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# Measuring the Quality of Credit Scoring Models

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August 2009

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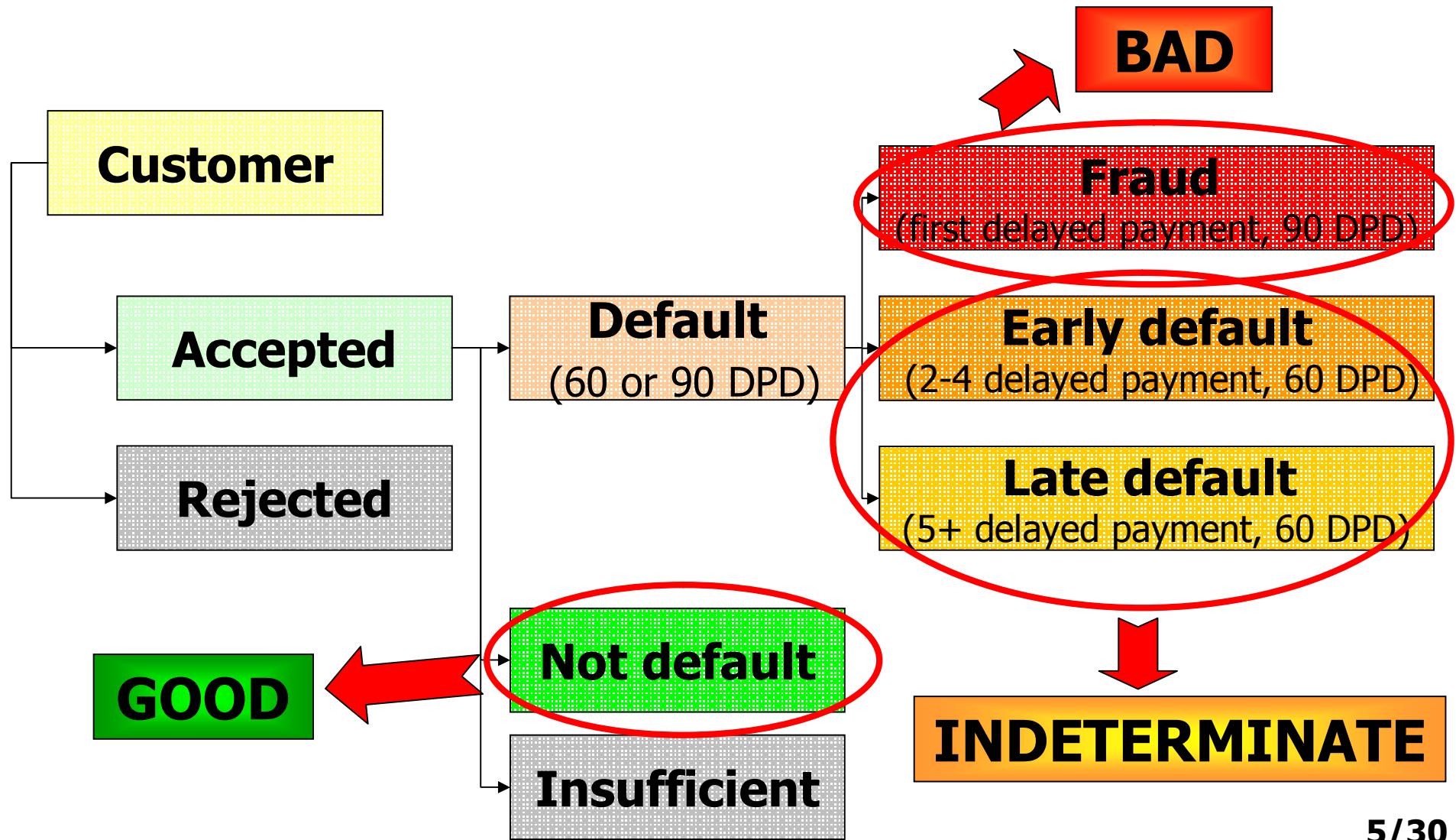
# Introduction

- It is impossible to use scoring model effectively without knowing how good it is.
- Usually one has several scoring models and needs to select just one. The best one.
- Before measuring the quality of models one should know (among other things):
  - good/bad definition
  - expected reject rate

# Good/bad client definition

- Good definition is the basic condition of effective scoring model.
- The definition usually depends on:
  - days past due (DPD)
  - amount past due
  - time horizon
- Generally we consider following types of client:
  - Good
  - Bad
  - Indeterminate
  - Insufficient
  - Excluded
  - Rejected.

# Good/bad client definition



# Measuring the quality

- Once the definition of good / bad client and client's score is available, it is possible to evaluate the quality of this score. If the score is an output of a predictive model (scoring function), then we evaluate the quality of this model. We can consider two basic types of quality indexes. First, indexes based on cumulative distribution function like
  - Kolmogorov-Smirnov statistics (KS)
  - Gini index
  - C-statistics
  - Lift.

The second, indexes based on likelihood density function like

- Mean difference (Mahalanobis distance)
- Informational statistics/value ( $I_{\text{Val}}$ ).

# Indexes based on distribution function

$$D_K = \begin{cases} 1, & \text{client is good} \\ 0, & \text{otherwise.} \end{cases}$$

Number of good clients:  $n$

Number of bad clients:  $m$

Proportions of good/bad clients:  $p_G = \frac{n}{n+m}$ ,  $p_B = \frac{m}{n+m}$

➤ Empirical distribution functions:

$$F_{n,GOOD}(a) = \frac{1}{n} \sum_{i=1}^n I(s_i \leq a \wedge D_K = 1)$$

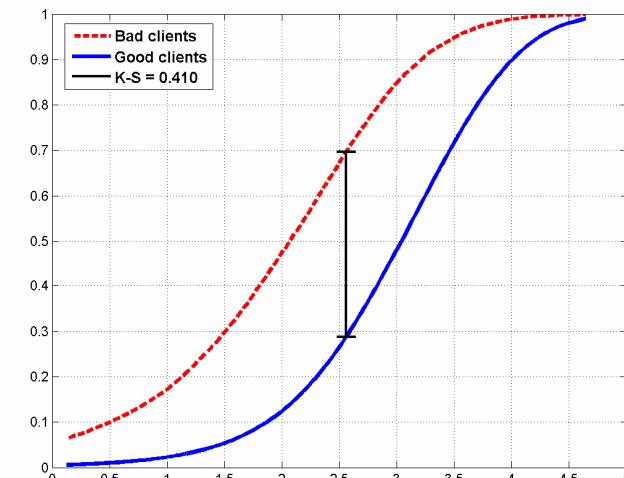
$$F_{m,BAD}(a) = \frac{1}{m} \sum_{i=1}^m I(s_i \leq a \wedge D_K = 0)$$

$$F_{N,ALL}(a) = \frac{1}{N} \sum_{i=1}^N I(s_i \leq a) \quad a \in [L, H]$$

$$I(A) = \begin{cases} 1 & A \text{ is true} \\ 0 & \text{otherwise} \end{cases}$$

➤ Kolmogorov-Smirnov statistics (KS)

$$KS = \max_{a \in [L, H]} |F_{m,BAD}(a) - F_{n,GOOD}(a)|$$



# Indexes based on distribution function

- Lorenz curve (LC)

