Alciun of York's „Propositiones ad Acuendos Juvenes"

("Propositions for Sharpening Youths")

Introduction and Commentary By Peter J. Burkholder

Introduction

In the year 782, Alcuin of York (735-804) was summoned to the court of Charlemagne in Frankia. By this point, the Frankish king's domain covered much of modern France; Lombardy had been subjected; authority had been established on the Spanish March; and Bavaria was soon to be Christianized. With his sphere of influence thus extended, Charlemagne was able to turn his interests to the revitalization of education among his peoples. It was for this reason that Alcuin's presence was requested on the Continent.

Alcuin, also known by his Latin name of Albinus, was born in Northumbria in the year of the Venerable Bede's death.[1] He spent time studying in Italy and taught at the cathedral school of York before assuming his place at the court of Charlemagne in 782. Alcuin played an integral part in the so-called "Carolingian Renaissance," founding the palace school at Aix-la- Chapelle where the seven liberal arts were taught according to the educational system of Cassiodorus (ca. 490-580). His most important writings were his revisions of the Vulgate and his voluminous letters,[2] the latter being collated in the ninth century as a model of Latin composition. Alcuin eventually assumed the position of abbot at the abbey of St-Martin of Tours where he founded an important library and school, and where he remained until his death on May 19, 804.

During the course of his tenure, Alcuin is credited with having written a set of mathematical exercises entitled "Propositiones ad acuendos juvenes" or "Propositions for Sharpening Youths." These problems and their solutions, 53 in number, serve as valuable evidence of the state of mathematical education under the Carolingian kings. To the best of my knowledge, a complete translation of, and commentary on, the Propositions has never been undertaken, while scholarly treatment of them has been cursory at best. It is hoped that such an endeavor will shed new light on our knowledge of medieval mathematics and mathematical education. Before delving into the Propositions themselves, however, discussion of the problem of authorship is offered.

The Problem of Authorship

The composition of the Propositions can only be tentatively attributed to Alcuin. The most compelling reason to ascribe them as such is the title given at the head of the manuscript used for the Migne edition: "Propositiones Alcuini doctoris Caroli Magni imperatoris ad acuendos juvenes."[3] This particular manuscript is a codex from the monastery Augia Dives, known today as Richenau near Constance, Switzerland. The monastery was secularized in 1803, with the manuscripts being dispersed between Karlsruhe, London, Stuttgart, St. Paul in Carinthia, and Zurich.[4] The manuscript is described by the editor as being "very old," but this is by no means conclusive evidence of its origin.[5] J.A. Giles, who edited Bede's works in which a version of the Propositions appears,[6] judges that the style of the queries is sufficiently like that of Alcuin to imply that he was indeed the original author.[7] Conversely, the literary manner in which the Propositions are stated is very unlike anything produced by Bede, and thus cannot be considered his. Corroborating evidence that Alcuin may have been the author of the Propositions comes from a letter sent to Charlemagne in which Alcuin states, "I have sent to your Excellency...some simple arithmetical problems for reason of pleasure."[8] Such testimony is, of course, tenuous, for Alcuin's authorship of the Propositions is in no way assured simply because he sent a copy of them to his king.

There is other evidence, though inconclusive, which indicates that the Propositions may have been penned by Alcuin. In an interesting mathematical correspondence which took place around 1025, two monks of Cologne and Liege make reference to a work entitled _Albinus_ the Latinized form of Alcuin's name.[9] The context in which the work is used is a debate over the relation of a square's side to its diagonal. Although the Propositions specifically treat no such problems, there are instances of geometrical methods employed for questions of land measurement and circumference. Thus, we have an instance where Alcuin's Propositions may have been widely utilized in the early eleventh century, and commonly known as his work.

As stated, there is evidence suggesting that the Propositions may have been the work of the Venerable Bede (672-735). An almost word-for-word version of this treatise appears under the heading "Incipiunt aliae propositiones ad acuendos juvenes" in Bede's works.[10] If this were indeed the case, Alcuin obviously could not have been the original author. However, it is worth noting that Bede never makes any mention of the Propositions, even in his own listing of his works. Moreover, Giles cites a number of scientific writings attributed to Bede, including the Propositions, which must be considered unauthentic.[11] For these reasons, Bede's version of the Propositions appears in Migne under the heading "Dubious and Spurious Works."

Based on a manuscript at Leyden,[12] Smith argues that the probable compiler of the Propositions was a monk named Ademar or Aymar of the ancient house of Chabanais, who lived from 988 to 1030. The problems contained therein seem to be based on Aesop's Fables, begun by Aesop himself in Samos during the seventh century B.C., and modified by Babrius around the third century. What the connection is between Aesop's and Alcuin's works is not readily apparent, and Smith fails to elaborate on his point. However, based in part on the method of presentation, Thiele believes that Ademar did indeed author the Propositions.[13]

Except for Giles, scholars are reluctant to give Alcuin credit for production of the Propositions, mainly on the grounds that he contributed little or nothing of originality to learning, and because the vast majority of his writings were works on theology. Thus, Alcuin assumes the typical medieval scholastic role as transmitter of knowledge, not producer of new material. A comprehensive study of the various manuscripts would no doubt help determine the actual author of the Propositions.

The Problems Themselves

The fifty-three problems which make up the Propositions follow a basic general pattern: a brief heading, a statement of the problem, a request for an answer to the problem, and a solution. There can be little doubt that the problems were read aloud,[14] possibly with the students copying them down on papyrus, tree bark or parchment.[15] A call for a response was then elicited of the form, "Let him say, he who is able..." Some of the problems such as those pertaining to logic exercises could have been deciphered with no recourse to writing; others involving drawn out arithmetic computations could have taken quite some effort to compute, particularly when working with clumsy Roman numerals.

There is no strict categorical framework for the problems, although clusters of certain types appear intermittently. Only two problems (1 & 26) pertain to rates and distances, the first being a very odd hypothetical situation involving a snail's arduous and drawn out trek to a luncheon; the second and more advanced problem involves a dog's pursuit of a hare,[16] and actually involves two rates over differing distances. It is as follows:

There is a field which is 150 feet long. At one end stood a dog, at the other, a hare. The dog advanced behind the hare, namely, to chase the hare. But whereas the dog went nine feet per stride, the hare went [only] seven. Let him say, he who wishes, How many feet and how many leaps did the dog take in pursuing the fleeing hare until it was caught?

Alcuin's solution is ingenious, though cryptic. Whereas we might solve such a problem by two equations and two unknowns, Alcuin notes that the differing rates of the animals is the key to the entire problem:

The length of the field was 150 feet. Taking half of 150 makes 75. The dog was covering nine feet per stride, and nine times 75 makes 675. The dog thus ran this many feet in chasing the rabbit until it caught the rabbit with its tenacious teeth. And indeed, because the rabbit went seven feet per stride, take 75 seven times. This is how many feet the fleeing rabbit travelled before being caught.

The reason for dividing the field in half may not be so clear, but it simply corresponds to partitioning the field by the difference of the animals' feet per stride, in this case, two. Alcuin then takes the measurement obtained by thus dividing and multiplies it by the respective rates of dog and hare to arrive at the correct answer.

This method can be generalized as follows. The dog must always cover the space between it and the hare ($d1$) plus the additional distance covered by the hare ($d2$). If the dog's rate is $r1$, then the equation describing the distance traversed by the dog is given by $d1+d2=r1t$. (1) In a similar fashion, the hare's flight is denoted by $d2=r2t$. (2) Substituting the value of $d2$ in (2) into (1) yields $d1+r2t=r1t$. Thus $d1=t(r1-r2)$. (3) From (1), we know that $t=(d1+d2)/r1$, and putting this value of t into (3) results in $d1=(d1+d2)(r1-r2)/r1$. Rearranging this equations yields $d1+d2=r1d1/(r1-r2)$. This is exactly what Alcuin's method does. It says that the total distance covered by, for instance, the dog is simply the intervening expanse divided by the difference of the two animals' rates, times the dog's rate. It is easy to see the advantages that such a method offers in an oral instruction setting.

A much larger corpus of problems (e.g. 2, 3, 4) might best be described as those of an unknown quantity. In each exercise, the reader is told that a certain quantity of people, animals or objects, if doubled, tripled, or in some other way arithmetically manipulated, adds up to 100. A typical example is problem 36:

A certain old man greeted a boy, saying to him: "May you live, boy, may you live for as long as you have [already] lived, and then another equal amount of time, and then three times as much. And may God grant you one of my years, and you shall live to be 100." Let him solve, he who can, How many years old was the boy at that time?

The answer is a bit trickier than it might appear at first glance, for it must be remembered that it is difficult to use base-10 arithmetic in solving a problem dealing with a 12-month year:

When [the old man] said "may you live for as long as you have lived," [the boy] had [already] lived eight years, three months. Another equal number of years would make 16 years, six months, while another equal span makes 33 years. Three times this makes 99 years, which with one more year added makes 100.

It would have been a rather simple affair for Alcuin to have invented such an exercise by starting with 100 and working backwards, and then extrapolating the procedure to other problems of the same genre. A slightly more complicated query of this type can be found in problem 40, where portions of the original quantity are doubled, halved, and then added:

A certain man saw from a mountain some sheep grazing and said, "O that I could have so many, and then just as many more, and then half of half of this [added], and then another half of this half. Then I, as the 100^{th} [member], might head back to my home together." Let him solve, he who can, How many sheep did the man see grazing?

Again, such a scenario could have easily been derived by beginning with 100, and then arithmetically manipulating it until the desired problem was in order:

36 sheep were first seen by the man when he said, "O that I could have so many." Adding an equal number makes 72, and a half of half of this, that is, of 36, makes 18. And again, a half of this, that is, of 18, makes nine. Therefore add 36 and 36, making 72. Add to this 18, which makes 90. Then add nine to 90, making 99. The man himself added to these will be the 100th one.

The only precaution which would be necessary would be to make sure that fractions do not occur, and this could be easily checked.

A third type of problem which Alcuin presents to the student is that of dividing quantities amongst various parties. This sometimes involves the division of an inheritance between sons, as in problem 12:

A certain father died and left as an inheritance to his three sons 30 glass flasks, of which 10 were full of oil; another 10 were half full, while another 10 were empty. Divide, he who can, the oil and flasks so that an equal share of the commoditites should equally come down to the three sons, both of oil and glass.

There is little doubt that anyone, whether trained in mathematics or not, could solve such a problem. One need only pour all of the oil into a central vat and divide the liquid and glass equally from there. However, as an exercise, Alcuin demonstrates how such a division might be accomplished without recourse to such crude means:

There are three sons and 30 glass flasks. However, of the flasks, 10 are full [of oil], 10 half full, and 10 empty. Take three times 10, which makes 30, so each son shall receive 10 flasks as his portion. Divide up the three portions, that is, give to the first son 10 half [filled] flasks, to the second son five full and five empty [flasks]. Do the same for the third son, and the brothers' portions of glass and oil shall be the same.

Questions pertaining to division of an estate are traceable back to Roman law and what is known as the Testament Problem.[17] Roman precepts made definite provisions for the division of property upon a father's death, and thus we find problems like number 35. Here, a father leaves behind a pregnant wife, with instructions for division of his inheritance in the case of either a boy or girl being born. To complicate matters, opposite sex twins are produced. A long-winded solution of how the father's possessions are to be divided follows.

These types of problems seem to stress logic more than arithmetic skills. The exercises involving distribution of corn by a head of household (paterfamilias) to his servants are slightly more complicated, as differing amounts of corn are allowed for men, women and children:

A certain head of household had 30 servants whom he ordered to be given 30 modia of corn as follows: The men should receive three modia; the women, two; and the children, a half [modium]. Let him solve, he who can, How many men, women and children were there?

As in the problems dealing with an unknown quantity, Alcuin had to be sure that his numbers worked out evenly in the end. Note, too, that he treats fractional measurements here, as each child receives half a modium of corn:

If you take thrice three, you get nine; if you take two five times, you get 10; and if you take half of 22, you get 11. Thus, three men received nine modia; five women received 10; and 22 children received 11 modia. Adding three and five and 22 makes 30 servants. Likewise, nine and 11 and 10 makes 30 modia. Hence there are 30 servants, and 30 modia [of corn].

Problems of exactly the same type, but with varying numbers of servants and corn, can be found in exercises 32 and 34, indicating that it was the procedure which Alcuin wished his pupils to understand.

Alcuin's logic problems, or slight variations of them, can still be found today in textbooks and on examinations. The most famous no doubt is the conundrum of the man, she-goat, wolf and cabbage which needed to be ferried across a river. (Problem 18) As only two passengers fit in the boat at once, and since certain combinations of animals and vegetable cannot be left alone, the reader is left to solve how a successful transport might take place. Alcuin assumes the role of ferryman and leads us through the problem step by step:

...I would first take the she-goat and leave behind the wolf and the cabbage. When I had returned, I would ferry over the wolf. With the wolf unloaded, I would retrieve the she-goat and take it back across. Then, I would unload the she-goat and take the cabbage to the other side. I would next row back and take the she-goat across. The crossing should go well by doing thus, and absent from threat of slaughter.

Problems of exactly this type appear in exercises 17, 19 and 20 as well. In each case, a long explanation of how a successful transnavigation might be performed is offered.

