**Věta (Výpočet torze).** *Pro křivku při parametrické reprezentaci $x = x(t), y = y(t), z = z(t)$ v prostoru platí*

$$\tau = \frac{\rho^2}{(x'^2 + y'^2 + z'^2)^3} \begin{vmatrix} x' & y' & z' \\ x'' & y'' & z'' \\ x''' & y''' & z''' \end{vmatrix}$$

*Důkaz.*

$$\mathbf{b} = \mathbf{t} \times \mathbf{n}$$

**t** ........normovaný tečný vektor $\mathbf{t} = \dot{k} = \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$

**n** ........hlavní normála $\mathbf{n} = \frac{\ddot{k}}{\|\ddot{k}\|} = \frac{1}{|\ddot{k}|} \begin{pmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{pmatrix}$

k ........funkce délky oblouku s

Z toho vyplývá:

$$\mathbf{b} = \mathbf{t} \times \mathbf{n} = \frac{\dot{k} \times \ddot{k}}{\|\ddot{k}\|} = \frac{1}{\|\ddot{k}\|} \left( \begin{vmatrix} \dot{y} & \dot{z} \\ \ddot{y} & \ddot{z} \end{vmatrix}, \quad - \begin{vmatrix} \dot{x} & \dot{z} \\ \ddot{x} & \ddot{z} \end{vmatrix}, \quad \begin{vmatrix} \dot{x} & \dot{y} \\ \ddot{x} & \ddot{y} \end{vmatrix} \right)^T$$

První souřadnice:

$$\frac{\begin{vmatrix} \dot{y} & \dot{z} \\ \ddot{y} & \ddot{z} \end{vmatrix}}{\|\ddot{k}\|} = \frac{\dot{y}\ddot{z} - \ddot{y}\dot{z}}{\|\ddot{k}\|} = \frac{\dot{y}\ddot{z} - \ddot{y}\dot{z}}{\sqrt{\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2}}$$

Derivace:

$$\dot{\mathbf{b}} = \frac{(\ddot{y}\ddot{z} + \dot{y}\dddot{z} - \dddot{y}\dot{z} - \ddot{y}\ddot{z}) \|\ddot{k}\| - (\dot{y}\ddot{z} - \ddot{y}\dot{z})\, 2\, (\ddot{x}\dddot{x} + \ddot{y}\dddot{y} + \ddot{z}\dddot{z})}{2 \|\ddot{k}\| (\ddot{x}^2 + \ddot{y}^2 + \ddot{z}^2)} =$$

$$= \frac{\begin{vmatrix} \dot{y} & \dddot{y} \\ \dot{z} & \dddot{z} \end{vmatrix}}{\|\ddot{k}\|} - \frac{\begin{vmatrix} \dot{y} & \ddot{y} \\ \dot{z} & \ddot{z} \end{vmatrix}}{\|\ddot{k}\|^3} (\ddot{x}\dddot{x} + \ddot{y}\dddot{y} + \ddot{z}\dddot{z}) = \frac{\dot{k} \times \dddot{k}}{\|\ddot{k}\|} - \frac{\dot{k} \times \ddot{k}}{\|\ddot{k}\|^3} \langle \ddot{k}, \dddot{k} \rangle$$

□