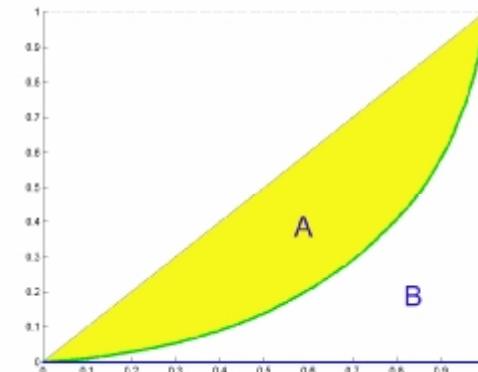
$$x = F_{m.BAD}(a)$$

$$y = F_{n.GOOD}(a), a \in [L, H].$$

- Gini index

$$Gini = \frac{A}{A + B} = 2A$$

$$Gini = 1 - \sum_{k=2}^{n+m} (F_{m.BAD_k} - F_{m.BAD_{k-1}}) \cdot (F_{n.GOOD_k} + F_{n.GOOD_{k-1}})$$

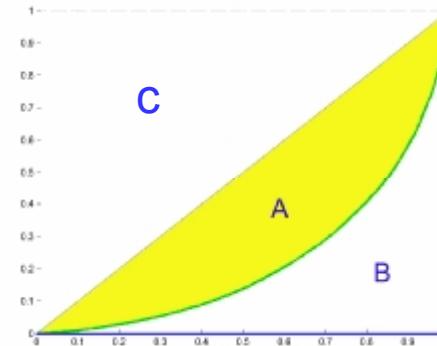


where  $F_{m.BAD_k}$  ( $F_{n.GOOD_k}$ ) is k-th vector value of empirical distribution function of bad (good) clients

# Indexes based on distribution function

- C-statistics:

$$c-stat = A + C = \frac{1+Gini}{2}$$



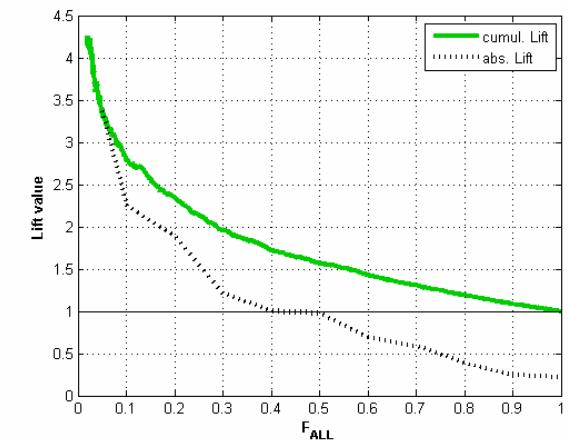
It represents the likelihood that randomly selected good client has higher score than randomly selected bad client, i.e.

$$c-stat = P(s_1 \geq s_2 \mid D_{K_1} = 1 \wedge D_{K_2} = 0)$$

# Indexes based on distribution function

□ Another possible indicator of the quality of scoring model can be *cumulative Lift*, which says, how many times, at a given level of rejection, is the scoring model better than random selection (random model). More precisely, the ratio indicates the proportion of bad clients with less than a score  $a$ ,  $a \in [L, H]$  , to the proportion of bad clients in the general population. Formally, it can be expressed by:

$$Lift(a) = \frac{CumBadRate(a)}{BadRate} = \frac{\frac{\sum_{i=1}^{n+m} I(s_i \leq a \wedge Y = 0)}{\sum_{i=1}^{n+m} I(s_i \leq a)}}{\frac{\sum_{i=1}^{n+m} I(Y = 0)}{\sum_{i=1}^{n+m} I(Y = 0 \vee Y = 1)}} = \frac{\frac{\sum_{i=1}^{n+m} I(s_i \leq a \wedge Y = 0)}{\sum_{i=1}^{n+m} I(s_i \leq a)}}{\frac{n}{N}}$$

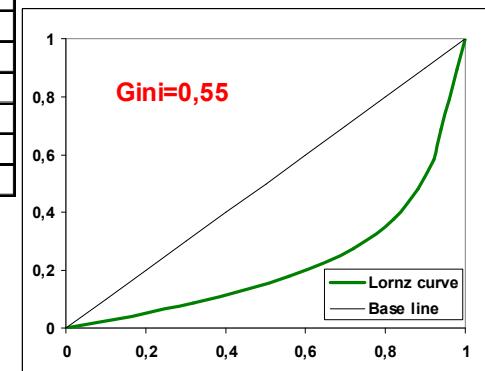
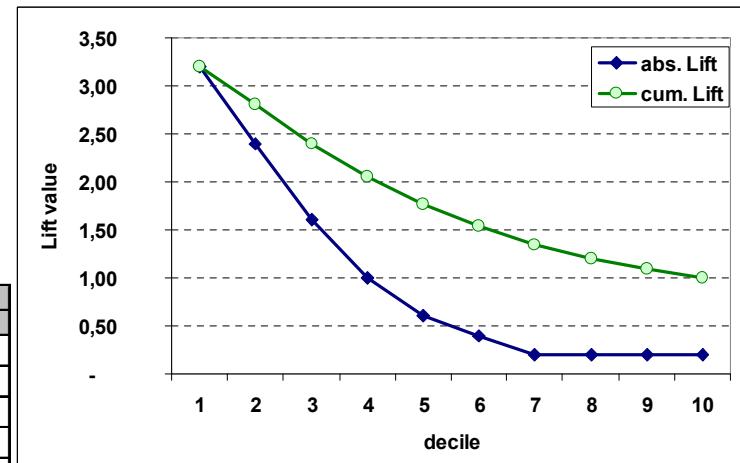


$$absLift(a) = \frac{BadRate(a)}{BadRate}$$

# Indexes based on distribution function

- Usually it is computed using table with numbers of all and bad clients in some bands (deciles).

decile	# cleints	absolutely			cumulatively		
		# bad clients	Bad rate	abs. Lift	# bad clients	Bad rate	cum. Lift
1	100	16	16,0%	3,20	16	16,0%	3,20
2	100	12	12,0%	2,40	28	14,0%	2,80
3	100	8	8,0%	1,60	36	12,0%	2,40
4	100	5	5,0%	1,00	41	10,3%	2,05
5	100	3	3,0%	0,60	44	8,8%	1,76
6	100	2	2,0%	0,40	46	7,7%	1,53
7	100	1	1,0%	0,20	47	6,7%	1,34
8	100	1	1,0%	0,20	48	6,0%	1,20
9	100	1	1,0%	0,20	49	5,4%	1,09
10	100	1	1,0%	0,20	50	5,0%	1,00
All	1000	50	5,0%				

