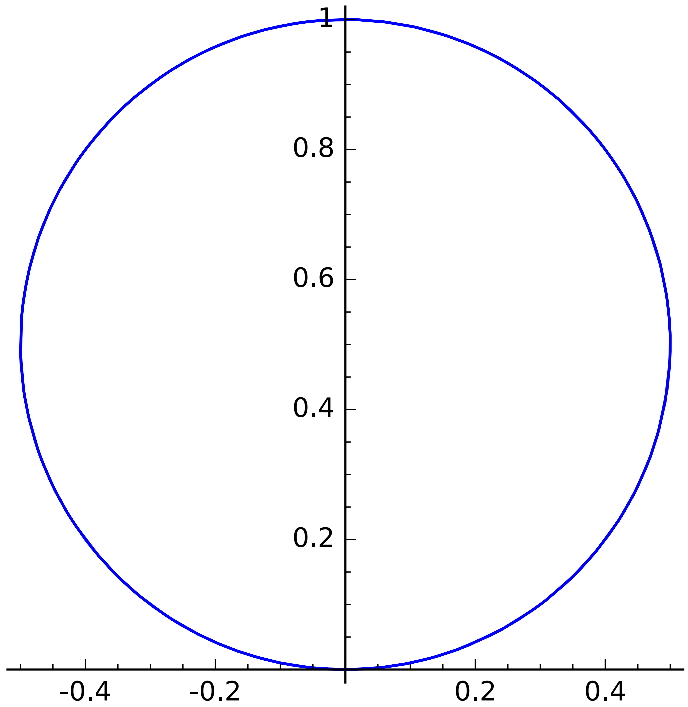
```
reset();
%typeset_mode True
var('x,y,t,u,v,a');
```

(x, y, t, u, v, a)

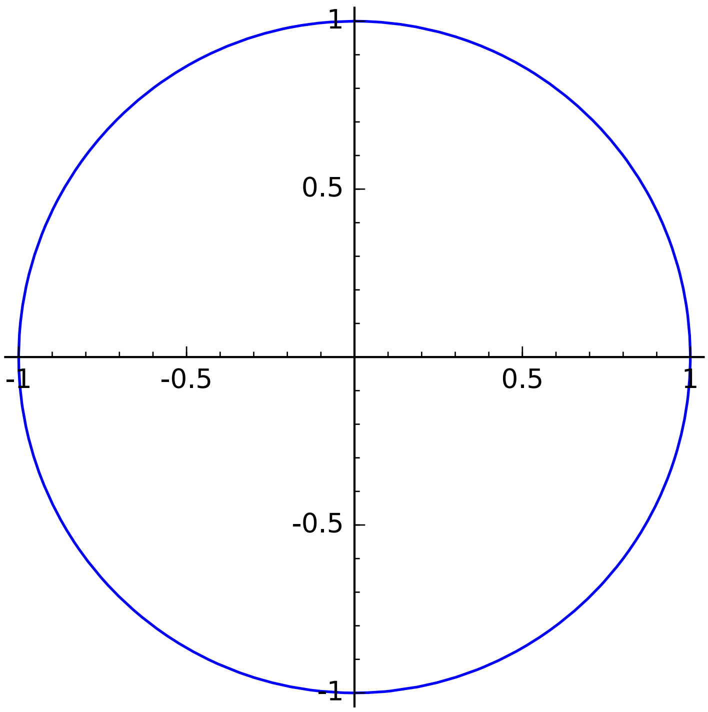
```
plot([(-x^2+1)^(1/2),-(-x^2+1)^(1/2)],(x,-1,1),figsize=4,\
    aspect_ratio=1); #V hranatých závorkách jsou ědv funkce, které \
    kreslí ědv ůplkrunice. Argument aspect_ratio=x je ěpomr os y/x.
```



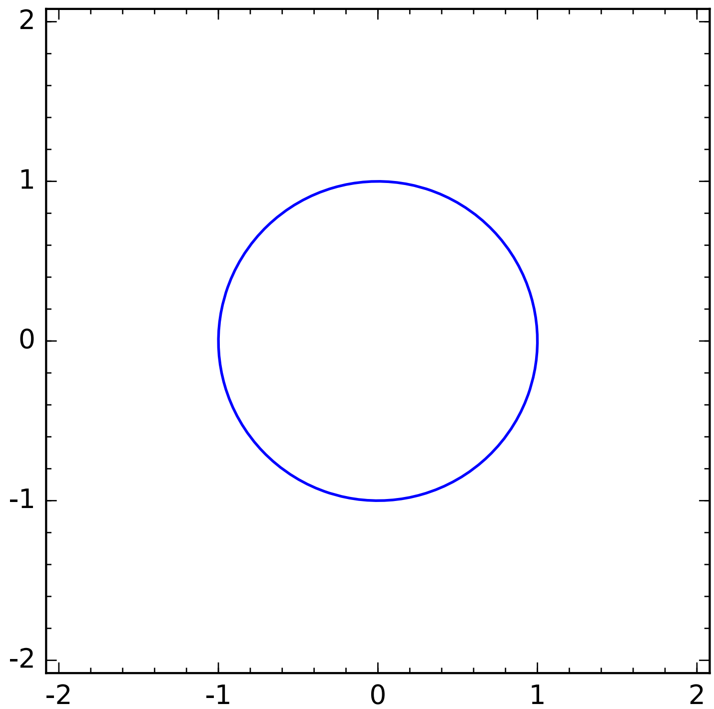

```
polar_plot(sin(t),(t,0,2*pi)); #Pomocí polárních řsouadnic vyklreslí\
     žkrunici! Try - cos(t)!
```