Other logic problems are more straightforward and exhibit a certain amount of humor. The answer to Alcuin's problem of how many footprints an ox makes in the last furrow is, of course, none, "because the ox goes in front of the plow and the plow follows it. For however many footprints the ox makes on the ploughed earth by going first, so many the plough following behind destroys by ploughing." (Problem 14) Another question of this type entails a man who wishes to slaughter 300 pigs in three days, but with a odd number being butchered per day -- a problem which Alcuin states "is indissoluble and composed for rebuking." (Problem 43) Alcuin's problems pertaining to area are of particular interest. They consist of queries as to how many measurements or objects can fit inside of a larger confine. Certain exercises we might define as dealing with acreage, although such a term is not entirely accurate for measurements of aripenna, the standard land quantity.[18] Other problems are of no immediate practical value whatsoever, and are thus clearly meant as purely mathematical exercises. Take for example the question of how many rectangular houses can fit within a circular city (problem 29):

There is a city which is 8000 feet in circumference. Let him say, he who is able, How many houses should the city contain, such that each [house] is 30 feet long, and 20 feet wide?

Solution:

The city measures 8000 feet around, which is divided into proportions of one-and-a-half to one, i.e. 4800 and 3200. The length and width of the houses are [also] of these [dimensions]. Thus, take half of each of the above [measurements], and from the larger number there shall remain 2400, while from the smaller, 1600. Then, divide 1600 into twenty [parts] and you will obtain 80 times 20. In a similar fashion, [divide] the larger number, i.e. 2400, into 30 pieces, deriving 80 times 30. Take 80 times 80, making 6400. This many houses can be built in the city, following the above-written proposal.

Essentially, what Alcuin does is to force the ratio of the length to width of each house onto the city. Thus, as each house measures 30x20, the ratio of length to width is 3:2. Alcuin breaks up the circumference of the town into two pieces such that their ratio is 3:2 as well. Having done this, he simply straightens out the pieces and sets them perpendicular to one another. This, however, yields an unclosed figure. He thus divides each side in two and rearranges the four sides in order to make a closed structure. The ratio of 3:2 is preserved since 4800/2:3200/2 equals 3:2. Now Alcuin has a rectangular town with a circumference of 8000 feet, and whose dimensions are proportional to the dimensions of the houses. From there, the problem of the number of houses which can fit in the town is trivial.

The most obvious shortcoming of Alcuin's method is that the area enclosed by different curves of equal length is not the same. The area of a circle is given by pi-r-squared, whereas the area for a rectangle is denoted by length times width. Thus, the area enclosed by a circle with a circumference of 8000 feet is roughly 5,092,958 square feet, whereas a rectangle measuring 1600 by 2400 encloses only 3,840,000 square feet – a difference of over 1.2 million square feet. The only conclusion which can be drawn is that Alcuin was unaware of the consequences of modifying shape, as he employs the same methodology in problems 27 and 28. In addition, we need only note the absence of allowance for streets to realize the purely hypothetical nature of such a problem.

There is further evidence that Alcuin's Propositions sought merely to stir the minds of their readers as opposed to serving as a handbook for quotidian problems. Glaring examples of this are exercises 13 and 41, both of which teach the lesson of geometric growth.[19] In the former, a servant is ordered by his king to assemble an army from 30 villages as follows:

He should bring back as many men [from each successive village] as he had taken there. Thus, [the servant] came to the first village alone; he came with one other person to the next; three people came to the third, etc...

Such a gathering can be mathematically modelled by the relation N=2^(v), where v is each successive village and N is the number of people assembled. Hence, the total number of villagers conscripted would be given as S 2^(v), with the summation beginning at v=0 and continuing to v=30 -- a figure representing an army which was far beyond the capabilities of even the richest or most ambitious of kings to field. Alcuin's solution gives figures up to v=15, while the edition ascribed to Bede continues all the way to v=30, although with errors beginning at v=22. Neither attempts to sum the figures, nor is it expected that a student would be expected to; rather, it was the process which undoubtedly lay at the heart of the problem.

A similar hypothetical problem demonstrates the idea of arithmetic progression. It has been related that when Gauss (1777-1855) was a young student, his mathematics teacher one day instructed the class to add the numbers one through 100. No sooner had the assignment been made than Gauss somehow magically produced the correct figure of 5050. How had he done it? The key to the problem is to realize that by adding corresponding low and high figures, a simple multiplication problem unfolds. Thus, 1+100=101; 2+99=101; 3+98=101;...;49+52=101; 50+51=101. It is manifest from this that one need only multiply the constant sum, 101, by 50, the number of sums. In this way, the correct response of 5050 is obtained.

Alcuin's ladder problem (42) shows that this concept was already known by the ninth century:

There is a ladder which has 100 steps. One dove sat on the first step, two doves on the second, three on the third, four on the fourth, five on the fifth, and so on up to the hundredth step. Let him say, he who can, How many doves were there in all?

Solution:

There will be as many as follows: Take the dove sitting on the first step and add it to the 99 doves sitting on the 99th step, thus getting 100. Do the same with the second and 98th steps and you shall likewise get 100. By combining all the steps in this order, that is, one of the higher steps with one of the lower, you shall always get 100. The 50th step, however, is alone and without a match; likewise, the 100th stair is alone. Add them all and you will have 5050 doves.

We can see that with only slight modification, the above-described concept was in place almost a thousand years before Gauss dazzled his schoolteacher. Perhaps the young Gauss wasn't so clever after all!

Conclusion and Topics for Further Study

Whether or not Alcuin himself authored the Propositions may never be known, but this is not of great consequence. The Propositions are interesting problems in their own right and reveal the general state and method of mathematical instruction around the time of Charlemagne. A thread of continuity with classical education can be discerned in these puzzles as well as the influence of Barbarian values of practical methods for everyday problems. However, it must be concluded that the Propositions sought only to instill various simple methods in its users, this being accomplished by repeated problems of the same genre.

It should not be concluded that the Propositions are indicative of the general state of mathematics during the eighth or ninth centuries. We have prima facie evidence[20] that these problems were utilized primarily for didactic purposes; thus, to argue that the Propositions are an example of the poor state of mathematics is erroneous. While such a conclusion may be justified, it by no means is a necessary deduction from the evidence at hand.

The Propositions are also potentially valuable for the economic and social insight they offer, and a spreadsheet of various weights and measures which appear throughout the problems is included as an appendix. Whether or not these values are consistent with contemporary conditions awaits another study.

An Introduction to the Translations

In translating the 53 problems and answers of the Propositions, I have utilized the Migne edition of Alcuin's works. I have annotated this text and supplied alternate or additional versions of problems as they appear in Bede's supposed previous work. Where differences occur, a footnote is provided beginning with "Bede," hence referring the reader to Bede's edition. A further comparison with Heruagius's edition of Bede's writings revealed only trivial discrepancies, and thus alternate readings from this work have been omitted.

A quality translation must be true both to the original language and the language into which the material is converted. With this is mind, I have tried to keep verb tenses consistent according to English usage despite Alcuin's variations within a given problem. English words which have been read into the Latin are contained within square brackets [ ] and are either interpretive or corroborated by Bede's edition. Certain words referring to weights and measures (e.g. aripennum, denarius, solidus) have been left in the original. Though aripennum might be rendered "arpent" and solidus a "sous," such translations either do little in helping us grasp what is involved in the usage, or are modernly deceitful.

References

[1] Since the Propositions to be discussed cannot be ascribed to Alcuin with certainty, I will offer only a very brief biographical account of the man. Secondary literature on Alcuin is plentiful. See for example Stephen Allott, _Alcuin of York_, (York, 1974); L. Wallach, _Alcuin and Charlemagne_, (New York, 1959); Eleanor Duckett, _Alcuin, A Friend of Charlemagne_, (New York, 1951); C.J.B. Gaskoin, _Alcuin: His Life and Works_, (Cambridge, 1904); Andrew West, _Alcuin and the Rise of the Christian Schools_, (London, 1893); and Frederick Lorenz, _The Life of Alcuin_, Jane Slee, trans., (London, 1837). For a listing of Alcuin's texts and translations, see George Sarton, _Introduction to the History of Science_, 3 vols., (Baltimore, 1927), vol. 1, part 1, pp. 528-529.

[2] See Rolph Page, _The Letters of Alcuin_, (New York, 1909).

[3] _Alcuini opera omnia_, J.P. Migne, ed., vol. 2, found in _Patrologiae latinae cursus completus..._, vol. 101, (Paris, 1863).

[4] Frederick Hall, _A Companion to Classical Texts_, (Oxford, 1913), pp. 294 & 342.

[5] D.E. Smith tells us that the oldest manuscript of the problems dates from the eleventh century. _History of Mathematics_, 2 vols., (New York, 1923; reprint, 1951), vol. 1, p. 186.

[6] One such manuscript ascribing the Propositions to Bede is _Codex Latinus Monacensis_, no. 14272. Its origin is either tenth or eleventh century. See _Catalogus codicum Latinorum bibliothecae regiae monacensis_, (Hildesheim, 1975), vol. 2,2, pp. 152-153. (This is the only ascription to Bede noted by Lynn Thorndike, _A Catalog of Incipits of Medieval Scientific Writings_, (Cambridge, MA, 1963).)

[7] Giles, ed., _The Miscellaneous Works of Venerable Bede, in the Original Latin_, 6 vols., (London, 1843), vol. 6, p. xiv.

[8] "Misi excellentiae vestrae...aliquas figuras arithmeticae subtilitatis, laetitiae causa." Migne, op. cit., vol. 100, letter 101, col. 314, dated anno 800.

[9] An edition of the correspondence, along with scholarly commentary, can be found in Paul Tannery's _Memoires scientifiques_, vol. 5 of _Sciences exactes au moyen-age-, (Paris, 1922), pp. 264-288. An earlier partial edition is contained in Jules Clerval's _Les Ecoles de Chartres au moyen- age, du ve au xvie siecle_, (Paris, 1895; reprint, Geneva, 1977), pp. 459- 464. I have studied these letters anew and hope to make my findings available in the near future in a paper entitled "Speculum geometricae undecimo saeculo: The Mathematical Correspondence of Ragimbold of Cologne and Radulf of Liege, ca. 1025."

[10] Migne, op. cit., vol. 90, cols. 667-676. These also appear in volume one of Joannes Hervagius's edition of Bede's _Opera Bedae Venerabilis..._, 8 vols. bound in 4, (Basil, 1563), but

are not included in Giles's edition. The most notable difference between Bede's version and that of Alcuin is the lack of solutions for problems 36-53 in the former.

[11] Giles, op. cit., vol. 6, pp. ix-xv.

[12] Georg Thiele, ed., _Der Illustrierte lateinische Aesop in der Handschrift des Ademar_, Codex Vossianus Lat. Oct. 15, Fol. 195-205, (Leiden, 1905).

[13] Ibid., pp. 23-25.

[14] Vera Sanford specifically places Alcuin's Propositions under the rubric "Verbal Problems." _A Short History of Mathematics_, (Boston, 1930), pp. 212-213.

[15] For the materials available to schoolchildren, see Pierre Riche, _Education and Culture in the Barbarian West, Sixth through Eighth Centuries_, trans. from the third edition by John Contreni, (Columbia, SC, 1976), pp. 458-462.

[16] Smith describes this problem as being "already ancient" by Alcuin's time, but fails to cite any precedents. Op. cit., 187. Sanford dates pursuit problems to Roman legionaries, whose stride was so uniform that time schedules could be worked out for marching from place to place. Op. cit., pp. 217-218.

[17] See Sanford, op. cit., pp. 218-219.

[18] The use of aripenna and the smaller perticae, of course, implies that such measurements were standard and well-known to all. From problem 25, we can deduce that one aripennum equals 184.53 perticae.

[19] Sanford regards problems of geometric progression as some of the oldest types of mathematical endeavors, and cites extant Babylonian tablets from ca. 2000 b.c. to this effect. Op. cit., pp. 174-176.

[20] See problem 43.

Propositiones Alcuini Doctoris Caroli Magni Imperatoris ad Acuendes Juvenes [1]

Propositions of Alciun, A Teacher of Emperor Charlemagne, for Sharpening Youths

Translation

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I. propositio de limace.
Limax fuit ab hierundine invitatus ad prandium infra leucam unam. In die autem non potuit plus quam unam unciam pedis ambulare. Dicat, qui velit, in quot diebus [2] ad idem prandium ipse limax perambulabat?

1. proposition concerning the snail.
A snail was invited by a swallow to lunch a league away. However, it could

not walk further than one inch per day. Let him say, he who wishes, How many [years and] days did it take for the snail to walk to that lunch?

Sequitur solutio de limace.
In leuca una sunt mille quingenti passus; vii d pedes xc unciae. Quot unciae, tot dies fuerunt, qui faciunt annos ccxlvi, et dies ccx.
Here follows the solution of the snail.
In one league, there are 1500 passus [3]. 7500 feet [equals] 90,000 inches. There are as many days as there are inches, that is, 246 years, 210 days.

II. propositio de viro ambulante in via. [4]
Quidam vir ambulans per viam vidit sibi alios homines obviantes, et dixit eis: Volebam [5], ut fuissetis alii tantum, quanti estis; et medietas medietatis; et hujus numeri medietas [et rursum de medietate medietas]; tunc una mecum c fuissetis. Dicat, qui velit, quanti fuerunt, qui in prima ab illo visi sunt?
2. proposition of the man walking in the street.
A certain man walking in the street saw other men coming towards him, and he said to them: "O that there were so many [more] of you as you are [now]; and then half of half of this [were added]; and then half of this number [were added], and again, a half of [this] half. Then, along with me, you would number 100 [men]." Let him say, he who wishes, How many men were first seen by the man?