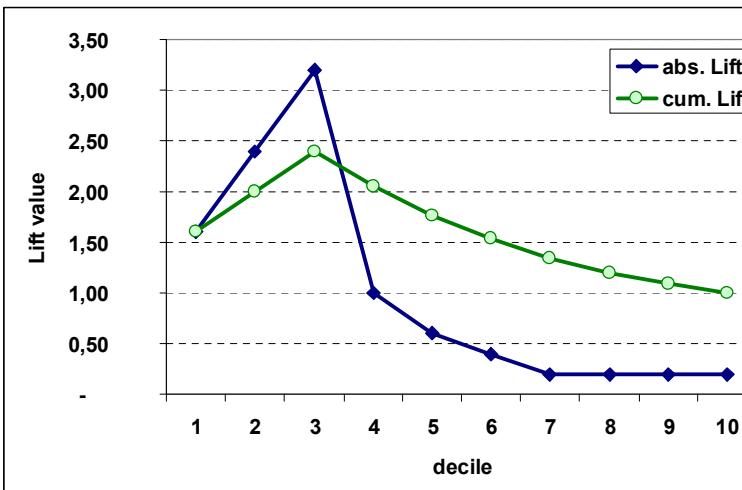
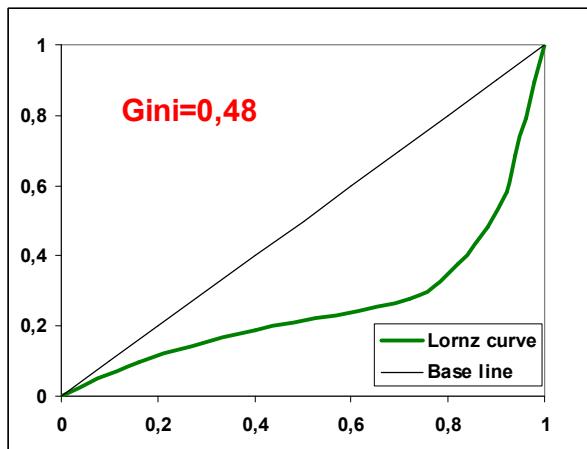


# Indexes based on distribution function

- When bad rates are not monotone:

- LC looks fine
- Gini is slightly lowered
- Lift looks strange

decile	# cleints	absolutely			cumulatively		
		# bad clients	Bad rate	abs. Lift	# bad clients	Bad rate	cum. Lift
1	100	8	8,0%	1,60	8	8,0%	1,60
2	100	12	12,0%	2,40	20	10,0%	2,00
3	100	16	16,0%	3,20	36	12,0%	2,40
4	100	5	5,0%	1,00	41	10,3%	2,05
5	100	3	3,0%	0,60	44	8,8%	1,76
6	100	2	2,0%	0,40	46	7,7%	1,53
7	100	1	1,0%	0,20	47	6,7%	1,34
8	100	1	1,0%	0,20	48	6,0%	1,20
9	100	1	1,0%	0,20	49	5,4%	1,09
10	100	1	1,0%	0,20	50	5,0%	1,00
All	1000	50	5,0%				



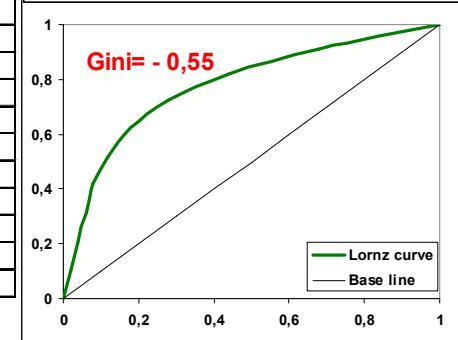
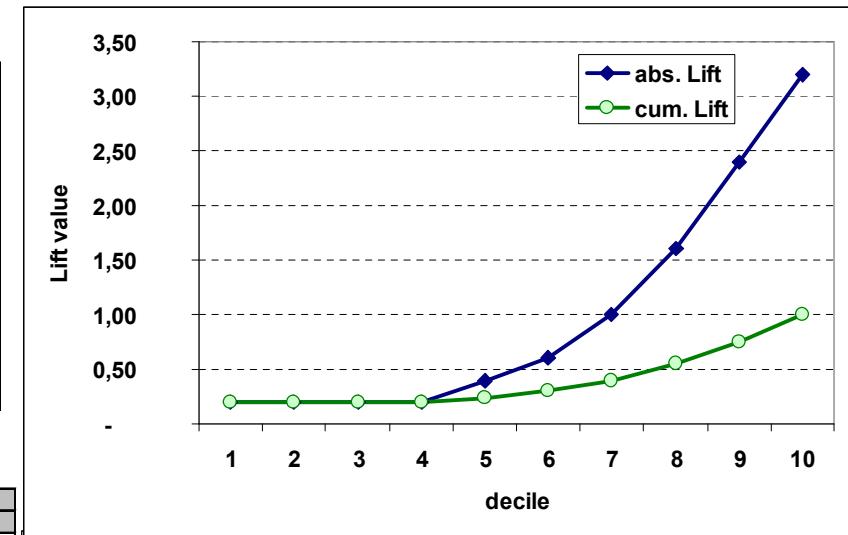
# Indexes based on distribution function

- When score is reversed, we obtain reversed figures.

decile	# cleints	absolutely			cumulatively		
		# bad clients	Bad rate	abs. Lift	# bad clients	Bad rate	cum. Lift
1	100	16	16,0%	3,20	16	16,0%	3,20
2	100	12	12,0%	2,40	28	14,0%	2,80
3	100	8	8,0%	1,60	36	12,0%	2,40
4	100	5	5,0%	1,00	41	10,3%	2,05
5	100	3	3,0%	0,60	44	8,8%	1,76
6	100	2	2,0%	0,40	46	7,7%	1,53
7	100	1	1,0%	0,20	47	6,7%	1,34
8	100	1	1,0%	0,20	48	6,0%	1,20
9	100	1	1,0%	0,20	49	5,4%	1,09
10	100	1	1,0%	0,20	50	5,0%	1,00
All	1000	50	5,0%				



decile	# cleints	absolutely			cumulatively		
		# bad clients	Bad rate	abs. Lift	# bad clients	Bad rate	cum. Lift
1	100	1	1,0%	0,20	1	1,0%	0,20
2	100	1	1,0%	0,20	2	1,0%	0,20
3	100	1	1,0%	0,20	3	1,0%	0,20
4	100	1	1,0%	0,20	4	1,0%	0,20
5	100	2	2,0%	0,40	6	1,2%	0,24
6	100	3	3,0%	0,60	9	1,5%	0,30
7	100	5	5,0%	1,00	14	2,0%	0,40
8	100	8	8,0%	1,60	22	2,8%	0,55
9	100	12	12,0%	2,40	34	3,8%	0,76
10	100	16	16,0%	3,20	50	5,0%	1,00
All	1000	50	5,0%				

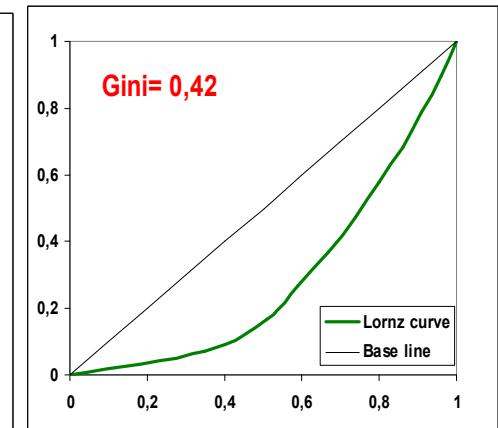
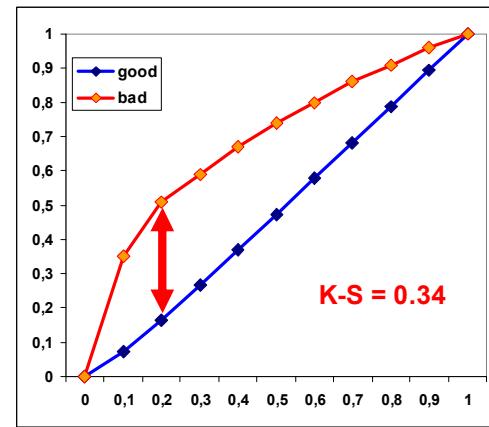


# Indexes based on distribution function

□ The Gini is not enough!!!