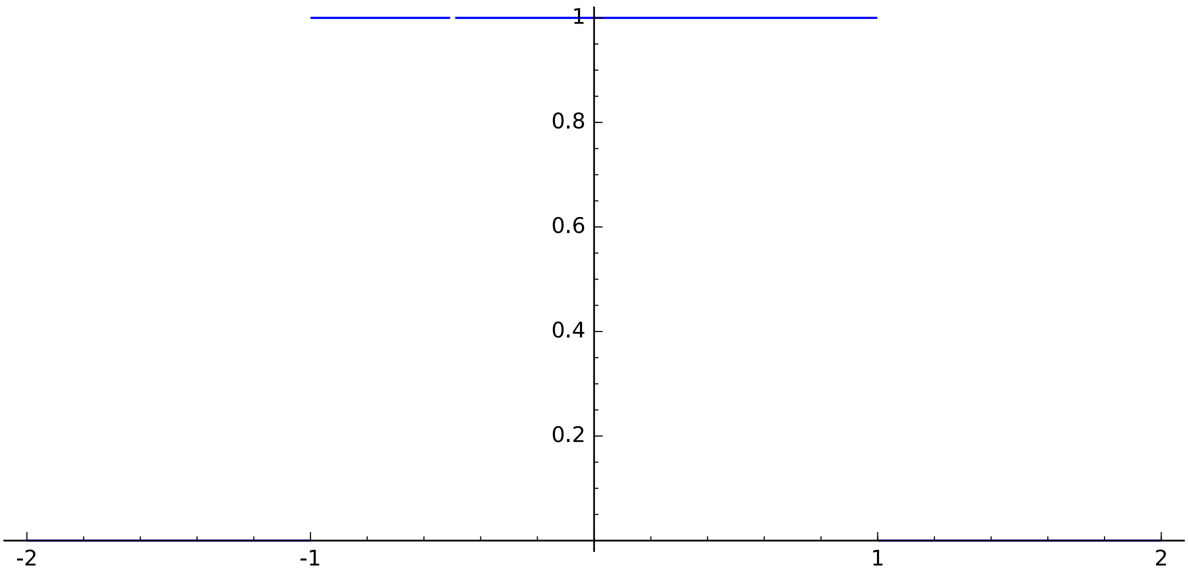

```
parametric_plot((sin(x),cos(x)), (x,0,2*pi)); #žKrunice pomocí \
    parametrických rovnic!
```




```
implicit_plot(x^2+y^2-1,(x,-2,2),(y,-2,2)); #žKrunice pomocí \
    implicitní funkce
```

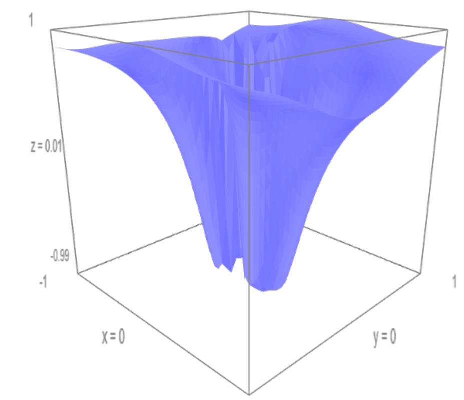



```
f=piecewise([[(-infinity,-1),0,],[[-1,1],1],[(1,infinity),0]],var=x)\
    ;
plot(f,(x,-2,2), exclude=[-1,1,-0.5]) #exclude čoznaí body \
    nespojistoti (Bod -0.5 je tam jen pro ilustraci!!!)
```