Solutio de eadem propositione.
Qui imprimis ab illo visi sunt, fuerunt xxxvi. Alii tantum lxxii. Medietas medietatis xviii. Et hujus numeri medietas sunt viiii. Dic ergo sic: lxxii et xviii fiunt xc. Adde viiii, fiunt xcviiii. Adde loquentem, et habebis c.[6]
Solution of the same proposition.
Those who were first seen by the man were 36 in number; double this would be 72. A half of half of this is 18, and a half of this number makes 9. Therefore, say this: 72 and 18 makes 90. Adding 9 to this makes 99. Include the speaker and you shall have 100.

III. propositio de duobus proficiscentibus.[7]
Duo homines ambulantes per viam, videntesque ciconias, dixerunt inter se: Quot sunt? Qui conferentes numerum dixerunt: Si essent aliae tantae; et ter tantae, et medietas tertii, adjectis duobus, c essent. Dicat, qui potest, quantae fuerunt, quae imprimis ab illis visae sunt?
3. proposition concerning the two travellers.
Two men were walking in the street when they noticed some storks. They asked each other, "How many are there?" Discussing the matter, they said: "If [the storks] were doubled, then taken three times, and then half of the third [were taken] and with two more added, there would be 100." Let him say, he who is able, How many [storks] were first seen by the men?

Solutio de ciconiis.
xxviii et xviii,[8] et tertio sic: fiunt lxxxiiii. Et medietas tertii

fiunt xiiii. Sunt in totum xcviii. Adjectis duobus, c apparent.

Solution concerning the storks.
28 taken three times makes 84. Half of a third makes 14. Thus, in total there are 98. By adding two, there are 100.

IV. propositio de homine et equis.[9]
Quidam homo vidit equos pascentes in campo, optavit dicens: Utinam essetis mei, et essetis alii tantum, et medietas medietatis; certe gloriarer super equos c. Discernat, qui vult, quot equos imprimis vidit ille homo pascentes?
4. proposition concerning the man and the horses.
A certain man saw some horses grazing in a field and said longingly: "O that you were mine, and that you were double in number, and then a half of half of this [were added]. Surely, I might boast about 100 horses." Let him discern, he who wishes, How many horses did the man originally see grazing?

Solutio de equis.
xl equi erant, qui pascebant. Alii tantum fiunt lxxx. Medietas medietatis hujus, id est, xx, si addatur, fiunt c.
Solution concerning the horses.
There were 40 horses grazing; double this makes 80. A half of half of this, i.e. 20, if added, makes 100.

V. propositio de emptore denariorum.[10]
Dixit quidam emptor:[11] Volo de centum denariis c porcos emere; sic tamen, ut verres x denariis ematur; scrofa autem v denariis; duo vero porcelli denario uno. Dicat, qui intelligit, quot verres, quot scrofae, quotve porcelli esse debeant, ut in neutris numerus nec superabundet, nec minuatur?
5. proposition concerning the buyer and his denarii.
A certain buyer said: "I want to buy 100 pigs with 100 denarii in such a way that a mature boar is bought for 10 denarii; a sow for five denarii; and two small female pigs for one denarius." Let him say, he who knows, How many boars, sows, and small female pigs should there be so that there are neither too many nor too few of either [pigs or denarii]?

Solutio de emptore.
Fac viiii scrofas et unum verrem in quinquaginta quinque denariis; et lxxx porcellos in xl. Ecce porci xc. In residuis v denariis, fac porcellos x, et habebis centenarium numerum in utrisque.
Solution concerning the buyer.
Buy nine sows and one boar with 55 denarii, and 80 small female pigs with 40; behold, 90 pigs. With the remaining five denarii, buy ten small female pigs, and you shall have 100 pigs for 100 denarii.

VI. propositio de duobus negotiatoribus c solidos habentis.
Fuerunt duo negotiatores, habentes c solidos communes, quibus emerent

porcos. Emerunt autem in solidis duobus porcos v, volentes eos saginare, atque iterum venundare, et in solidis lucrum facere. Cumque vidissent tempus non esse ad saginandos porcos, et ipsi eos non valuissent tempore hiemali pascere, tentavere venundando, si potuissent, lucrum facere, sed non potuerunt; quia non valebant eos amplius venundare, nisi ut empti fuerant, id est, ut de v porcis duos solidos acciperent. Cum hoc conspexissent, dixerunt ad invicem: Dividamus eos. Dividentes autem et vendentes, sicut emerant, fecerunt lucrum. Dicat, qui valet, imprimis quot porci fuerunt; et dividat ac vendat et lucrum faciat, quod facere de simul venditis non valuit.

6. proposition of the two businessmen who had 100 solidi.

There were two businessmen who had 100 solidi between them, with which they bought some pigs. For two solidi, they bought five pigs, wishing to fatten them and to sell them again at a profit. But when they saw that the time was not right to fatten the pigs, and being unable to pasture them over the winter, they tried to make a profit by selling them. However, they were unsuccessful because they could only sell the pigs for what they had paid (i.e., five pigs for two solidi). When they realized this, they said to each other, "We shall divide the pigs." But by dividing and selling the pigs for as much as they had paid, they made a profit. Let him say, he who can, How many pigs were there at first, and how did the men divide and sell for a profit that which they could not do together?

Solutio de porcis.

Imprimis ccl porci erant, qui c solidis sunt comparati, sicut supra dictum est, in duobus solidis v porcos: quia sive quinquagies quinos, sive quinquies l dixeris, ccl numerabis. Quibus divisis unus tulit cxxv, alter similiter. Unus vendidit deteriores tres semper in solido; alter meliores duos in solido. Sic evenit, ut is qui deteriores vendidit, de cxx porcis xl solidos est consecutus.[12] Qui vero meliores, lx solidos est consecutus; quia de inferioribus xxx semper in x solidis; de melioribus viginti autem in x solidis sunt venundati: et remanserunt utrisque v porci, ex quibus ad lucrum iiii solidos et duos denarios facere potuerunt.

Solution concerning the pigs.

There were 250 pigs to begin with. These were bought for 100 solidi, as stated above, at the price of two solidi per five pigs. Because whether you say "50 times five" or "five times 50," you arrive at 250. One man sold three inferior pigs at a price of one solidi; the other, two better pigs per solidi. Thus it happened that he who sold the inferior pigs obtained 40 solidi for 120 pigs, whereas the better pigs brought in 60 solidi. This is because it was always 30 inferior pigs for ten solidi, and 20 better pigs for ten solidi. For each man, there remained five pigs, from which they could make four solidi and two denarii in profit.

VII. propositio de disco pensante libras xxx.

Est discus qui pensat libras xxx sive solidos dc, habens in se aurum, argentum, aurichalcum, et stannum. Quantum habet auri, ter tantum habet argenti. Quantum argenti, ter tantum aurichalci. Quantum aurichalci, ter tantum stanni. Dicat, qui potest, quantum in unaquaque specie pensat?

7. proposition concerning the plate weighing 30 pounds.

There is a plate weighing 30 pounds or 600 solidi. In it, there is gold, silver, brass and tin. It has three times are much silver as gold, three times as much brass as silver, and three times as much tin as brass. Let him say, he who can, How much does each type of metal weigh?

Solutio.
Aurum pensat uncias novem: argentum ter incias viiii, id est, libras duas et tres uncias. Aurichalcum pensat ter libras duas et [ter] iii uncias, id est, libras vi et viiii uncias. Stannum pensat ter libras vi, et ter uncias viiii, hoc est, libras xx, et iii uncias. viiii unciae, et ii librae cum iii unciis: et vi librae cum viiii unciis: et xx librae cum iii unciis adunatae, xxx libras efficiunt.
Solution.
The gold weighs nine ounces. The silver weighs three times this, i.e. two pounds, three ounces. The brass weighs three times two pounds, three ounces, i.e. six pounds, nine ounces. The tin weighs three times six pounds, nine ounces, i.e. 20 pounds, three ounces. Nine ounces, and two pounds, three ounces, and six pounds, nine ounces, and 20 pounds, three ounces, taken together, make 30 pounds.

Item aliter ad solidum. Aurum pensat solidos argenteos xv. Argentum ter xv, id est, xlv. Aurichalcum ter xlv, id est, cxxv. Stannum ter cxxxv, hoc est, ccccv. Junge ccccv, et cxxxv: et xlv: et xv; et invenies dc, qui sunt librae xxx.
Another method. The gold weighs 15 silver solidi. The silver is three times the gold, i.e. 45. The brass is three times 45, i.e. 125 [sic]. The tin is three times 135, i.e. 405. Add 405 and 135 and 45 and 15, and you will get 600 [solidi], which equals 30 pounds.

VIII. propositio de cupa.
Est cupa una, quae c metretis impletur capientibus singulis modia tria; habens fistulas iii. Ex numero modiorum tertia pars et vi per unam fistulam currit: per alteram tertia pars sola: per tertiam sexta tantum. Dicat nunc, qui vult, quot sextarii per unamquamque fistulam cucurrissent.
8. proposition concerning the cask.
There is a cask which has three cracks in it. It is filled with 100 metretae, each holding three modia. Of the modia, a third and sixth part run out through one crack. Through another [crack], only a third part runs out. Only a sixth part runs out of the third crack. Let him say now, he who wishes, how many sextarii ran out through each crack.

Solutio.
Per primam fistulam iii dc sextarii cucurrerunt. Per secundum ii cccc.[13] Per tertiam i cc.
Solution.
3600 sextarii run through the first crack; 2400 through the second; and 1200 through the third.

IX. propositio de sago.
Habeo sagum habentem in longitudine cubitos c, et in latitudine lxxx. Volo

exinde per portiones sagulos facere, ita ut unaquaeque portio habeat in longitudine cubitos v, et in latitudine cubitos iiii. Dic, rogo, sapiens, quot saguli exinde fieri possint?
9. proposition concerning the cloak material.
I have a material for cloaks which is 100 cubits long, 80 cubits wide. From it, I wish to make smaller cloaks from portions in such a way that each portion is five cubits in length and four cubits wide. I ask you to tell me, wise one, How many smaller cloaks can be made from [the material]?

Solutio.
De quadrigentis octogesima pars v sunt; et centesima iiii. Sive ergo octuagies v, sive centies iiii duxeris, semper cccc invenies. Tot sagi erunt.[14]
Solution.
An eightieth part of 400 is five, and a hundredth part, four. Therefore, whether you measure off 80 [lengths] of five [cubits], or 100 of four, you shall always arrive at 400. There shall be this many cloaks.

X. propositio de linteo.[15]
Habeo linteamen unum longum cubitorum lx, latum cubitorum xl. Volo ex eo portiones facere, ita ut unaquaeque portio habeat in longitudine cubitos senos, et in latitudine quaternos, ut sufficiat ad tunicam consuendam. Dicat, qui vult, quot tunicae exinde fieri possint?
10. proposition concerning the linen cloth.
I have a single linen cloth which is 60 cubits long, 40 cubits wide. I wish to make it into smaller portions, each being six cubits in length, four cubits in width, so that each piece is ample for making a tunic. Let him say, he who wishes, How many tunics can be made [from the larger piece]?

Solutio. [16]
Decima pars sexagenarii vi sunt. Decima vero quadragenarii iiii sunt. Sive ergo decimam sexagenarii, sive decimam quadragenarii decies miseris, centum portiones vi cubitorum longas; et iiii cubitorum latas invenies.
Solution.
One tenth of 60 is six, and a tenth of 40 is four. Therefore, whether you shall have taken ten times a tenth of 60 [cubits] or ten times a tenth of 40, you will arrive at 100 portions of six cubits in length, and four cubits wide.

XI. propositio de duobus hominibus sorores accipientibus.
Si duo homines ad invicem, alter alterius sororem in conjugium sumpserit; dic, rogo, qua propinquitate filii eorum sibi pertineant?
11. proposition concerning the two men marrying [one another's] sister.
If two men should marry one another's sister, tell me, I ask, What will be the sons' relations to each other?

Solutio ejusdem. [17]
Verbi gratia: si ego accipiam sororem socii mei, et ille meam, et ex nobis procreentur filii; ego denique sum patruus filii sororis meae; et illa

amita filii mei. Et ea propinquitate sibi invicem pertinent.
Solution of the same [proposition].
As stated, if I should marry my friend's sister, and he should marry mine, sons would be produced by us. Thus, I shall be the paternal uncle of my sister's son, and she shall be my son's maternal aunt. The relation of the two men [to the sons] shall be the same.

XII. propositio de quodam patrefamilias et tribus filiis ejus.
Quidam paterfamilias moriens dimisit [18] haereditatem tribus filiis suis, xxx ampullas vitreas, quarum decem fuerunt plenae oleo. Aliae decem dimidiae. Tertiae decem vacuae. Dividat, qui potest, oleum et ampullas, ut unicuique eorum de tribus filiis aequaliter obveniat tam de vitro, quam de oleo.
12. proposition concerning a certain father and his three sons.
A certain father died and left as an inheritance to his three sons 30 glass flasks, of which 10 were full of oil; another 10 were half full, while another 10 were empty. Let him divide, he who can, the oil and flasks so that an equal share of the commodities should equally come down to the three sons, both of oil and glass.

Solutio.
Tres igitur sunt filii, et xxx ampullae. Ampullarum autem quaedam x sunt plenae, et x mediae, et x vacuae. Duc ter decies; fiunt xxx. Unicuique filio veniunt x ampullae in portionem. Divide autem per tertiam partem, hoc est, da primo filio x semis ampullas, ac deinde da secundo v plenas et v vacuas. Similiter dabis tertio, et erit trium aequa germanorum divisio tam in oleo, quam in vitro.
Solution.
There are three sons and 30 glass flasks. However, of the flasks, 10 are full [of oil], 10 half full, and 10 empty. Take three times 10, which makes 30, so each son shall receive 10 flasks as his portion. Divide up the three portions, that is, give to the first son 10 half [filled] flasks, to the second son five full and five empty [flasks]. Do the same for the third son, and the brothers' portions of glass and oil shall be the same.