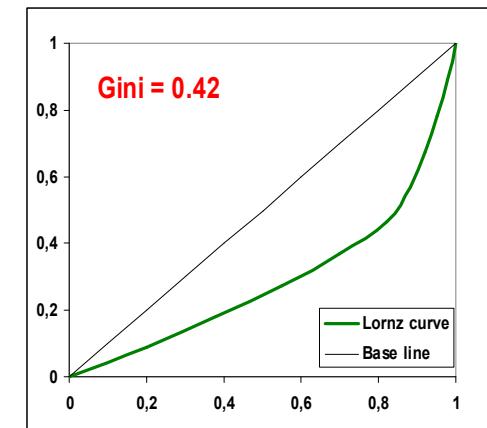
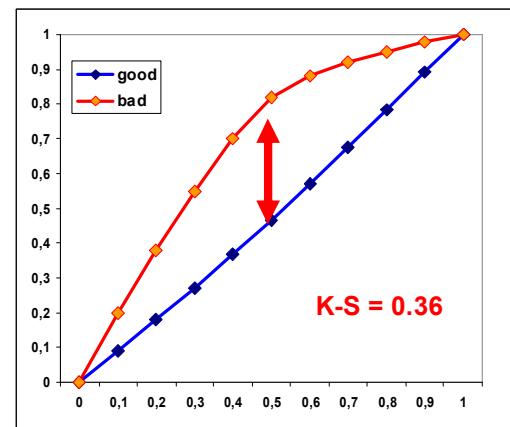
➤ SC 1:

decile	# cleints	# bad clients	Bad rate
1	100	35	35,0%
2	100	16	16,0%
3	100	8	8,0%
4	100	8	8,0%
5	100	7	7,0%
6	100	6	6,0%
7	100	6	6,0%
8	100	5	5,0%
9	100	5	5,0%
10	100	4	4,0%
All	1000	100	10,0%



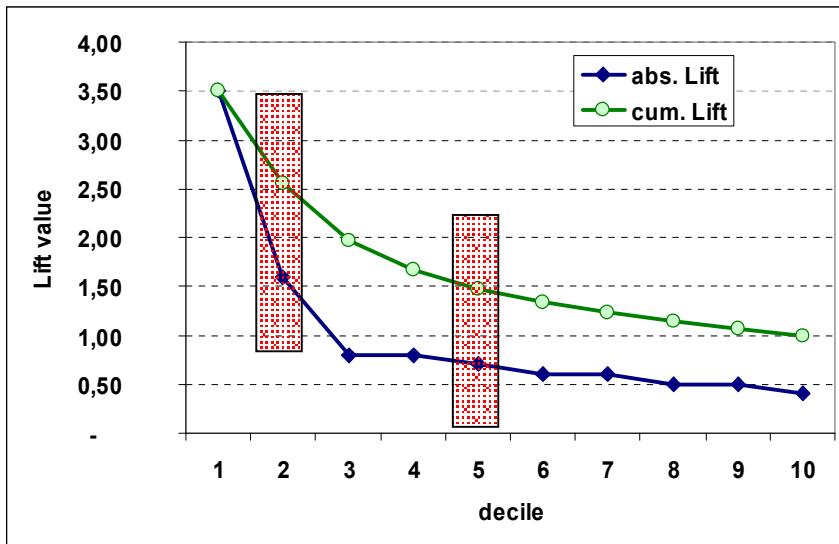
➤ SC 2:

decile	# cleints	# bad clients	Bad rate
1	100	20	20,0%
2	100	18	18,0%
3	100	17	17,0%
4	100	15	15,0%
5	100	12	12,0%
6	100	6	6,0%
7	100	4	4,0%
8	100	3	3,0%
9	100	3	3,0%
10	100	2	2,0%
All	1000	100	10,0%

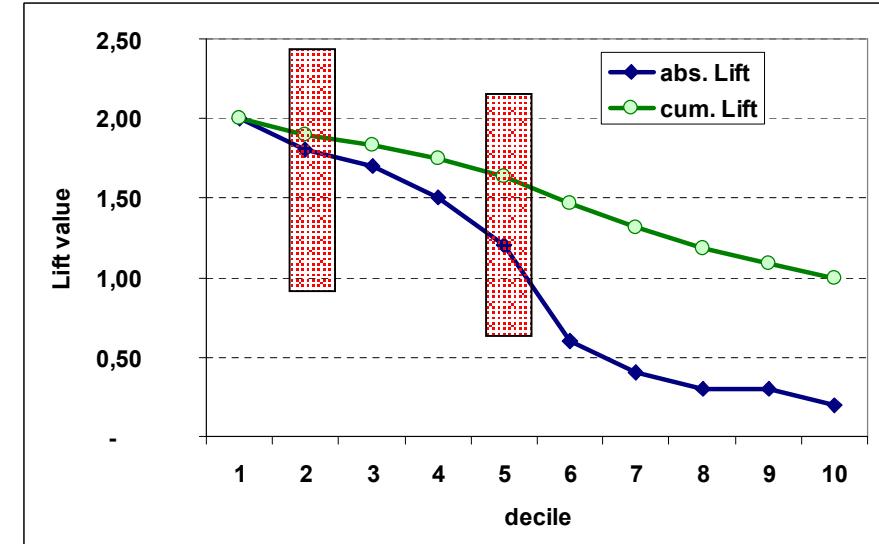


# Indexes based on distribution function

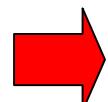
➤ SC 1:



➤ SC 2:



$$\begin{aligned} \text{Lift}_{20\%} &= 2.55 &> & \text{Lift}_{20\%} = 1.90 \\ \text{Lift}_{50\%} &= 1.48 &< & \text{Lift}_{50\%} = 1.64 \end{aligned}$$



SC 2 is better if reject rate is expected around 50%.

SC 1 is much more better if reject rate is expected by 20%.