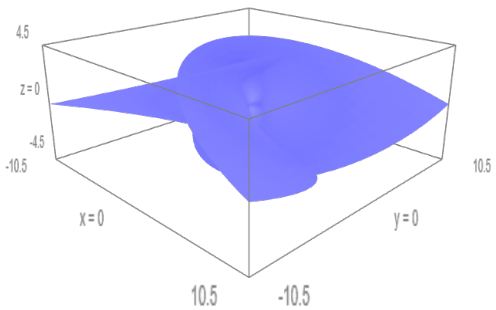



```
#k.save('graph8Sage.ps') žUloí graf k!
```

```
plot3d((cos(x/(x^2+y^2))),(x,-1,1),(y,-1,1)); #3D graf
```




```
parametric_plot3d((u/2-u^3/6+u*v^2/2, -v/2+v^3/6-v*u^2/2, u^2/2-v\
    ^2/2),(u,-3,3),(v,-3,3)); #Parametrická 3D grfika
```




```
reset();
var('y,x,a')
m=animate([plot((a*x^2), (x,-5,5)) for a in srange(-5,5,0.1)],xmin\
    =-4,xmax=4,ymin=-15,ymax=15);m #Animace
```

(y, x, a)

[/fb063b90-7e23-4c5e-9918-95f4c9474f8d/raw/.sage/temp/project-fb063b90-7e23-4c5e-9918-95f4c9474f8d/36

```
%latex
\subsection*{Analýza}
```

Analýza

```
reset();
var('x,y')
```

(x, y)

```
#Limity
limit(ln(x)/exp(x),x=0,dir='+') #Zprava
limit(ln(x)/exp(x),x=0,dir='-') #Zleva
limit(ln(x)/exp(x),x=Infinity)  #Do čnekonena
```

$-\infty$
∞
0

```
#Derivace podle x a y.
diff(x^3+y^3,x)
diff(x^3+y^3,y)
diff(x^3+y^3,x,x)
diff(x^3+y^3,y,y)
diff(x^3+y^3,x,y)
```

$3\,x^2$
$3\,y^2$
$6\,x$
$6\,y$
0

```
#Implicitní derivace
reset();
%typeset_mode True
var('x,c');
y = function('y')(x);y
dy=y.diff(x);dy
eq=sqrt(x)+sqrt(y)==1; eq
df=diff(eq,x);df
yx=solve(df,dy);yx
yx[0].rhs().simplify_full() #Řšeení první derivace posdle x
dff=diff(eq,x,2);dff
dyy=y.diff(x,2);dyy
yxx=solve(dff,dyy);yxx
yxx[0].rhs().subs(yx).simplify_full() #Řšeení druhé derivace podle x\
   !
```

$(x,\ c)$
$y(x)$
$\frac{\partial}{\partial x}y(x)$
$\sqrt{x} + \sqrt{y(x)} = 1$
$\frac{\frac{\partial}{\partial x}y(x)}{2\,\sqrt{y(x)}} + \frac{1}{2\,\sqrt{x}} = 0$
$[\frac{\partial}{\partial x}y(x) = -\frac{\sqrt{y(x)}}{\sqrt{x}}]$
$-\frac{\sqrt{y(x)}}{\sqrt{x}}$

$$-\frac{\frac{\partial}{\partial x}y(x)^2}{4\,y(x)^{\frac{3}{2}}}+\frac{\frac{\partial^2}{(\partial x)^2}y(x)}{2\sqrt{y(x)}}-\frac{1}{4\,x^{\frac{3}{2}}}=0$$

$$\frac{\partial^2}{(\partial x)^2}y(x)$$

$$[\frac{\partial^2}{(\partial x)^2}y(x)=\frac{x^{\frac{3}{2}}\frac{\partial}{\partial x}y(x)^2+y(x)^{\frac{3}{2}}}{2\,x^{\frac{3}{2}}y(x)}]$$

$$\frac{\sqrt{x}+\sqrt{y(x)}}{2\,x^{\frac{3}{2}}}$$

```
a=taylor(sin(x),x,0,2);a; b=taylor(sin(x),x,0,4);b; c=taylor(sin(x),\
   x,0,6);c #ůTaylorv rozvoj
```

$$x$$

$$-\frac{1}{6}x^3+x$$

$$\frac{1}{120}x^5-\frac{1}{6}x^3+x$$

```
#Integrování
a=integrate(sqrt(exp(x)-1),x);a
```

$$2\sqrt{e^x-1}-2\arctan\left(\sqrt{e^x-1}\right)$$

```
%latex
\subsection*{Programování a procedury}
```

Programování a procedury

```
reset();
%typeset_mode True
def proc(n):
    s=0;
    if n==1:
        print('Jsi čjednika!')
    elif n>1:
        for i in [1..n, step=2]:
            s=s+i
        print(s)
    elif n<-1:
        print('Nikdo nemá rád záporáky!')
```

```
proc(1); proc(-6); proc(12)
```

Jsi jednička!
Nikdo nemá rád záporáky!
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