XIII. propositio de rege.
Quidam rex jussit famulo suo colligere de xxx villis exercitum, eo modo ut ex unaquaque villa tot homines sumeret quotquot illuc adduxisset. Ipse tamen ad villam primam solus venit; ad secundam cum altero; jam ad tertiam tres venerunt. Dicat, qui potest, quot homines fuissent collecti de xxx villis.
13. proposition concerning the king.
A certain king ordered his servant to gather an army from 30 villages as follows: He should bring back as many men [from each successive village] as he had taken there. Thus, [the servant] came to the first village alone; he came with one other person to the next; three people came to the third. Let him say, he who is able, how many men were collected from the 30 villages.

Solutio. [19]

In prima igitur mansione duo fuerunt; [20] in secunda iiii, in tertia viii, in quarta xvi, in quinta xxxii, in sexta lxiiii, in septima cxxviii, in octava cclvi, in nona dxii, in decima i xxiiii, in undecima ii xlviii, in duodecima iiii xcvi, in quarta decima xvi ccclxxxiiii. In quinta decima xxxii dcclxviii, etc.

Solution.
In the first village, there were two [people]; in the second, four; in the third, eight; in the fourth, 16; in the fifth, 32; in the sixth, 64; in the seventh, 128; in the eighth, 256; in the ninth, 512; in the 10th, 1024; in the 11th, 2048; in the 12th, 4096; in the 14th, 16, 384; in the 15th, 32, 768; etc.

XIV. propositio de bove.
Bos qui tota die arat, quot vestigia faciat in ultima riga?
14. proposition concerning the ox.
How many footprints in the last furrow does an ox make which has been plowing all day?

Solutio.
Nullum omnino vestigium facit bos in ultima riga, eo quod ipse praecedit aratrum, et hunc aratrum sequitur. Quotquot enim hic praecedendo in exculta terra vestigia figit,[21] tot ille subsequens excolendo resolvit. Propterea illius nullum reperitur vestigium in ultima riga.
Solution.
An ox makes no footprints whatsoever in the last furrow. This is because the ox goes in front of the plow, and the plow follows it. For however many footprints the ox makes on the ploughed earth by going first, so many the plough following behind destroys by ploughing. On account of this, no footprints appear in the last furrow.

XV. propositio de homine.
Quaero a te ut dicas mihi quot rigas factas habeat homo in agro suo, quando de utroque capite campi tres versuras factas habuerit?
15. proposition concerning the man.
I ask you in order that you might tell me, How many furrows might a man have in his field if he shall have made three turns at each head of the field?

Solutio.
Ex uno capite campi iii. Ex altero iii, quae faciunt rigas versuras vi.
Solution.
Three [furrows] from one head of the field, and three from the other, making six plowed furrows.[22]

XVI. propositio de duobus hominibus boves ducentibus.
Duo homines ducebant boves per viam, e quibus unus alteri dixit: Da mihi boves duos; et habeo tot boves quot et tu habes. At ille ait: Da mihi et tu duos boves, et habeo duplum quam tu habes. Dicat, qui vult, quot boves

fuerunt, quot unusquisque habuit.
16. proposition concerning the two men leading oxen.
Two men were leading oxen along the road when one said to the other, "Give me two oxen, and I shall have as many oxen as you." Then the other said, "You give me two oxen, and I shall have twice as many as you." Let him say, he who wishes, how many oxen there were, and how many each man had.

Solutio.
Prior, qui dari sibi duos rogavit, boves habebat iiii. At vero, qui rogabatur, habebat viii. Dedit quippe rogatus postulanti duos, et habuerunt uterque sex. Qui enim prius acceperat, reddidit duos danti priori, qui habebat sex, et habuit viii, quod est duplum a quator, et illi remanserunt iiii, quod est simplum ab viii.
Solution.
At first, the man who asked for two to be given to him had four oxen. But indeed, the man who was asked had eight. Of course, having been asked, he gave two to the one asking, and each of the two had six. For the man who first asked returned two to the one first giving (who now had six), and he had eight, which is double four, and four remained to that one, which is half of eight.

XVII. propositio de tribus fratribus singulas habentibus sorores.
Tres fratres erant qui singulas sorores habebant, et fluvium transire debebant (erat enim unicuique illorum concupiscentia in sorore proximi sui), qui venientes ad fluvium non invenerunt nisi parvam naviculam, in qua non potuerunt amplius nisi duo ex illis transire. Dicat, qui potest, qualiter fluvium transierunt, ne una quidem earum ex ipsis maculata sit?
17. proposition concerning the men [23] who had unmarried sisters.
There were three men, each having an unmarried sister, who needed to cross a river. Each man was desirous of his friend's sister. Coming to the river, they found only a small boat in which only two persons could cross at a time. Let him say, he who is able, How did they cross the river, so that none of the sisters were defiled by the men?

Solutio.
Primo omnino ego et soror mea introissemus in navem et transfretassemus ultra; transfretatoque fluvio dimisissem sororem meam de nave, et reduxissem navem ad ripam. Tunc vero introissent sorores duorum virorum, illorum videlicet, qui ad littus remanserant. Illis igitur feminis navi egressis, soror mea [quae prima transierat], intraret, navemque reduceret ad nos. Illa egrediente foras, duo in navem fratres intrassent, ultraque venissent. Tunc unus ex illis una cum sorore sua navem ingressi ad nos transfretassent. Ego autem et ille, qui navigaverat, sorore mea remanente foras, ultra venissemus. Nosque ad littora vectos, una ex illis duabus quaelibet mulieribus, ultra navem reduceret, sororeque mea secum recepta pariter ad nos ultra venissent. Et ille, cujus soror ultra remanserat, navem ingressus eam secum reduceret. Et fieret expleta transvectio nullo maculante contagio. [24]

Solution.
First of all, my sister and I got into the boat and crossed. Having

crossed the river, I let my sister out and recrossed the river. Then the sisters of the two men who remained on the bank got in. When these women had gotten out of the boat, my sister, who had already gone across, got in and brought the boat back to us. She then got out, and the two brothers crossed in the boat. Then, one of the brothers and his sister crossed over to us. However, I and the brother who piloted the boat went across while my sister remained behind. When we had been taken to the [other] side, one of the other women took the boat back across, and my sister came across to us with her at the same time. Then the man whose sister had remained on the other side got in the boat and and brought it back with her. Thus the crossing was accomplished, with no one being defiled.

XVIII. propositio de homine et capra et lupo.
Homo quidam debebat ultra fluvium transferre [25] lupum, capram, et fasciculum cauli. Et non potuit aliam navem invenire, nisi quae duos tantum ex ipsis ferre valebat. Praeceptum itaque ei fuerat ut omnia haec ultra illaesa transire potuit? [26]
18. proposition concerning the man, the she-goat, and the wolf.
A certain man needed to take a wolf, a she-goat and a load of cabbage across a river. However, he could only find a boat which would carry two of these [at a time]. Thus, what rule did he employ so as to get all of them across unharmed?

Solutio.
Simili namque tenore ducerem prius capram et dimitterem foris lupum et caulum. Tum deinde venirem, lupumque transferrem: [27] lupoque foris misso capram navi receptam ultra reducerem; capramque foris missam caulum transveherem ultra; atque iterum remigassem, capramque assumptam ultra duxissem. Sicque faciendo facta erit remigatio salubris, absque voragine lacerationis.
Solution.
In a similar manner, I would first take the she-goat and leave behind the wolf and the cabbage. When I had returned, I would ferry over the wolf. With the wolf unloaded, I would retrieve the she-goat and take it back across. Then, I would unload the she-goat and take the cabbage to the other side. I would next row back, and take the she-goat across. The crossing should go well by doing thus, and absent from the threat of slaughter.

XIX. propositio de viro et muliere ponderantibus [plaustri pondus onusti].
De viro et muliere, quorum uterque pondus habebat plaustri onusti, duos habentes infantes inter utrosque plaustrali pondere pensantes fluvium transire debuerunt. Navem invenerunt quae non poterat ferre plus nisi unum pondus plaustri. Transfretari faciat, qui se putat posse, ne navis mergatur.
19. proposition concerning the man and his wife, [each] weighing as much as a loaded cart.
A man and his wife, each the weight of a loaded cart, who had two children each the weight of a small cart, needed to cross a river. However, the boat they came across could only carry the weight of one cart. Let him devise [a way] of crossing in order that the boat should not sink.

Solutio.
Eodem quoque ordine, ut superius. Prius intrassent duo infantes et transissent unusque ex illis reduceret navem. Tunc mater navem ingressa transisset. Deinde filius ejus reduceret navem. Qua transvecta frater illius navim ingressus ambo ultra transissent, rursusque unus ex illis ad patrem reduceret navem. Qua reducta, filio foris stante, pater transiret: rursusque filius, qui ante transierat, ingressus navim eamque ad fratrem reduceret: jamque reductam ingrediantur ambo et transeant. Tali subremigante ingenio erit expleta navigatio forsitan sine naufragio.
Solution.
Also in the same manner, first, the two children get in [the boat] and cross; one of them then brings the boat back. Then the mother gets in the boat and crosses; her son brings the boat back. With the boat back, the brother of this one gets in the boat and both cross; one of them then brings the boat back to the father. When the boat has returned and with the son on the bank, the father may cross. Then the brother who had gone across before get in the boat and brings it back to his brother. Now with the boat returned, both brothers get in and cross. By such a clever plan of crossing, the navigation can perhaps take place without the boat sinking.

XX. propositio de hirtitiis.[28]
De hirtitiis masculo et femina habentibus duos natos libram ponderantibus, flumen transire volentibus.
20. proposition concerning the hirtitii.
A masculine and feminine [....] who had two children weighing [29] a pound wished to cross a river.

Solutio.
Similiter, ut superius, transissent prius duo infantes, et unus ex illis navem reduceret; in quam pater ingressus ultra transisset; et ille infans, qui prius cum fratre transierat, navim ad ripam reduceret, in quam frater illius rursus ingressus ambo ultra venissent; unusque propterea ex illis foras egressus; et alter ad matrem reduceret navim: in quam mater ingressa ultra venisset: qua egrediente foras, filius ejus, qui ante cum patre transierat, navim rursus ingressus eam ad fratrem ultra reduceret; in quam ambo ingressi ultra venissent, et fieret expleta transvectio nullo formidante naufragio.
Solution.
Again, as above, first the two children go across. One of them brings back the boat, in which the father crosses. Then, the child who had first gone across with his brother brings the boat back to the river, and he and his brother both go across. One of them gets out on the [opposite] shore; the other takes the boat back to the mother. The mother gets in and crosses. When she has unloaded at the [opposite] shore, her son, who had previously crossed with his father, gets in the boat again and takes it over to his brother. Both brothers get in and cross. A crossing can be carried out thusly, free from dread of accident.

XXI. propositio de campo et ovibus in eo locandis.
Est campus qui habet in longitudine pedes cc, et in latitudine pedes c. Volo ibidem mittere oves; sic tamen ut unaquaeque ovis habeat in longo pedes v, et in lato pedes iv. Dicat, rogo, qui valet, quot oves [30] ibidem locari possint?
21. proposition concerning the field and the sheep to be placed in it.
There is a field which is 200 feet long, 100 feet wide. I want to put sheep in it as follows: Each sheep should have [an area] five feet long and four feet wide. Let him say, I ask he who is able, How many sheep can be put in such a place?

Solutio.
Ipse campus habet in longitudine pedes cc. Et in latitudine pedes c. Duc bis [31] quinquenos de cc, fiunt xl. At deinde c divide per iiii. Quarta pars centenarii xxv. Sive ergo xl vicies quinquies; sive xxv quadragies ducti, [32] millenarium implent numerum. Tot ergo ibidem oves colfocari [33] possunt.

Solution.
The field is 200 feet long and 100 feet wide. Divide 200 by five, making 40. Then, divide 100 by four, a fourth part of which is 25. Hence, whether 40 times 25, or 25 times 40, the number 1000 is obtained. This many sheep can inhabit such a place.

XXII. propositio de campo fastigioso.
Est campus fastigiosus, qui habet in uno latere perticas c, et in altero latere perticas c, et in fronte perticas l, et in medio perticas lx, et in altera fronte perticas l. Dicat, qui potest, quot aripennas [34] claudere debet?
22. proposition concerning the slanting field.
There is a slanting field which is 100 perticae on each side, 50 perticae on one front, 60 perticae in the middle, and 50 perticae on the other front. Let him say, he who is able, How many aripennae does [this field] enclose?

Solutio.
Longitudo hujus campi c perticis, et utriusque frontis latitudo l, medietas vero lx includitur. Junge utriusque frontis numerum cum medietate, et fiunt clx. Ex ipsis assume tertiam partem, id est, liii, et multiplica centies, fiunt v ccc. Divide [35] in xii aequas partes, et inveniuntur ccccxli. [36] Item eosdem divide in xii partes, et reperiuntur xxxvii. Tot sunt in hoc campo aripenni. [37]

Solution.
The field is 100 perticae in length, 50 perticae on each front, and 60 perticae in the middle. Add the length of each front with the middle, making 160. Take one third of this, that is, 53, and multiply it by 100, making 5300. Divide this into 12 equal parts, and you arrive at 441. Likewise, divide this into 12 equal parts, and you get 37. There are this many aripenni in the field.