# Indexes based on distribution function

- Lift can be expressed and computed by formulae:

$$Lift(a) = \frac{F_{n.BAD}(a)}{F_{N.ALL}(a)} \quad a \in [L, H]$$

$$Lift_q = \frac{F_{n.BAD}(F_{N.ALL}^{-1}(q))}{F_{N.ALL}(F_{N.ALL}^{-1}(q))} = \frac{1}{q} F_{n.BAD}\left(F_{N.ALL}^{-1}(q)\right)$$

$$F_{N.ALL}^{-1}(q) = \min\{a \in [L, H], F_{N.ALL}(a) \geq q\}$$

$$Lift_{10\%} = 10 \cdot F_{n.BAD}\left(F_{N.ALL}^{-1}(0.1)\right).$$

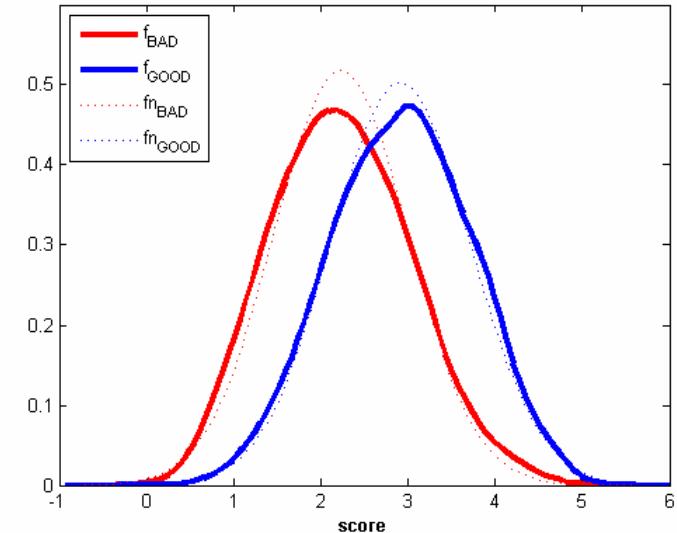
# Indexes based on density function

- Likelihood density functions:  $f_{GOOD}(x)$     $f_{BAD}(x)$
- Kernel estimates:  $\tilde{f}_{GOOD}(x, h) = \sum_{\substack{i=1, \\ D_k=1}}^n \frac{1}{n} K_h(x - s_i)$     $\tilde{f}_{BAD}(x, h) = \sum_{\substack{i=1, \\ D_k=0}}^m \frac{1}{m} K_h(x - s_i)$
- Optimal bandwidth (maximal smoothing):

$$h_{OS,k} = \left[ \frac{(2k+1)! k (2k+5)^{k+3/2}}{(2k+3)!} \right]^{\frac{1}{2k+1}} \cdot \tilde{\sigma} \cdot n^{\frac{1}{2k+1}}$$

where:

- $k$  is the order of kernel function  
(e.g. 2 for Epanechnikov kernel)
- $n$  is number of actual cases
- $\tilde{\sigma}$  is an estimate of standard deviation



# Indexes based on density function

- Mean difference (Mahalanobis distance):

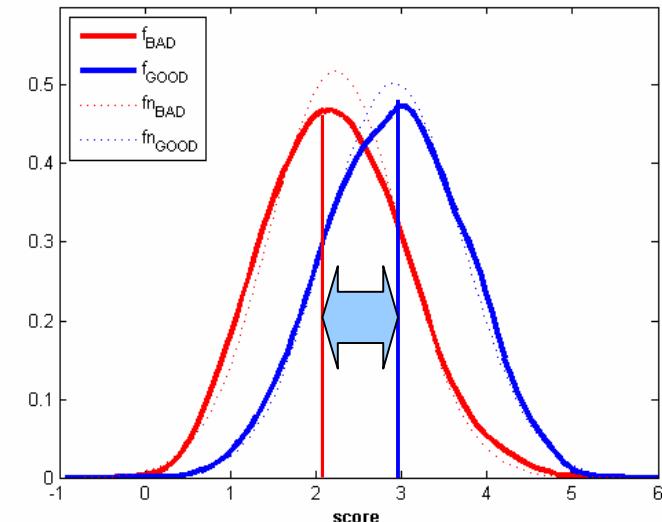
$$D = \frac{M_g - M_b}{S}$$

where  $S$  is pooled standard deviation:

$$S = \left( \frac{nS_g^2 + mS_b^2}{n+m} \right)^{\frac{1}{2}}$$

$M_g, M_b$  are means of good (bad) clients

$S_g, S_b$  are standard deviations of good (bad) clients



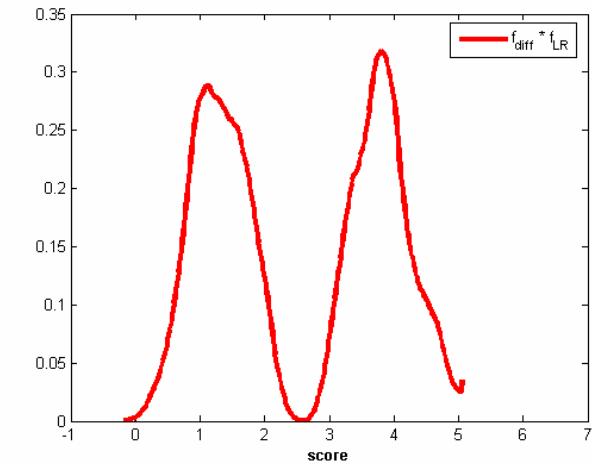
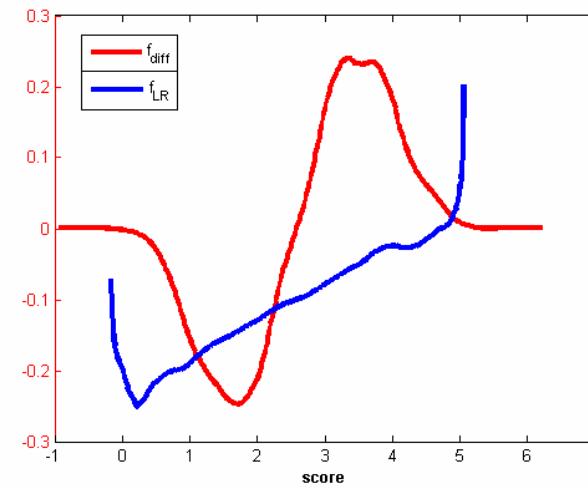
# Indexes based on density function

- Information value ( $I_{val}$ ) – continuous case (Divergence):

$$I_{val} = \int_{-\infty}^{\infty} (f_{GOOD}(x) - f_{BAD}(x)) \ln \left( \frac{f_{GOOD}(x)}{f_{BAD}(x)} \right) dx$$

$$f_{diff}(x) = f_{GOOD}(x) - f_{BAD}(x)$$

$$f_{LR}(x) = \ln \left( \frac{f_{GOOD}(x)}{f_{BAD}(x)} \right)$$

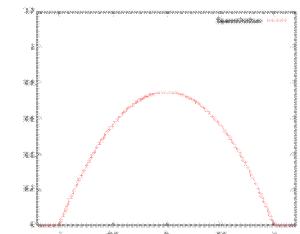


# Indexes based on density function

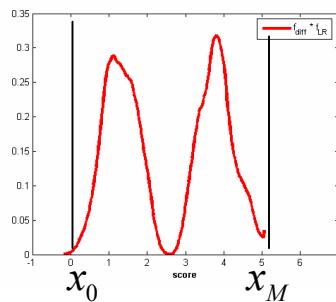
## ➤ Information value ( $I_{val}$ ) – discretized continuous case:

