

Algoritmus(de Casteljaou, pro trojúhelníkové Bezierovy segmenty)

Vstup Bezierovy body $\mathbf{b}_{i,j,k}, i, j, k \geq 0, i + j + k = n$, parametrická hodnota (u^*, v^*) .

Výstup: Bod křivky $\mathbf{b}(u^*, v^*)$

Postup:

Begin

FOR i:=0 TO n DO $\mathbf{b}_i^0 := \mathbf{b}_i$;

FOR j:=1 TO n-i DO

Begin

k:=n-i-j;

$\mathbf{b}_{i,j,k}^0 := \mathbf{b}_{i,j,k}$

END;

FOR l:=1 TO n DO

FOR i:=1 TO n-l DO

FOR j:=1 TO n-l-i DO

Begin

k:=n-l-i-j;

$\mathbf{b}_{i,j,k}^l := u^* \mathbf{b}_{i+1,j,k}^{l-1} + v^* \mathbf{b}_{i,j+1,k}^{l-1} + (1-u^*-v^*) \mathbf{b}_{i,j,k+1}^{l-1}$

END

$\mathbf{b}(u^*, v^*) := \mathbf{b}_n^{0,0,0}$

END.

Pro pomocné body $\mathbf{b}_{i,j,k}^l$ platí: $\mathbf{b}_{i,j,k}^l =$

$$\sum_{i^*+j^*+k^*=l} \mathbf{b}_{i+i^*,j+j^*,k+k^*}^0 B_{i^*,j^*,k^*}^l(u^*, v^*, 1-u^*-v^*)$$

.

Důkaz indukci vzhledem k l .

1. $l = 0$: $\mathbf{b}_{i,j,k}^0 = \mathbf{b}_{i,j,k}^0 \frac{0!}{0!0!0!} (u^*)^0 (v^*)^0 (1-u^*-v^*)^0 = \mathbf{b}_{i,j,k}^0 B_{0,0,0}^0(u^*, v^*, 1-u^*-v^*)$.

$i^* + j^* + k^* = 0$

2. předpokládejme, že vzorec platí pro $(l-1)$

platí:

$$\mathbf{b}_{i,j,k}^{l-1} = \sum_{i^*+j^*+k^*=l-1} \mathbf{b}_{i+i^*,j+j^*,k+k^*}^0 B_{i^*,j^*,k^*}^{l-1}(u^*, v^*, 1-u^*-v^*).$$

$$\begin{aligned}
\mathbf{b}_{i,j,k}^l &= u^* \mathbf{b}_{i+1,j,k}^{l-1} + v^* \mathbf{b}_{i,j+1,k}^{l-1} + (1 - u^* - v^*) \mathbf{b}_{i,j,k+1}^{l-1} = \\
&= u^* \left(\sum_{i^*+j^*+k^*=l-1} \mathbf{b}_{i+1+i^*,j+j^*,k+k^*}^0 B_{i^*,j^*,k^*}^{l-1}(u^*, v^*, 1 - u^* - v^*) \right) \\
&+ v^* \left(\sum_{i^*+j^*+k^*=l-1} \mathbf{b}_{i+i^*,j+1+j^*,k+k^*}^0 B_{i^*,j^*,k^*}^{l-1}(u^*, v^*, 1 - u^* - v^*) \right) + \\
&+ (1 - u^* - v^*) \left(\sum_{i^*+j^*+k^*=l-1} \mathbf{b}_{i+i^*,j+j^*,k+1+k^*}^0 B_{i^*,j^*,k^*}^{l-1}(u^*, v^*, 1 - u^* - v^*) \right) = \\
&= \sum_{i'+j'+k'=l} u^* \mathbf{b}_{i+i',j+j',k+k'}^0 B_{i'-1,j',k'}^{l-1}(u^*, v^*, 1 - u^* - v^*) + \\
&+ v^* \mathbf{b}_{i+i',j+j',k+k'}^0 B_{i',j'-1,k'}^{l-1}(u^*, v^*, 1 - u^* - v^*) + \\
&+ (1 - u^* - v^*) \mathbf{b}_{i+i',j+j',k+k'}^0 B_{i',j',k'-1}^{l-1}(u^*, v^*, 1 - u^* - v^*) \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 u^* B_{i'-1,j',k'}^{l-1}(u^*, v^*, 1 - u^* - v^*) + \\
&+ v^* B_{i',j'-1,k'}^{l-1}(u^*, v^*, 1 - u^* - v^*) + (1 - u^* - v^*) B_{i',j',k'-1}^{l-1}(u^*, v^*, 1 - u^* - v^*) \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 [u^* \frac{(l-1)!}{(i'-1)!j'!k'!} (u^*)^{i'-1} (v^*)^{j'} (1 - u^* - v^*)^{k'} + \\
&+ v^* \frac{(l-1)!}{i'!(j'-1)!k'!} (u^*)^{i'} (v^*)^{j'-1} (1 - u^* - v^*)^{k'} + \\
&+ (1 - u^* - v^*) \frac{(l-1)!}{i'!j'!(k'-1)!} (u^*)^{i'} (v^*)^{j'} (1 - u^* - v^*)^{k'-1} \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 (u^*)^{i'} (v^*)^{j'} (1 - u^* - v^*)^{k'} \left[\frac{(l-1)!}{(i'-1)!j'!k'!} + \frac{(l-1)!}{i'!(j'-1)!k'!} + \frac{(l-1)!}{i'!j'!(k'-1)!} \right] \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 (u^*)^{i'} (v^*)^{j'} (1 - u^* - v^*)^{k'} \frac{(l-1)!}{(i'-1)!(j'-1)!(k'-1)!} \left[\frac{1}{j'k'} + \frac{1}{i'k'} + \frac{1}{i'j'} \right] \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 (u^*)^{i'} (v^*)^{j'} (1 - u^* - v^*)^{k'} \frac{(l-1)!}{(i'-1)!(j'-1)!(k'-1)!} \frac{i'+j'+k'}{i'j'k'} \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 (u^*)^{i'} (v^*)^{j'} (1 - u^* - v^*)^{k'} \frac{l!}{i'!j'!k'!} \\
&= \sum_{i'+j'+k'=l} \mathbf{b}_{i+i',j+j',k+k'}^0 B_{i',j',k'}^l(u^*, v^*, 1 - u^* - v^*)
\end{aligned}$$