XXIII. propositio de campo quadrangulo.

Est campus quadrangulus qui habet in uno latere perticas xxx, et in alio perticas xxxii, et in fronte perticas xxxiiii, et in altera perticas xxxii. Dicat, qui potest, quot aripenni in eo concludi debent?
23. proposition concerning the quadrangular field.
There is a field which is 30 perticae on one side, 32 perticae on another, 34 perticae in the front, and 32 perticae on the remaining side. Let him say, he who can, How many aripenni are contained in such a field?

Solutio.
Duae ejusdem campi longitudines faciunt lxii. Duc dimidiam lxii, fiunt xxxi. Ac duae ejusdem campi latitudines junctae fiunt lxvi. Duc vero mediam de lxvi, fiunt xxxiii. Duc vero [38] terties semel, fiunt i xx. Divide per duodecimam partem bis sicut superius, hoc est, de mille viginti, duc per xii, fiunt lxxxv, rursusque lxxxv divide per xii, fiunt vii. Sunt ergo in hoc aripenni numero septem.

Solution.
Two lengths of this field make 62 [perticae]. Half of 62 makes 31. But [the other] two sides of the field added together make 66. Half of 66 makes 33. Take [33] 31 times, making 1020. Divide [1020] twice by 12 as above, first getting 85, then 85 by 12, making 7. Thus there are seven aripenni in this field.

XXIV. propositio de campo triangulo.
Est campus triangulus qui habet in uno latere perticas xxx, et in alio perticas xxx, et in fronte perticas xviii. [39] Dicat, qui potest, quot aripennos concludere debet?
24. proposition concerning the triangular field.
There is a field which is 30 perticae on one side, 30 perticae on another, and 18 perticae in the front. Let him say, he who can, How many aripenni must be contained [in such a field]?

Solutio.
Junge duas longitudines istius campi, et fiunt lx. Duc mediam de lx, fiunt xxx, et quia in fronte perticas xviii habet, duc mediam de xviii, fiunt viiii. Duc vero novies triginta, fiunt cclxx. Fac exinde bis xii, id est, divide cclxx, per duodecimam, fiunt xxii et semis; atque iterum xxii et semis per duodecimam divide partem....[40] fit aripennis unus et perticae x, et dimidia.
Solution.
Adding two lengths of the field makes 60. Removing half of 60 makes 30. Because there are 18 perticae in front, take half of this away, making nine. Taking nine times 30 makes 270. Then, divide [270] by twelve, making 22-and-a-half. Again, divide 22-and-a-half by twelve, [making two, [41] with four left over, which is a third of 12. Thus there are two aripenna in this amount and three parts of a third aripennum.] This makes one aripennum, and 10-and-a-half perticae.

XXV. propositio de campo rotundo.
Est campus rotundus, qui habet in gyro perticas cccc. Dic quot aripennos capere debet?

25. proposition concerning the round field.
There is a round field which contains 400 perticae in its circle. Tell me, How many aripenni ought it to hold?

Solutio.
Quarta quidem pars hujus campi, qui cccc includitur perticis est c, hos si per semetipsos [42] multiplicaveris, id est, si centies duxeris, x millia fiunt, hos in xii partes dividere debes; etenim de x millibus duodecima est dcccxxxiii, quam cum item in xii partitus fueris, invenies lxviiii. Tot enim aripennis hujusmodi campus includitur. [43]

Solution.
A quarter of this field, which contains 400 perticae, is 100. If you multiply [100] by 100, you get 10,000, which you must divide into 12 parts. For indeed, a twelfth of 10,000 is 833, which when again partitioned into twelfths gives 69. [44] This many aripenni are included in the field.

XXVI. propositio de cursu cbnks. bc. fvgb. lfp:rks. [45]
Est campus qui habet in longitudine pedes cl. In uno capite stabat canis, et in alio stabat lepus. Promovit namque canis ille post illum, [46] scilicet leporem currere. Ast ubi ille canis faciebat in uno saltu pedes viiii, lepus transmittebat vii. Dicat, qui velit, quot pedes quotque saltus canis persequendo, et lepus fugiendo, quoadusque comprehensus est, fecerunt? [47]
26. proposition concerning the chase of the dog and the flight of the hare.
There is a field which is 150 feet long. At one end stood a dog, at the other, a hare. The dog advanced behind [the hare], namely, to chase the hare. But whereas the dog went nine feet per stride, the hare went [only] seven. Let him say, he who wishes, How many feet and how many leaps did the dog take in pursuing the fleeing hare until it was caught?

Solutio.
Longitudo hujus videlicet campi habet pedes cl. Duc mediam de cl, fiunt lxxv. Canis vero faciebat in uno saltu pedes viiii, quippe lxxv novies ducti fiunt dclxxv, tot pedes leporem consequendo [48] canis cucurrit, quoadusque eum comprehendit dente tenaci. At vero quia lepus faciebat pedes vii, in uno saltu, duc ipsos lxxv septies. [49] Tot vero pedes lepus fugiendo peregit, donec consecutus est.

Solution.
The length of this field was 150 feet. Taking half of 150 makes 75. The dog was covering nine feet per stride, and nine times 75 makes 675. The dog thus ran this many feet in chasing the rabbit until it caught the rabbit with its tenacious teeth. And indeed, because the rabbit went seven feet per stride, take 75 seven times. This is how many feet the fleeing rabbit travelled before being caught.

XXVII. propositio de civitate quadrangula.
Est civitas quadrangula quae habet in uno latere pedes mille centum; et in alio latere pedes mille; et in fronte pedes dc, et in altera pedes dc. Volo

ibidem tecta domorum ponere, sic, ut habeat unaquaeque casa in longitudine pedes xl, et in latitudine pedes xxx. Dicat, qui velit, quot casas capere debet?
27. proposition concerning the quadrangular city.
There is a quadrangular city which has one side of 1100 feet, another side of 1000 feet, a front of 600 feet, and a final side of 600 feet. I want to put some houses there so that each house is 40 feet long and 30 feet wide. Let him say, he who wishes, How many houses ought the city to contain?

Solutio.
Si fuerunt duae hujus civitatis longitudines junctae, facient ii c. Similiter duae, si fuerunt latitudines junctae, faciunt i cc. Ergo duc mediam de i cc, faciunt [50] dc, rursusque duc mediam de ii c, fiunt i l. Et quia unaquaeque domus habet in longitudine [51] pedes xl, et in lato xxx: deduc [52] quadragesimam partem de mille l, fiunt xxvi. Atque iterum assume tricesimam de dc, fiunt xx. Vicies ergo xxvi ducti, fiunt dxx. Tot domus capiendae sunt.

Solution.
If the two lengths of this city were joined together, they would measure 2100 [feet]. Likewise, if the two sides were joined, they would measure 1200. Therefore, take half of 1200, i.e. 600, and half of 2100, i.e. 1050. Because each house is 40 feet long and 30 feet wide, take a fourtieth part of 1050, making 26. Then, take a thirtieth of 600, which is 20. 20 times 26 is 520, which is the number of houses to be contained in the city.

XXVIII. propositio de civitate triangula.
Est civitas triangula quae in uno habet latere pedes c, et in alio latere pedes c, et in fronte pedes xc, volo enim ibidem aedificia domorum construere, [53] sic tamen, ut unaquaeque domus habeat in longitudine pedes xx, et in latitudine pedes x. Dicat, qui potest, quot domus capi debent?
28. proposition concerning the triangular city.
There is a triangular city which has one side of 100 feet, another side of 100 feet, and a third of 90 feet. Inside of this, I want to build a structure of houses, however, in such a way that each house is 20 feet in length, 10 feet in width. Let him say, he who can, How many houses should be contained [within this structure]?

Solutio.
Duo igitur hujus civitatis latera juncta fiunt cc, atque duc mediam de cc, fiunt c. Sed quia in fronte habet pedes xc, duc mediam de xc, fiunt xlv. Et quia longitudo uniuscujusque domus habet pedes xx, et latitudo ipsarum pedes x, duc xx partem in [54] c, fiunt v. Et pars decima quadragenarii iv sunt. Duc itaque quinquies iiii, fiunt xx. Tot domos hujusmodi captura [55] est civitas.

Solution.
Two sides of the city joined together make 200; taking half of 200 makes 100. But because the front is 90 feet, take half of 90, making 45. And since the length of each house is 20 feet while the width is 10, take 20 into 100, making five. A tenth part of 40 is four; thus, take four five

times, making 20. The city is to contain this many houses in this way.

XXVIIII. propositio de civitate rotunda.
Est civitas rotunda quae habet in circuitu pedum viii millia. Dicat, qui potest, quot domos capere debet, ita ut unaquaeque habeat in longitudine pedes xxx, et in latitudine pedes xx?
29. proposition concerning the round city.
There is a city which is 8000 feet in circumference. Let him say, he who is able, How many houses should the city contain, such that each [house] is 30 feet long, and 20 feet wide?

Solutio.
In hujus civitatis ambitu viii millia pedum numerantur, qui sesquialtera proportione dividuntur in xxxx dccc, et in iii cc. In illis autem longitudo domorum; in istis latitudo versatur. Subtrahe itaque de utraque summa medietatem, et remanent de majori ii cccc: de minore vero i dc. Hos igitur i dc divide in vicenos et invenies octoagies viginti, rursumque major summa, id est ii cccc, in xxx partiti, octoagies triginta dinumerantur. Duc octoagies lxxx, et fiunt vi millia cccc. Tot in hujusmodi civitate domus, secundum propositionem supra scriptam, construi [56] possunt.

Solution.
This city measures 8000 feet around, which is divided into proportions of one-and-a-half to one, i.e. 4800 and 3200. The length and width of the houses are to be of these [dimensions]. Thus, take half of each of the above [measurements], and from the larger number there shall remain 2400, while from the the smaller, 1600. Then, divide 1600 into twenty [parts] and you will obtain 80 times 20. In a similar fashion, [divide] the larger number, i.e. 2400, into 30 pieces, deriving 80 times 30. Take 80 times 80, making 6400. This many houses can be built in the city, following the above-written proposal.

XXX. propositio de basilica.
Est basilica quae habet in longitudine pedes ccxl, et in lato pedes cxx. Laterculi vero stratae ejusdem unus laterculus habet in longitudine uncias xxiii, hoc est, pedem unum et xi uncias. Et in latitudine uncias xii, hoc est, pedem i. Dicat, qui velit, quot laterculi eamdem debent implere?
30. proposition concerning the basilica.
There is a basilica which is 240 feet long, 120 feet wide. One tile of the tiled basilica is 23 inches long, that is, one foot, 11 inches, while being 12 inches wide, i.e. one foot. Let him say, he who wishes, How many tiles are needed to cover the basilica?

Solutio.
cxl pedes longitudinis implent cxxvi laterculi; et cxx pedes latitudinis cxx laterculi; quia unusquisque laterculus in latitudine pedis mensuram habet. Multiplica itaque centum vicies cxxvi, in xv cxx [57] summa concrescit. Tot igitur in hujusmodi basilica laterculi pavimentum contegere possunt.

Solution.
126 tiles build 140 [sic] feet of length, [58] and 120 tiles, 120 feet of width, because each brick measures one foot in length. Thus, multiply 120 by 126, obtaining 15,120. Therefore in this way so many tiles are able to cover the ground of the basilica.

XXXI. propositio de canava. [59]
Est canava quae habet in longitudine pedes c, et latitudine pedes lxiiii. Dicat, qui potest, quot cupas capere debet? ita tamen, ut unaquaeque cupa habeat in longitudine pedes vii, et in lato, hoc est in medio pedes iiii, et pervius unus habeat pedes iiii. [60]
31. proposition concerning the wine cellar.
There is a wine cellar which is 100 feet long and 64 feet wide. Let him say, he who can, How many casks can it hold, given that each cask is seven feet long and four feet wide, and given that there is an aisle four feet wide in the middle [of the cellar]?

Solutio.
In centum autem quaterdecies vii numerantur, in lxiiii vero sedecies quaterni continentur, ex quibus iiii ad pervium reputantur, [61] quod in longitudinem ipsius canavae ducitur. [62] Quia ergo in lx quindecies quaterni sunt; et in centum quaterdecies septeni; duc quindecies xiiii, [63] fiunt ccx. Tot cupae juxta suprascriptam magnitudinem in hujusmodi canava [64] contineri possunt.

Solution.
There are fourteen sevens in 100, and sixteen fours in 64, of which four are needed for the aisle which runs the length of this cellar. And since there are fifteen fours in 60, and since there are fourteen sevens in 100, take 15 times 14, making 210. This many casks can be stored in the type of wine cellar described above. [65]

XXXII. propositio de quodam patrefamilias.
Quidam paterfamilias habuit familias xx. Et jussit eis dare [66] de annona modios xx. Sic jussit, ut viri acciperent [67] modios ternos, et mulieres binos, et infantes singula semodia. Dicat, qui potest, quot viri, aut quot mulieres, vel quot infantes esse debent? [68]
32. proposition concerning a certain head of household.
A certain head of household had 20 servants. He ordered them to be given 20 modia of corn as follows: The men should receive three modia; the women, two; and the children, half a modium. Let him say, he who can, How many men, women and children must there have been?

Solutio.
Duc semel ternos, fiunt iii, hoc est, unus vir ut modios accepit. Similiter et quinquies bini, fiunt x, hoc est, quinque mulieres acceperunt modia [69] x. Duc vero septies binos, fiunt xiiii, hoc est xiiii infantes acceperunt modios vii. Junge ergo i et v et xiiii, fiunt xx. Hae sunt familiae xx. Ac deinde junge iii et vii et x, fiunt xx, haec sunt modia xx. Sunt ergo simul familiae xx, et modia [70] xx.
Solution.