- Replace density functions by their kernel estimates and compute integral numerically (e.g. by composite trapezoidal rule).
- Using Epanechnikov kernel, given by  $K(x) = \frac{3}{4}(1-x^2) \cdot I(x \in [-1, 1])$  and optimal bandwidth  $h_{OS,k}$  we have

$$\tilde{f}_{IV}(x) = (\tilde{f}_{GOOD}(x, h_{OS,2}) - \tilde{f}_{BAD}(x, h_{OS,2})) \ln \left( \frac{\tilde{f}_{GOOD}(x, h_{OS,2})}{\tilde{f}_{BAD}(x, h_{OS,2})} \right)$$



- For given  $M+1$  points  $x_0, \dots, x_M$  we obtain



$$I_{val} = \frac{x_M - x_0}{2M} \left( \tilde{f}_{IV}(x_0) + 2 \sum_{i=1}^{M-1} \tilde{f}_{IV}(x_i) + \tilde{f}_{IV}(x_M) \right)$$

# Indexes based on density function

## ➤ Information statistics/value ( $I_{val}$ ) – discrete case:

- Create intervals of score – typically deciles. Number of goods (bads) in i-th interval is marked by  $g_i (b_i)$ .
- It must holds  $g_i > 0, b_i > 0 \quad \forall i$

- Then we have

$$I_{val} = \sum_i \left( \frac{g_i}{n} - \frac{b_i}{m} \right) \ln \left( \frac{g_i m}{b_i n} \right)$$

score int.	# bad clients	#good clients	% bad [1]	% good [2]	[3] = [2] - [1]	[4] = [2] / [1]	[5] = ln[4]	[6] = [3] * [5]
1	1	10	2,0%	1,1%	-0,01	0,53	-0,64	0,01
2	2	15	4,0%	1,6%	-0,02	0,39	-0,93	0,02
3	8	52	16,0%	5,5%	-0,11	0,34	-1,07	0,11
4	14	93	28,0%	9,8%	-0,18	0,35	-1,05	0,19
5	10	146	20,0%	15,4%	-0,05	0,77	-0,26	0,01
6	6	247	12,0%	26,0%	0,14	2,17	0,77	0,11
7	4	137	8,0%	14,4%	0,06	1,80	0,59	0,04
8	3	105	6,0%	11,1%	0,05	1,84	0,61	0,03
9	1	97	2,0%	10,2%	0,08	5,11	1,63	0,13
10	1	48	2,0%	5,1%	0,03	2,53	0,93	0,03
All	50	950				Info. Value	0,68	

# Indexes based on density function

- Information value for our example of two scorecards:

➤ **SC 1:**

decile	# cleints	# bad clients	#good	% bad [1]	% good [2]	[3] = [2] - [1]	[4] = [2] / [1]	[5] = ln[4]	[6] = [3] * [5]	cum. [6]
1	100	35	65	35,0%	7,2%	-0,28	0,21	-1,58	0,44	0,44
2	100	16	84	16,0%	9,3%	-0,07	0,58	-0,54	0,04	0,47
3	100	8	92	8,0%	10,2%	0,02	1,28	0,25	0,01	0,48
4	100	8	92	8,0%	10,2%	0,02	1,28	0,25	0,01	0,49
5	100	7	93	7,0%	10,3%	0,03	1,48	0,39	0,01	0,50
6	100	6	94	6,0%	10,4%	0,04	1,74	0,55	0,02	0,52
7	100	6	94	6,0%	10,4%	0,04	1,74	0,55	0,02	0,55
8	100	5	95	5,0%	10,6%	0,06	2,11	0,75	0,04	0,59
9	100	5	95	5,0%	10,6%	0,06	2,11	0,75	0,04	0,63
10	100	4	96	4,0%	10,7%	0,07	2,67	0,98	0,07	0,70
All	1000	100	900						Info. Value	0,70

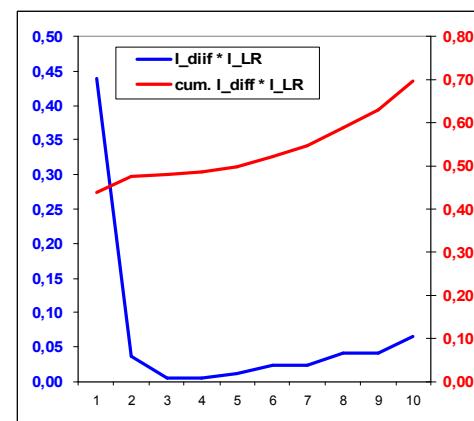
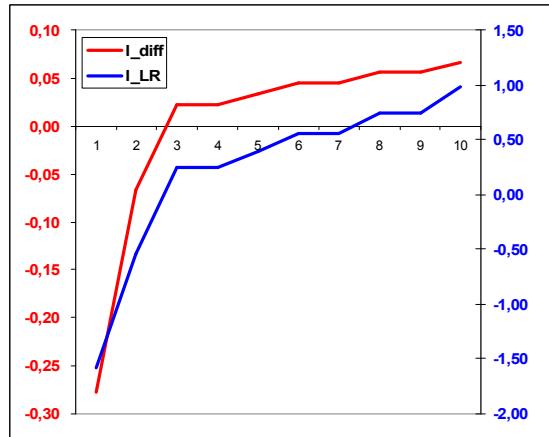
➤ **SC 2:**

decile	# cleints	# bad clients	#good	% bad [1]	% good [2]	[3] = [2] - [1]	[4] = [2] / [1]	[5] = ln[4]	[6] = [3] * [5]	cum. [6]
1	100	20	80	20,0%	8,9%	-0,11	0,44	-0,81	0,09	0,09
2	100	18	82	18,0%	9,1%	-0,09	0,51	-0,68	0,06	0,15
3	100	17	83	17,0%	9,2%	-0,08	0,54	-0,61	0,05	0,20
4	100	15	85	15,0%	9,4%	-0,06	0,63	-0,46	0,03	0,22
5	100	12	88	12,0%	9,8%	-0,02	0,81	-0,20	0,00	0,23
6	100	6	94	6,0%	10,4%	0,04	1,74	0,55	0,02	0,25
7	100	4	96	4,0%	10,7%	0,07	2,67	0,98	0,07	0,32
8	100	3	97	3,0%	10,8%	0,08	3,59	1,28	0,10	0,42
9	100	3	97	3,0%	10,8%	0,08	3,59	1,28	0,10	0,52
10	100	2	98	2,0%	10,9%	0,09	5,44	1,69	0,15	0,67
All	1000	100	900						Info. Value	0,67

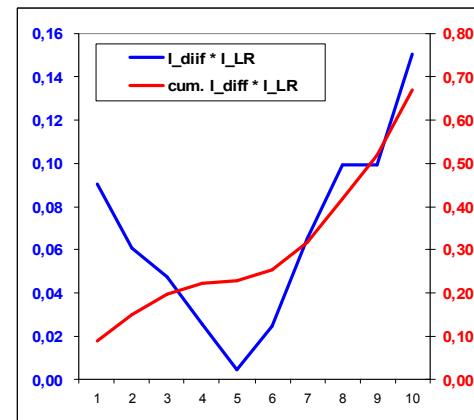
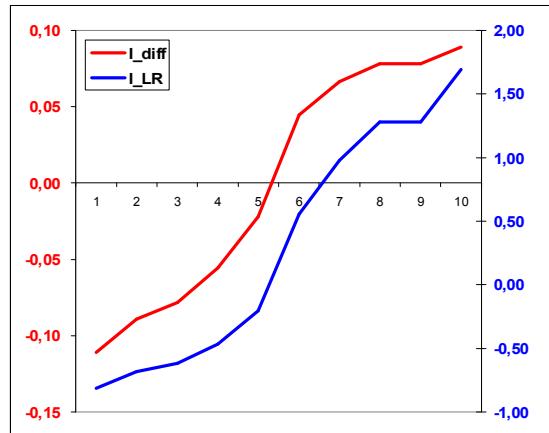
# Indexes based on density function

- Using markings  $I_{diff_i} = \left( \frac{g_i}{n} - \frac{b_i}{m} \right)$   $I_{LR_i} = \ln\left(\frac{g_i m}{b_i n}\right)$  we have:

➤ SC 1:



➤ SC 2:



K-S	= 0.34
Gini	= 0.42
Lift <sub>20%</sub>	= 2.55
Lift <sub>50%</sub>	= 1.48
I <sub>val</sub>	= 0.70
I <sub>val20%</sub>	= 0.47
I <sub>val50%</sub>	= 0.50

K-S	= 0.36
Gini	= 0.42
Lift <sub>20%</sub>	= 1.90
Lift <sub>50%</sub>	= 1.64
I <sub>val</sub>	= 0.67
I <sub>val20%</sub>	= 0.15
I <sub>val50%</sub>	= 0.23

# Some results for normally distributed scores

- Assume that the scores of good and bad clients are normally distributed, i.e. we can write their densities as

$$f_{GOOD}(x) = \frac{1}{\sigma_g \sqrt{2\pi}} e^{-\frac{(x-\mu_g)^2}{2\sigma_g^2}} \quad f_{BAD}(x) = \frac{1}{\sigma_b \sqrt{2\pi}} e^{-\frac{(x-\mu_b)^2}{2\sigma_b^2}}$$

- Estimates of parameters  $\mu_g$ ,  $\mu_b$ ,  $\sigma_g$  and  $\sigma_b$ :

$M_g$ ,  $M_b$  are means of good (bad) clients

$S_g$ ,  $S_b$  are standard deviations of good (bad) clients

- Pooled standard deviation:

$$S = \left( \frac{nS_g^2 + mS_b^2}{n+m} \right)^{\frac{1}{2}}$$

- Estimates of mean and standard dev. of scores for all clients  $\mu_{ALL}$ ,  $\sigma_{ALL}$ :

$$S_{ALL} = \left( \frac{n^2 S_g^2 + m^2 S_b^2}{(n+m)^2} \right)^{\frac{1}{2}} \quad M = M_{ALL} = \frac{nM_g + mM_b}{n+m}$$

# Some results for normally distributed scores

- Assume that standard deviations are equal to a common value  $\sigma$  :

$$D = \frac{\mu_g - \mu_b}{\sigma}$$

$$D = \frac{M_g - M_b}{S}$$

$$KS = \Phi\left(\frac{D}{2}\right) - \Phi\left(\frac{-D}{2}\right) = 2 \cdot \Phi\left(\frac{D}{2}\right) - 1$$

$$Gini = 2 \cdot \Phi\left(\frac{D}{\sqrt{2}}\right) - 1$$

$$Lift_q = \frac{1}{q} \Phi\left(\frac{\sigma_{ALL}}{\sigma} \cdot \Phi^{-1}(q) + p_G \cdot D\right)$$

$$Lift_q = \frac{1}{q} \Phi\left(\frac{S_{ALL}}{S} \Phi^{-1}(q) + p_G \cdot D\right)$$

$$I_{val} = D^2$$

Where  $\Phi(\cdot)$  is the standardized normal distribution function,  $\Phi_{\mu, \sigma^2}(\cdot)$  the normal distribution function with parameters  $\mu, \sigma^2$  and  $\Phi^{-1}(\cdot)$  is the standard quantile function.

# Some results for normally distributed scores

➤ Generally (i.e. without assumption of equality of standard deviations):

$$D^* = \frac{\mu_g - \mu_b}{\sqrt{\sigma_g^2 + \sigma_b^2}}$$

$$D^* = \frac{M_g - M_b}{\sqrt{S_g^2 + S_b^2}}$$

$$KS = \Phi\left(\frac{a}{b}\sigma_b \cdot D^* - \frac{1}{b}\sigma_g \sqrt{a^2 D^{*2} + 2b \cdot c}\right) - \Phi\left(\frac{a}{b}\sigma_g \cdot D^* - \frac{1}{b}\sigma_b \sqrt{a^2 D^{*2} + 2b \cdot c}\right)$$

where  $a = \sqrt{\sigma_b^2 + \sigma_g^2}$ ,  $b = \sigma_b^2 - \sigma_g^2$ ,  $c = \ln\left(\frac{\sigma_g}{\sigma_b}\right)$

$$\begin{aligned} KS &= \Phi\left(\frac{\sqrt{S_b^2 + S_g^2}}{S_b^2 - S_g^2} S_b \cdot D^* - \frac{1}{S_b^2 - S_g^2} S_g \sqrt{(S_b^2 + S_g^2) D^{*2} + 2 \cdot (S_b^2 - S_g^2) \ln\left(\frac{S_g}{S_b}\right)}\right) \\ &\quad - \Phi\left(\frac{\sqrt{S_b^2 + S_g^2}}{S_b^2 - S_g^2} S_g \cdot D^* - \frac{1}{S_b^2 - S_g^2} S_b \sqrt{(S_b^2 + S_g^2) D^{*2} + 2 \cdot (S_b^2 - S_g^2) \ln\left(\frac{S_g}{S_b}\right)}\right) \end{aligned}$$

# Some results for normally distributed scores

- Generally (i.e. without assumption of equality of standard deviations):

$$Gini = 2 \cdot \Phi(D^*) - 1$$

$$Lift_q = \frac{1}{q} \Phi_{\mu_b, \sigma_b^2}(\mu_{ALL} + \sigma_{ALL} \cdot \Phi^{-1}(q)) = \frac{1}{q} \Phi \left( \frac{\sigma_{ALL} \cdot \Phi^{-1}(q) + \mu_{ALL} - \mu_b}{\sigma_b} \right)$$

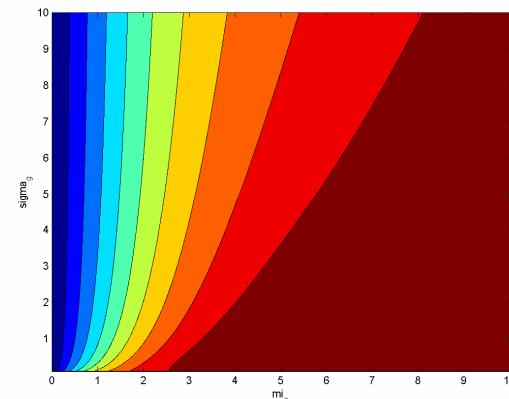
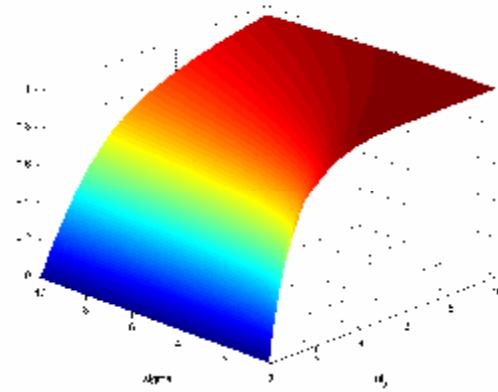
$$Lift_q = \frac{1}{q} \Phi \left( \frac{S_{ALL} \cdot \Phi^{-1}(q) + M - M_b}{S_b} \right)$$

$$I_{val} = (A+1)D^{*2} + A - 1, \quad A = \frac{1}{2} \left( \frac{\sigma_b^2}{\sigma_g^2} + \frac{\sigma_g^2}{\sigma_b^2} \right)$$