Take one three times which makes three; that is, each man received this many modia. Likewise, take five twice, making 10; in this way, five women received 10 modia. Then, take two seven times, making 14; thus, 14 children received seven modia. Add one and five and 14, making 20; this is the number of servants. Then, add three and seven and 10, this being the number of modia. Thus there are 20 servants and 20 modia [of corn].

XXXIII. propositio de alio patrefamilias erogante suae familiae annonam.
Quidam paterfamilias habuit familias xxx, quibus jussit dari de annona modios xxx. Sic vero jussit, ut viri acciperent modios ternos, et mulieres binos, et infantes singula semodia. Sovat, qui potest, quot viri, aut quot mulieres, quotve infantes fuerunt?
33. proposition concerning another head of household distributing corn to his servants.
A certain head of household had 30 servants whom he ordered to be given 30 modia of corn as follows: The men should receive three modia; the women, two; and the children, a half modium. Let him solve, he who can, How many men, women and children were there?

Solutio.
Si duxeris ternos ter, fiunt viiii. Et si duxeris quinquies binos, fiunt x, ac deinde duc vicies bis semis, fiunt xi, hoc est, viri iii acceperunt modia viiii, et quinque mulieres acceperunt x, et xxii infantes acceperunt xi modia. Simul juncti iii et v, et xxii faciunt familias xxx. Rursusque viiii et xi, et x, simul juncti faciunt modia xxx. Quod sunt simul familiae xxx, et modii xxx. [71]

Solution.
If you take thrice three, you get nine; if you take two five times, you get 10; and if you take half of 22, you get 11. Thus, three men received nine modia; five women received 10; and 22 children received 11 modia. Adding three and five and 22 makes 30 servants. Likewise, nine and 11 and 10 makes 30 modia. Hence there are 30 servants, and 30 modia [of corn].

XXXIV. propositio altera de patrefamilias partiente familiae suae annonam.
Quidam paterfamilias habuit familias c, quibus praecepit dare de annona modios c, eo vero tenore, ut viri acciperent modios ternos, mulieres binos, et infantes singula semodia. Dicat ergo, qui valet, quot viri, quot mulieres, aut quot infantes fuerunt?
34. another proposition concerning a head of household distributing corn to his servants.
A certain head of household had 100 servants. He ordered that they be given 100 modia of corn as follows: The men should receive three modia; the women, two; and the children, half a modium. Thus let him say, he who can, How many men, women, and children were there?

Solutio.
Undecim terni fiunt xxxiii. Et xv bis ducti fiunt xxx, [72] id est, xi viri acceperunt xxxiii modios; et xv mulieres acceperunt xxx et lxxiiii infantes acceperunt xxxvii, qui simul juncti, id est, xi et xv, et lxxiiii fiunt c, quae sunt familiae c. Similiter junge xxxiii, et xxx et xxxvii

faciunt [73] c, qui sunt modii c. His ergo simul junctis habes familias c et modios c.

Solution.

11 times three makes 33, and twice 15 makes 30; that is, 11 men received 33 modia [of corn]. 15 women received 30 [modia], and 74 children received 37. Adding these together, that is, 11 and 15 and 74, makes 100, which is the number of servants. Likewise, adding 33 and 30 and 37 makes 100, which is the number of modia. Thus with these sums, you have 100 servants, and 100 modia [of corn].

XXXV. propositio de obitu cujusdam patrisfamilias.

Quidam paterfamilias moriens reliquit infantes, et in facultate sua, solidorum dcccclx, [74] et uxorem praegnantem. Qui jussit ut si ei masculus nasceretur, acciperet de omni massa dodrans, hoc est, uncias viiii. Et mater ipsius acciperet quadrans, hoc est, uncias iii. Si autem filia nata esset, [75] acciperet septunx, hoc est vii [76] uncias, et mater ipsius acciperet quincunx, hoc est, v uncias. Contigit autem ut geminos parturiret, id est, puerum et puellam. Solvat, qui potest, quantum accepit mater, et quantum filius, quantumve filia?

35. proposition concerning the death of a certain father.

A certain father died and left behind children, a pregnant wife, and 960 solidi from his estate. [However, on his deathbed], he stipulated that if a son should be born to her, then the son should receive three fourths of the inheritance -- that is, nine twelfths. The mother should get a quarter [of the estate], that is, three twelfths. However, if a daughter were born, she should receive seven twelfths, and the mother, five twelfths. But as it happened, she gave birth to twins -- both a boy and a girl. Let him solve, he who can, How much did the mother, son and daughter each receive?

Solutio. [77]

Junge ergo viiii et iii, fiunt xii, xii namque unciae libram faciunt. Rursusque junge similiter vii et v, fiunt iterum xii. Ideoque bis xii faciunt xxiiii, xxiiii autem faciunt duas libras, id est, solidos xl. Deinde ergo [duc] per vicesimam quartam partem dcccclx solidos, et vicesima quarta pars eorum fiunt xl. Deinde duc, quia facit [78] dodrans sive dodrans, xl in nonam partem, ideo novies xl accepit filius, hoc est, xviii libras, quae faciunt solidos ccclx. Et quia mater tertiam partem contra filium accepit, et quintam contra filiam, iii et v, fiunt viii. Itaque duc, quia legitur, quod faciat bis seu bisse xl in parte octava; octies ergo xl accepit mater, hoc est, libras xvi, quae faciunt solidos cccxx. Deinde duc, quia legitur, quod faciat septunx, xl in vii partibus: postea duc septies xl, fiunt xiiii librae, quae faciunt solidos cclxxx, hoc filia accepit. Junge ergo ccclx et cccxx et cclxxx, fiunt dcccclx solidi et xlviii librae.

Solution.

Add nine and three, making 12. 12 ounces make a pound. Then add seven and five which make another 12. 12 taken twice makes 24 [ounces], equaling two pounds, itself equal to 40 solidi. Then take a twenty-fourth part of the 960 solidi which is 40. Then, because the son received three fourths or nine twelfths [of the inheritance], take a ninth of 40. The son received nine times 40 [ounces], that is, 18 pounds, which equals 360 solidi. And since the mother received a third as much as the son received and a fifth as much as the daughter, [she got] three and five which makes eight. Therefore, as prescribed, take twice 40 and divide it into eight parts. Thus the mother received eight times 40 [ounces], that is, 16 pounds, which is 320 solidi. Then, as stipulated, divide 40 into seven parts so as to get seven twelfths. After this, take seven times 40, that is, 14 pounds, which equals 280 solidi. This is what the daughter received. Add 360 and 320 and 280, making 960 solidi, 48 pounds.

XXXVI. propositio de salutatione cujusdam senis ad puerum.
Quidam senior salutavit puerum, cui et dixit: Vivas, filii, vivas, inquit, quantum vixisti, et aliud tantum, et ter tantum. Addatque tibi Deus unum de annis meis, et impleas annos centum. Solvat, qui potest, quot annorum tunc tempore puer erat?
36. proposition concerning a certain old man's greeting to a boy.
A certain old man greeted a boy, saying to him: "May you live, boy, may you live for as long as you have [already] lived, and then another equal amount of time, and then three times as much. And may God grant you one of my years, and you shall live to be 100." Let him solve, he who can, How many years old was the boy at that time?

Solutio.
In eo vero, quod dixit, vivas, quantum vixisti, vixerat ante annos viii et menses tres: et aliud tantum fiunt anni xvi et menses vi, et alterum tantum fiunt anni xxxiii, qui ter multiplicati fiunt anni xcviiii, unum ipsis additum fiunt c.
Solution.
When [the old man] said "may you live for as long as you have lived," [the boy] had [already] lived eight years, three months. Another equal number of years make 16 years, six months, while another equal span makes 33 years. Three times this makes 99 years, which with one more year added makes 100.

XXXVII. propositio de quodam homine volente aedificare domum.
Homo quidam, volens aedificare domum, locavit artifices vi, ex quibus v magistri et unus discipulus erat, et convenit inter eum, qui aedificare volebat; et artificies, ut per singulos dies xxv denarii eis in mercede darentur, sic tamen, ut discipulus medietatem de eo, quod unus ex magistris accipiebat, acciperet. Dicat, qui potest, quantum unusquisque de illis per unamquamque diem accepit?
37. proposition concerning a certain man wishing to build a house.
A certain man, wanting to build a house, found six workmen, of whom five were masters and one an apprentice. It was agreed between the man who wanted to build and the workmen that 25 denarii should be given to them per day as pay, and that the apprentice should receive half what the masters receive. Let him say, he who can, How much did each of them receive per day?

Solutio.
Tolle primum xxii denarios et divide eos in vi partes. Sic unicuique de

magistris, qui quinque sunt, iiii denarios; nam quinquies quatuor xx sunt. Duos, qui remanserunt, quae est medietas de uno, tolle et da discipulo; et sunt adhuc iii denarii desidui; quos sic distribues. Fac de unoquoque denario partes xi, ter undecim fiunt xxxiii, tolle illas triginta partes, divide eas inter magistros v. Quinquies seni fiunt xxx. Accidunt ergo unicuique magistro partes vi. Tolle tres partes, quae super xxx remanserunt, quod est medietas senarii, et da discipulo.
Solution.
First, take 22 denarii and divide them into six parts. Give four denarii to each of the five masters, since five times four is 20. Take the remaining two denarii, which is half of [a share], and give them to the apprentice. There are still three denarii remaining which you distribute thusly: Divide each denarius into 11 parts, making 33. Take 30 of them and divide them amongst the five masters, as five times six makes 30. Hence, six parts go to each master. Take the remaing three parts, that is, half of the six [which the masters received], and give them to the apprentice.

XXXVIII. propositio de quodam emptore in animalibus centum. [79]
Voluit quidam homo emere animalia promiscua c de solidis c, ita ut equus tribus solidis emeretur; bos vero in solido i, et xxiiii [80] oves in sol. i. Dicat, qui valet, quot caballi, vel quot boves, quotve fuerunt oves?
38. proposition concerning a certain purchaser and [his] 100 animals.
A certain man wanted to buy 100 various animals for 100 solidi. He wished to pay three solidi per horse, one solidus per cow, and one solidus per 24 sheep. Let him say, he who can, How many horses, cows and sheep were there?

Solutio.
Duc ter vicies tria i, fiunt lxviiii. Et duc bis vicies quatuor, fiunt xlviii. Sunt ergo caballi xxiii, et solidi lxviiii. Et oves xlviii, et solidi ii. Et boves xxviiii, in solidis xxviiii. Junge ergo xxiii et xlviii et xxviiii, fiunt animalia c. Ac deinde junge lxviiii et ii et xxviiii, fiunt solidi c. Sunt ergo simul juncta animalia c, et solidi c.

Solution.
Take three times 23, making 69. Then, take two times 24, making 48. There are thus 23 horses [which cost] 69 solidi, 48 sheep [costing] two solidi, and 29 cows [which cost] 29 solidi. Therefore, add 23 and 48 and 29, making 100 animals. Then, add 69 and two and 29, making 100 solidi. Thus there are 100 animals and just as many solidi.

XXXVIIII. propositio de quodam emptore in oriente.
Quidam homo voluit de c solidis animalia promiscua emere c in oriente; qui jussit famulo suo, ut camelum v solidis acciperet; asinum solido i. xx oves in solido compararet. Dicat, qui vult, quot cameli, vel asini, sive oves in negotio c solidorum fuerunt?
39. proposition concerning a certain purchaser in the east.
A certain man wished to buy 100 assorted animals for 100 solidi in the East. He ordered his servant to pay five solidi per camel, one solidus per

ass, and one solidus per 20 sheep. Let him say, he who wishes, How many camels, asses and sheep were obtained for 100 solidi?

Solutio.
Si duxeris x novies, [et] v fiunt xcv, hoc est, cameli xviiii sunt empti in solidis xcv. Adde cum ipsis unum, hoc est, in solido i asinum i, fiunt xcvi. Ac deinde duc vicies quater, fiunt lxxx, hoc est, in quatuor solidis oves lxxx. Junge ergo xviiii et i et lxxx, fiunt c. Haec sunt animalia. Ac deinde junge xcv, et i et iiii, fiunt solid. c. Simul ergo juncti faciunt pecora c, et solidos c.

Solution.
If you take 10 nine times and add five, you get 95; that is, 19 camels are bought for 95 solidi. Add to this one solidus for an ass, making 96. Then, take 20 times four, making 80 -- that is, 20 sheep for four solidi. Add 19 and one and 80, making 100 -- this is the number of animals. Then add 95 and one and four, making 100 solidi. Hence there are 100 beasts and 100 solidi.

XL. propositio de homine et ovibus in monte pascentibus.
Quidam homo vidit de monte oves pascentes, et dixit, utinam haberem tantum, et aliud tantum et medietatem de medietate, et de hac medietate aliam medietatem, [81] atque ego centesimus una cum ipsis ingrederer meam domum. Solvat, qui potest, quot oves vidit ibidem pascentes?
40. proposition concerning a man and [some] sheep grazing on a mountain.
A certain man saw from a mountain some sheep grazing and said, "O that I could have so many, and then just as many more, and then half of half of this [added], and then another half of this half. Then I, as the 100th [member], might head back to my home together with them." Let him solve, he who can, How many sheep did the man see grazing?

Solutio.
In hoc ergo, quod dixit; haberem tantum; xxxvi oves primum ab illo visae sunt. Et aliud tantum fiunt lxxii, atque medietas de hac videlicet medietate, hoc est, de xxxvi, fiunt x et viii. Rursusque de hac secunda scilicet medietate assumpta medietas, id est, de xviii fiunt viiii. Junge ergo xxxvi et xxxvi, fiunt lxxii. Adde cum ipsis xviii, fiunt xc. Adde vero viiii cum xc, fiunt xcviiii. Ipse vero homo cum ipsis additus erit centesimus.

Solution.
36 sheep were first seen by the man when he said, "O that I could have so many." Adding an equal number makes 72, and a half of half of this, that is, of 36, makes 18. And again, a half of this, that is, of 18, makes nine. Therefore add 36 and 36, making 72. Add to this 18, which makes 90. Then add nine to 90, making 99. The man himself added to these will be the 100th one.

XLI. propositio de sode et scrofa.
Quidam paterfamilias stabilivit curtem novam, [82] in qua posuit scrofam, quae peperit porcellos vii in media sode, qui83 una cum matre, quae octava est, pepererunt igitur unusquisque in omni angulo vii. Et ipsa iterum in

media sode cum omnibus generatis peperit vii. Dicat, qui vult, una cum matribus quot porci fuerunt?

41. proposition concerning the pigsty and the sow.

A certain head of household set up a new [quadrangular] enclosure in which he placed a sow. The sow gave birth to seven piglets in the middle of the sty. The offspring, along with the mother, the eighth pig, each gave birth to another seven piglets in each corner [of the sty]. Then, in the middle of the sty, the mother and all her offspring [each] gave birth to seven more. Let him say, he who wishes, How many pigs were there [in the end], including the mother?

Solutio.

In prima igitur parturitione, quae fuit facta in media sode, fuerunt porcelli vii, et mater eorum octava. Octies igitur octo ducti fiunt lxiiii. Tot porcelli una cum matribus fuerunt in i angulo. Ac deinde sexagies quater octo ducti fiunt dxii. Tot cum matribus suis porcelli in angulo ii. Rursusque dxii octies ducti fiunt i.ii xcvi. Tot in tertio angulo cum matribus suis fuerunt. Qui si octies multiplicentur, fiunt xxxii dcclxxxviii, tot cum matribus in quarto fuerunt angulo. Multiplica quoque octies xxxii dcclxxxviii, fiunt cc lxii et ccciiii. Tot enim creverunt, cum in media sode novissime partum fecerunt.

Solution.

In the first birth, which took place in the middle of the sty, there were seven piglets, with the mother being the eighth [member]. Eight taken eight times is 64 -- this many piglets, along with the mother, were in the first corner. Then, 64 taken eight times makes 512 -- this many piglets, including their mothers, were in the second corner. 512 taken eight times yields 4096 -- this many piglets, along with their mother, were in the third corner. If [4096] is multiplied eight times, one gets 32,788 [sic] [84] -- this many piglets, including the mother, were in the fourth corner. Taking eight times 32,788 [sic] makes 262,304 [sic]. [85] There grew to be this many [pigs] in the last stage in the middle of the sty.

XLII. propositio de scala habente gradus centum.

Est scala una habens gradus c. In primo gradu sedebat columba una; in secundo duae; in tertio tres; in quarto iiii; in quinto v. Sic in omni gradu usque ad centesimum. Dicat, qui potest, quot columbae in totum fuerunt?

42. proposition concerning the ladder having 100 steps.

There is a ladder which has 100 steps. One dove sat on the first step, two doves on the second, three on the third, four on the fourth, five on the fifth, and so on up to the hundredth step. Let him say, he who can, How many doves were there in all?

Solutio.

Numerabitur autem sic: a primo gradu in quo una sedet, tolle illam, et junge ad illas xcviiii, quae nonagesimo [nono] gradu consistunt, et erunt c. Sic secundum ad nonagesimum octavum et invenies similiter c. Sic per singulos gradus, unum de superioribus gradibus, et alium de inferioribus, hoc ordine conjunge, et reperies semper in binis gradibus c. Quinquagesimus autem gradus solus et absolutus est, non habens parem; similiter et centesimus solus remanebit. Junge ergo omnes et invenies columbas vl.

Solution.
There will be as many as follows: Take the dove sitting on the first step and add to it the 99 doves sitting on the 99th step, thus getting 100. Do the same with the second and 98th steps and you shall likewise get 100. By combining all the steps in this order, that is, one of the higher steps with one of the lower, you shall always get 100. The 50th step, however, is alone and without a match; likewise, the 100th stair is alone. Add them all and you will find 5050 doves.

XLIII. propositio de porcis.
Homo quidam habuit ccc porcos, et jussit, ut tot porci numero impari in iii dies occidi deberent. [86] Similis est et de xxx sententia. Dicat, qui potest, quot porci impares sive de ccc sive de xxx, inter tres dies occidendi sunt? Haec ratio indissolubilis ad increpandum composita est.
43. proposition concerning the pigs.
A certain man had 300 pigs. He ordered all of them slaughtered in three days, but with an uneven number being killed each day. He wished the same thing to be done with 30 pigs. Let him say, he who can, What odd number of pigs out of 300 or 30 were to be killed in three days? (This ratio is indissoluble and was composed for rebuking.)

Solutio.
Ecce fabula! quae a nemine solvi potest, ut ccc porci, sive triginta in tribus diebus impari numero occidantur. Haec fabula est tantum ad pueros increpandos.

Solution.
Behold an impossibility which is able to be solved by nobody!, in such a way that 30 [pigs] be killed in three days by an odd number. Such an implausible story is only for teasing young boys.

XLIIII. propositio de salutatione pueri ad patrem.
Quidam puer salutavit patrem; Ave, inquit, pater! Cui pater: Valeas, fili! vivas, quantum vixisti, quos annos geminatos triplicatos; [87] et sume unum de annis meis; et habebis annos c. Dicat, qui potest, quot annorum tunc tempore puer erat?
44. proposition concerning the boy's greeting to his father.
A certain boy addressed his father, saying, "Greetings, father!" The father responded, "May you fare well, my son, and may you live three times twice your years. Then, adding one of my own years, you will live to be 100." Let him say, he who can, How many years was the boy at the time?

Solutio.
Erat enim puer annorum xvi, et mensium vi, qui geminati cum mensibus fiunt anni xxxiii, qui triplicati fiunt xcviiii. Addito uno patris anno c apparent.
Solution.
They boy was 16 years, six months. Double this makes 33 years, which

tripled is 99. Having added one year of the father, there are 100.

XLV. propositio.
Columba sedens in arbore vidit alias volantes; dixit eis: Utinam fuissetis aliae tantum et ternae tantum, [88] tunc una mecum fuissetis c. Dicat, qui potest, quot columbae erant in primis volantes?
45. proposition.
A dove sitting in a tree saw some other doves flying and said to them, "O that you were doubled, and then tripled. Then, along with me, you would number 100." Let him say, he who can, How many doves were initially flying?

Solutio.
Triginta iii erant columbae, quas prius conspexit volantes. Item aliae tantae fiunt lxvi. Et tertiae tantum, fiunt xcviiii. Adde sedenteni, et erunt c.
Solution.
There were 33 doves flying at first. Double this makes 66, while three times [33] makes 99. Adding in the sitting dove makes 100.

XLVI. propositio de sacculo ab homine invento.
Quidam homo ambulans per viam invenit sacculum cum talentis duobus. Hoc quoque alii videntes dixerunt ei: Frater, da nobis portionem inventionis tantum. [89] Qui renuens noluit eis dare. Ipsi vero irruentes diripuerunt sacculum, et tulit sibi quisque solidos quinquaginta. Et ipse postquam vidit se resistere non posse, misit manum et rapuit solidos quinquaginta. Dicat, qui vult, quot homines fuerunt?
46. proposition concerning the small bag found by the man.
A certain man walking in the street found a small bag containing two talents. Some other people saw this and said to him: "Brother, give us a portion of your discovery." But the man shook his head and did not want to give them any. The others then rushed at him and tore apart the sack, each obtaining for himself 50 solidi. And when the man saw that he could no longer resist [their attack], he grabbed 50 solidi for himself. Let him say, he who wishes, How many men were there?

Solutio.
Apud quosdam talentum lxxii vel pondo vel habet libras. Libra vero habet solidos aureos lxxii. Sexagies quinquies lxxii ducti fiunt v cccc, qui numerus duplicatus fiunt decies dccc. In x millibus et octingentis sunt quinquagenarii ccxvi. Tot homines idcirco fuerunt.

Solution.
Each talent has 72 pounds in it by weight, and a pound equals 72 gold solidi. 65 times 72 equals 5400 [sic], [90] twice which makes 10,800. 50 goes into 10,800 216 times, which is the number of men [in the problem]. [91]

XLVII. propositio de episcopo qui jussit xii panes dividi.
Quidam episcopus jussit xii panes dividi in clero. Praecepit enim sic ut

singuli presbyteri binos acciperent panes; diaconus dimidium, lector quartam partem: ita tamen fiat, ut clericorum et panum unus sit numerus. Dicat, qui vult, quot presbyteri, vel quot diacones, aut quot lectores esse debent?

47. proposition concerning the bishop who ordered 12 loaves of bread to be divided.

A certain bishop ordered 12 loaves of bread divided amongst the clergy. He stipulated that each priest should receive two loaves; a deacon, half a loaf; and a lector, a quarter part. Hence, it should turn out that the number of clerics and loaves is the same. Let him say, he who can, How many priests, deacons and lectors must there have been?

Solutio.

Quinquies bini fiunt x, id est, v presbyteri decem panes receperunt: et diaconus unus dimidium panem: et inter lectores vi habuerunt panem et dimidium. Junge v et i et vi in simul, et fiunt xii. Rursusque junge x et semis et unum et semis, fiunt xii. Et illi sunt xii panes; qui simul juncti faciunt homines xii et panes xii. Unus est ergo numerus clericorum et panum.

Solution.

Twice five is 10; that is, five priests received 10 loaves. The deacon got half a loaf, and there was a loaf and a half for the six lectors. Add five and one and six, making 12. Then add 10-and-a-half and one-and-a-half, making 12, this being the number of loaves. Hence, there are 12 men altogether and 12 loaves. Therefore, the number of clerics and loaves is the same.

XLVIII. propositio de homine qui obviavit scholaribus.

Quidam homo obviavit scholaribus, [92] et dixit eis: Quanti estis in schola? Unus ex eis respondit dicens: Nolo hoc tibi dicere, tu numera nos bis, multiplica ter; tunc divide in quatuor partes. Quarta pars numeri, [93] si me addis cum ipsis, centenarium explet numerum. Dicat qui potest, quanti fuerunt, qui pridem obviaverunt ambulanti per viam?

48. proposition concerning the man who met [some] students.

A certain man met some students and asked them, "How many of you are there in school?" One of [the students] responded to him: "I do not want to tell you [except as follows]: double the number of us, then triple that number; then, divide that number into four parts. If you add me to one of the fourths, there will be 100." Let him say, he who can, How many [students] first met the man?

Solutio.

Terties ter bini [id est, bis xxxiii] fiunt lxvi: tanti erant, qui pridem obviaverunt ambulanti; qui numerus bis ductus cxxxii reddit. Hos multiplica ter, fiunt cccxcvi, horum quarta pars xcviiii sunt. Adde puerum respondentem et reperies c.

Solution.

Twice 33 makes 66; this is the number [of students] who first met the man. Twice this number yields 132, and three times this number gives 396, a quarter part of which is 99. Add in the responding boy and you will get

XLVIIII. propositio de carpentariis.
Septem carpentarii septenas rotas fecerunt. Dicat, qui potest, quot carrae rexerunt? [94]
49. proposition concerning the carpenters.
Seven carpenters [each] made seven wheels. Let him say, he who can, How many carts did they build?

Solutio.
Duc septies vii fiunt xlviiii, tot rotas fecerunt. xii vero quater ducti xlviii reddunt. Super xl et viiii rotas xii carra sunt erecta, et una superfuit rota.
Solution.
Take seven times seven, making 49, this being the number of wheels. 12 taken four times yields 48. 12 carts were assembled from the 49 wheels, with one wheel left over.

L. propositio de vino in vasculis.
Centum metra vini, rogo, ut dicat, qui vult, quot sextarios capiunt? vel ipsa etiam centum metra quot meros habent?
50. proposition concerning the wine in small vessels.
I ask so that one who wishes might respond: How many sextarii do 100 metra of wine contain, and how many meri do 100 metra have?

Solutio.
Unum metrum capit sectarios xl et viii. Duc centies xlviii, fiunt quatuor millia dccc. Tot sextarii sunt. Similiter et unum metrum habet meros cclxxxviiii, duc centies cclxxxviiii fiunt xxviii dcccc. Tot sunt meri.

Solution.
One metrum containes 48 sextarii. Take 48 a hundred times, making 4800 -- this is the number of sextarii [in 100 metra]. Likewise, one metrum contains 289 meri. 100 times 289 is 28,900 -- this is the number of meri [in 100 metra].

LI. propositio de vini in vasculis a quodam patre divisione. [95]
Quidam paterfamilias moriens dimisit [96] iiii filiis, iiii vascula vini: in primo vase erant modia xl, in secundo xxx, in tertio xx, et in quarto x; qui vocans dispensatorem domus suae ait: Haec quatuor vascula cum vino intrinsecus manente divide inter quatuor filios meos; sic tamen, ut unicuique eorum una [97] sit portio tam in vino, quam in vasis. Dicat, qui intelligit, quomodo dividendum est, ut omnes aequaliter ex hoc accipere possint?
51. proposition concerning the wine in small vessels divided by a certain father.
A certain dying father left four small vessels of wine to his four sons. In the first vessel, there were 40 modia [of wine]; in the second, 30; in the third, 20; and in the fourth, 10. Calling his house treasurer, he said:

"Divide these four vessels containing wine amongst my four sons in such a way that each son receives an equal portion of wine and vessels." Let him say, he who can, How must the vessels have been divided so that all [the sons] received an equal amount from this?