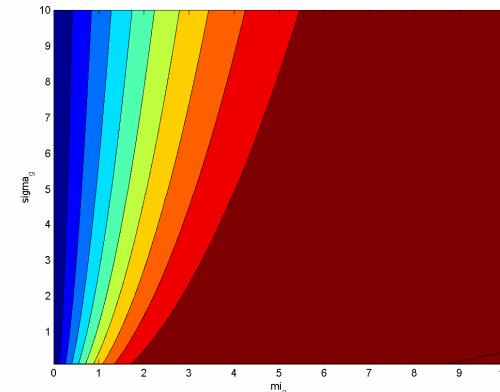
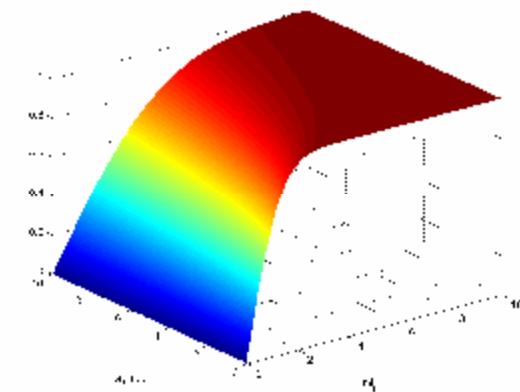
$$I_{val} = (A+1)D^{*2} + A - 1, \quad A = \frac{1}{2} \left( \frac{S_b^2}{S_g^2} + \frac{S_g^2}{S_b^2} \right)$$

# Some results for normally distributed scores

➤ **KS:**  $\mu_b = 0, \sigma_b^2 = 1$

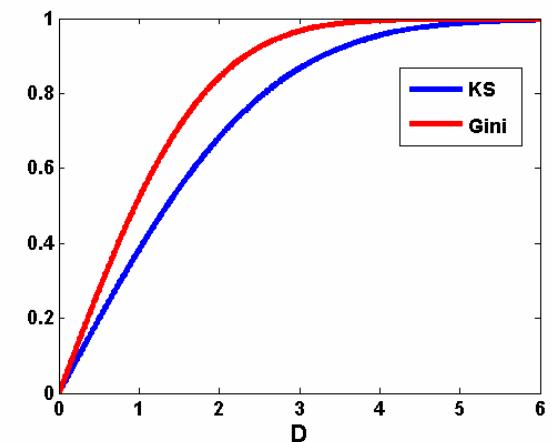


➤ **Gini**  $\mu_b = 0, \sigma_b^2 = 1$



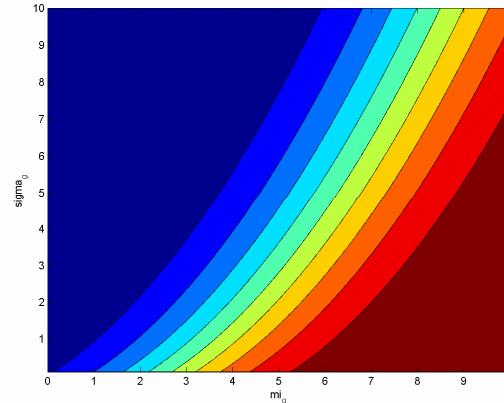
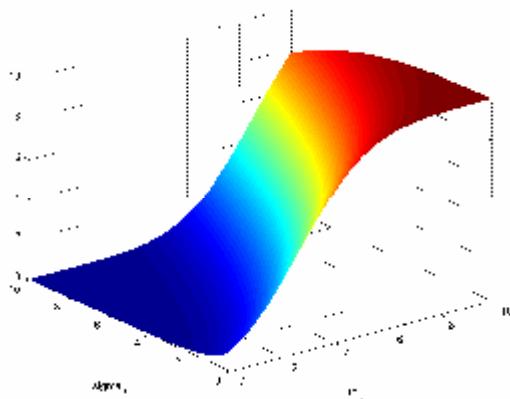
□ KS and the Gini react much more to change of  $\mu_g$  and are almost unchanged in the direction of  $\sigma_g^2$ .

• **Gini > KS**



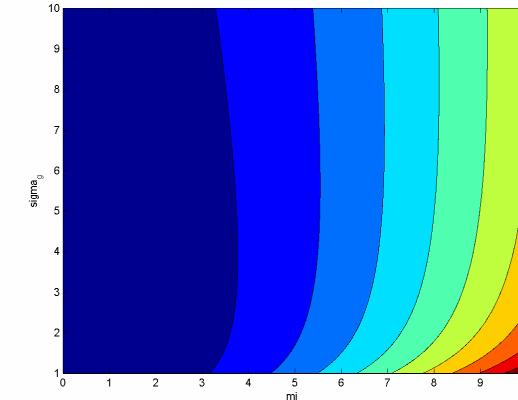
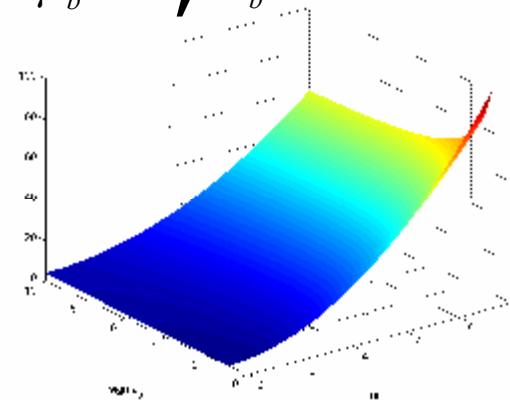
# Some results for normally distributed scores

➤ **Lift<sub>10%</sub>:**  $\mu_b = 0$ ,  $\sigma_b^2 = 1$



□ In case of Lift<sub>10%</sub> it is evident strong dependence on  $\mu_g$  and significantly higher dependence on  $\sigma_g^2$  than in case of KS and Gini.

➤ **I<sub>val</sub>:**  $\mu_b = 0$ ,  $\sigma_b^2 = 1$



□ Again strong dependence on  $\mu_g$ . Furthermore value of I<sub>val</sub> rises very quickly to infinity when  $\sigma_g^2$  tends to zero.

# Conclusions

- It is impossible to use scoring model effectively without knowing how good it is.
- It is necessary to judge scoring models according to their strength in score range where cutoff is expected.
- The Gini is not enough!
- Results concerning Lift and Information value can be used to obtain the best available scoring model.
- Results for normally distributed scores can help with computation of referred indexes. Furthermore they can help to understanding how those indexes behave.