Solutio.
In primo siquidem vasculo fuerunt modia xl, in secundo xxx, in tertio xx, in quarto x. Junge igitur xl et xxx et xx et x, fiunt c. Tunc deinde centenarium idcirco numerum per quartam divide partem. Quarta namque pars centenarii xxv reperitur, qui numerus bis ductus quinquagenarium de se reddit numerum. Eveniunt ergo unicuique filio in portione sua xxv modia; et inter duos l. In primo xl, et in quarto sunt modii x, hi juncti faciunt l, hoc dabis inter duos. Similiter junge xxx et xx modia, quae fuerunt in secundo et tertio vascula, et fiunt l et hoc quoque, similiter ut superius, dabis inter duos, et habebunt singuli xxv modia; eritque id faciendo singulorum aequa filiorum divisio, tam in vino, quam et in vasis.
Solution.
In the first vessel, there were 40 modia [of wine]; in the second, 30; in the third, 20; and in the fourth, 10. Thus add 40 and 30 and 20 and 10, making 100. Then, divide 100 into four parts, by which 25 is ascertained. This number, taken twice, makes 50. Thus 25 modia go to each son as a portion, and between two [sons], 50 [modia]. In the first [vessel], there are 40 [modia], and in the fourth, 10. Together, these make 50 [modia], which you should divide among two [of the sons]. In a similar fashion, add the 30 and 20 modia which are in the second and third vessels, making 50 [modia]. As above, divide this among the two [other] sons, they will each have portions of 25 modia. By doing this, there shall be an equal division of wine and vessels between the sons.

LII. propositio de homine patrefamilias.
Quidam pater familias jussit xc modia frumenti de una domo sua ad alteram deportari; quae distabat leucas xxx: et vero ratione ut uno camelo totum illud frumentum deportaretur in tribus subvectionibus, [98] et in unaquaque leuca comedat [99] modium unum. Dicat, qui velit, quot modii residui fuissent? [100]
52. proposition concerning the head of household.
A certain head of household ordered that 90 modia of grain be taken from one of his houses to another 30 leagues away. Given that this load of grain can be carried by a camel in three trips, and that [the camel] eats one modium per league, Let him say, he who wishes, How many modia were left over [at the end of the transport]?

Solutio.
In prima subvectione portavit camelus modios xxx super leucas x, et comedit in unaquaque leuca modium unum, id est, modios xx comedit et remanserunt x. In secunda subvectione similiter deportavit modios xxx, et ex his comedit xx, et remanserunt x, in tertia vero subvectione fecit similiter; deportavit medios xxx, et ex his comedit xx, et remanserunt decem. Sunt vero de his, qui remanserunt, modia xxx, et de itinere leucae x. Quos xxx, in quarta subvectione domum detulit, et ex his x in itinere comedit, et remanserunt de tota illa summa modia tantum xx.

Solution.
On the first trip, the camel carried 30 modia for 10 leagues, eating a modium [of grain] per league; that is, it ate 20 modia, leaving 10. On the second trip it also carried 30 modia, eating 20, and leaving 10. On the third trip it did the same, carrying 30 modia, eating 20, and leaving 10. Thus there were 30 modia [of grain] remaining and 10 leagues of the journey. [The camel] carried these 30 [modia] in a fourth trip [101] to the house, of which it ate 10 [sic] on the way, leaving only 20 [sic] modia outof the original amount. [102]

LIII. propositio de homine patrefamilias monasterii xii monachorum.
Quidam Pater monasterii habuit xii monachos, qui vocans [103] dispensatorem domus suae dedit illis ova cciiii, jussitque, ut singulis aequalem daret ex eis portionem. Sic tamen jussit, ut inter v presbyteros daret ova lxxxv. [104] Dicat, rogo, qui valet, quot ova unicuique ipsorum in portionem venerunt, [105] ita ut in nullo nec superabundet numerus, nec minuatur; sed omnis, ut supra diximus, aequalem in omni accipiat portionem? [106]
53. proposition concerning the head of a monastery with 12 monks.
A certain Father of a monastery had 12 monks. Calling the treasurer of his chapter, he gave them [the priests] 204 eggs, and he ordered that [the treasurer] should give an equal portion to each individual. He further stipulated that [the treasurer] give 85 eggs to the five priests,[68 to the four deacons, and 51 to the three lectors]. Let him say, I ask he who can, How many eggs did each [monk] receive as his portion, so that no one received too many, nor too little, but so that as we stated above, he willtake an equal portion to all?

Solutio.
Ducentos igitur quatuor per xii partem divide. Horum quippe pars xii in septima decima resolvitur parte; quia sive duodecies xvii, sive decies septies xii miseris, cciiii reperies. Sicut enim octogenarius quintus numerus septimum decimum quinarium reddit numerum de se, ita et sexagenarius octavus quadrifarie, et quinquagesimus primus trifarie. Junge v et iiii et iii, fiunt xii. Isti sunt homines xii. Rursusque junge lxxxv et lxviii et li, fiunt cciiii. Haec sunt ova cciiii. Veniunt ergo singulorum ex his in partes ova xvii per duodecimam partem. Septimum decimum aequa lance dividi fiunt....
Solution.
Divide 204 into 12 parts. 12 parts of this leaves 17 in each part, because whether you take 12 times 17, or 17 times 12, you will arrive at 204. For just as the number 85 contains 17 parts of five within it, thus 68 [contains 17 parts] of four, and 51 [contains 17 parts] of three. Adding five and four and three makes 12 -- this is the number of men. Then, add 85 and 68 and 51, making 204 -- this is the number of eggs. Therefore, the eggs will be divided into 17 parts of 12 each. The 17 [parts] are then divided equally....

References

[1] As given in cod. ms. _Augiae Divitis_. Bede's title reads: "Incipiunt aliae propositiones ad

acuendos juvenes."

[2] Bede: "...in quot annis vel diebus..."

[3] One passus equals five feet.

[4] Bede: "De homine et aliis hominibus in via sibi obviantibus."

[5] Bede: "Utinam."

[6] Bede gives the following alternate solution: "Alia, 28 et 28, et tertio sic, fiunt 84, et medietas tertiae fiunt 14; sunt in totum 98: adjectis duabus, 100 apparent."

[7] Bede: "De duobus profiscientibus visis ciconiis."

[8] Bede gives this number as 28. Alcuin's solution is only Bede's alternative. Bede's first solution is as follows: "Qui primis ab illo visi sunt fuerunt 36, et hujus medietas medietatis sunt 18, et hujus numeri medietas sunt 9. Dic ergo sic, 72 et 18 fiunt 90; adde 9, fiunt 99; adde loquentem, et habebis 100."

[9] Bede adds "in campo pascentibus."

[10] Bede: "De emptore in denariis centum."

[11] Bede: "mercator."

[12] Bede: "...est adeptus."

[13] Bede: "...1400."

[14] Bede continues: "Decima pars sexagenarii, 6 sunt; decima vero quadragenarii, 4 sunt. Sive ergo decimam sexagenarii, sive decimam qua[d]ragenarii decies miseris, 100 portiones 6 cubitorum longas, et 4 cubitorum latas invenies."

[15] Bede: "De linteamine."

[16] Bede fails to give a solution for this problem.

[17] Bede provides no answer for this problem.

[18] Bede: "divisit."

[19] Bede's solution is in columnar form with the number of villages in one column, the number of people in the other. He gives values for all 30 villages. However, his answers are incorrect starting at v = 22, where the number 4,194,214 appears instead of the correct 4,194, 304.

[20] Bede: "In villa 1 fuerunt collecti milites 2."

[21] Bede: "facit."

[22] The correct answer should be seven. Bede answers properly, but his explanation is unsupported.

[23] I have translated "fratres" as men instead of brothers, since the men are only brothers relative to their respective sisters, not each other. If the men were indeed brothers, it would mean that they desired an incestuous relation with their own sisters-a situation I highly doubt Alcuin intended.

[24] Bede: "Tali igitur sicque sollicitante studio facta est navigatio, nullo fufcante inquinationis contagio."

[25] Bede: "transire."

[26] Bede continues: "Dicat qui potest quomodo eos illaesos ultra transiret."

[27] Bede: "ultra transirem."

[28] Bede: "hiriciis." I have been utterly unable to find any likely translation for this word.

[29] Grammatically, the present active participle "ponderantibus" modifies the man and woman. It does not seem likely, however, that the man and woman would weigh only a pound each. The topic of weight does not come up in the solution; thus, it is impossible to know the reasoning behind giving the weights of the subjects.

[30] Bede: "quod omnes."

[31] The meaning of "bis" here is not understood.

[32] Bede: "duxeris."

[33] Bede: "collocari."

[34] Bede: "aripennos."

[35] Bede: "deinde."

[36] Bede: "351."

[37] Bede: "Tot sunt in hujus aripenni numero."

[38] Bede: "namque."

[39] Bede: "novemdecem."

[40] Bede continues: "...et fiunt 2, et remanent 4, quae est 3 pars 12. Sunt ergo aripenni in hoc numero 2 et 3 pars de aripenno 3."

[41] 22.5 divided by 12 is 1.875.

[42] This should probably be "semel ipsos."

[43] Bede: "includit."

[44] Such a scenario implies that one aripennum equals 184.53 perticae.

[45] An anagram which substitutes vowels with following consonants. Thus, the heading should read "Propositio de cursu canis ac fuga leporis." Bede's heading is "De campo et cane ac fuga leporis."

[46] Bede: "post leporem currere."

[47] Bede: "confecerint."

[48] Bede: "persequendo."

[49] Bede continues: "...fiunt 526."

[50] Bede: "fiunt."

[51] Bede: "...in longo."

[52] Bede: "duc."

[53] Bede: "Volo ut fiat ibi domorum constuctio..."

[54] Bede: "de."

[55] Bede: "capienda."

[56] Bede: "constitui."

[57] Bede: "1512."

[58] The correct answer should be 241.5 feet. Notice, too, that the length of the basilica has changed from 240 feet to 140 feet. Since Alcuin's (and Bede's) figures are inconsistent, his final answer will be wrong as well. The final number of tiles needed, assuming a length of 240 feet, should be 1253; assuming a length of 140 feet, 731.

[59] Bede: "cavana."

[60] Bede: "...ut unaquaeque cupa habeat in longitudine pedes 7 et in lato pedes 4, et pervius unus habeat pedes 4, et unaquaeque cupa habeat pedes 7."

[61] Bede: "deputantur."

[62] Bede: "cavanae."

[63] Bede: "10."

[64] Bede: "cavana."

[65] If the aisle runs down the middle of the cellar, only 196 casks can be stored.

[66] This should no doubt be the passive infinitive "dari." See problem 33.

[67] Bede: "accipiant."

[68] Bede: "...quotve infantes fuerunt."

[69] Bede: "modios."

[70] Bede: "modii."

[71] Bede continues "...tantum 36."

[72] Bede continues as follows: "...duc vero octogies quatuor semis, fiunt 37, id est 11 acceperunt 17, quod simul..."

[73] Bede: "fiunt."

[74] Bede: "860."

[75] Bede: "nasceretur."

[76] Bede: "quinque."

[77] Bede's heading reads "De animalibus emptis," which is clearly incorrect. Such a heading would seem to be appropriate for problem 38 or 39.

[78] At this point in Bede's text, the scribe apparently no longer saw any reason for providing answers, saying: "Reliquae solutiones desiderantur: potest autem quisque ratione arithmetica propositiones illas solvere; ita ad exercendum ingenium omissa valebant."

[79] Bede: "De animalibus emptis."

[80] Bede: "35."

[81] Bede: "...et de hac medietate aliam idcirco medietatem..."

[82] Bede: "...curtem novam quadrangulam..."

[83] Bede: "quia."

[84] 8 x 4096 = 32,768.

[85] 32,768 x 8 = 262,144.

[86] Bede: "occiderentur."

[87] Bede: "triplicabis."

[88] Bede: "...aliae tantae et adhuc tantae."

[89] Bede: "...inventionis tuae."

[90] 65 x 72 = 4680.

[91] There are two possible alternate interpretations here. Working backwards: 10,800/2 = 5400; 5400/72 = 75, while 5400/65 = 83.076923. Since Alcuin probably intended only to deal with

whole numbers, it is reasonable to assume that 65 is a mistake for the correct figure of 75. However, is 75 the number of pounds in a talent, or the number of gold solidi in a pound?

[92] Bede: "scholariis."

[93] Bede: "nostrum."

[94] Bede: "carra fecerunt."

[95] Bede: "De patre familias distribuente."

[96] Bede: "divisit."

[97] Bede: "aequalis."

[98] Bede's presentation is slightly different: "Quidam paterfamilias habebat de una domo sua ad alteram domum leucas 30, et habens camelum qui debebat in tribus subjectionibus ex una domo sua ad alteram de annona fere modia 90..."

[99] Bede: "comedebat."

[100] Bede: "...modia residua fuerint."

[101] A fourth trip contradicts the earlier statement that the entire transport can be completed in only three trips.

[102] The camel carried the remaining 30 modia for 20 leagues. At the rate of one modium per league, only 10 modia would reach the intended destination.

[103] Bede: "convocans."

[104] Bede continues: "...et inter 4 diaconos 68, et inter tres [lectores] 51."

[105] Bede: "evenerunt."

[106] Bede: "...sed omnes, ut supra diximus, aequalem in omnibus accipiant portionem."