$\boxed{1.\ CV.}$ ~~se~~ střední hodnoty se neskončí !

- povolené 2 neomluvené absence
- 2 zápočtovky – každá aspoň na 50 % (nepřenáší se)
- variace, permutace, kombinace (' s opak.)

$$V(k,n) = \frac{n!}{(n-k)!}$$

$$V'(k,n) = n^k$$

$$P(n) = n! = (V(n,n))$$

~~P'(n) =~~

$$P'(n_1,n_2,n_3) = \frac{(n_1+n_2+n_3)!}{n_1!\,n_2!\,n_3!}$$

$$K(k,n) = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

$$K'(k,n) = \binom{n+k-1}{k}$$

n … počet přihrádek

k … co vybírám

1) a) $5! = 120$

b) $\frac{5!}{2} = 60$

c) $4! = 24$

3) a) $531441 = 9^6$

b) $10^6 - 9^6 = 468559$

4) a) $\frac{(1+2+4+4)!}{1!\cdot 2!\cdot 4!\cdot 4!} = 34650$

b) $\frac{(1+1+2+4)!}{1!\,1!\,2!\,4!} = 840$

c) $34650 - 840 = 33810$

h) $34650 - 2\cdot 840 + 60 = \cancel{32910}\ 33030$

$$60 = \frac{5!}{2} = \frac{(1+1+1+2)!}{1!\,1!\,1!\,2!}$$

5) $3\cdot 8\cdot 4\cdot 5 = 480$

6) a) $6\cdot 6 = 36$

b) $6+5+4+3+2+1 = 21$

7) a) $2^8 = 256$

b) $2\cdot 2^5 = 64$

8) $\binom{5}{2} = \frac{5!}{3!\,2!} = 10$

9) $\binom{12}{3}\cdot\binom{8}{2}\cdot\binom{3}{1} = 12320$

10) a) $4 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 96$

b) $4 \cdot 3 \cdot 2 \cdot 1 \cdot 1 + 3 \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 42$ (podle toho, či je na konci 0 nebo 5)

11) a)

□ | □ || □ □ □ □

$\binom{9}{3} = 84$

b)

□ || □ | □ ~~X~~ ryba

$\binom{6}{3} = 20$

– 12 piv 4 druhy

• | • • • || • • • • • • • • •

$\binom{15}{3}$

12) $\binom{15}{7} \cdot \binom{10}{2}$

• ||||||| • ||

13)

• | • • • | • • | • • • •

$\binom{9}{3} \cdot \binom{7}{3} \cdot \binom{14}{3} = 1070160$

2) a) $7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 = 2520$

c) $7^5 = 16807$

b) $\binom{7}{5} = 21$

d)

• ||| • | • | • | • •

$\binom{11}{5} = 462$

$\boxed{1. \text{ PŘ.}}$

$\boxed{2. \text{ CV.}}$

– pokus př. hozím kostkou

– elementární jevy: ω_i : padlo mi i na kostce $i = 1, \ldots, 6$

– $\Omega = \{\omega_1, \ldots, \omega_6\}$ … množina všech el. jevů

A … padlo sudé číslo $A = \{\omega_2, \omega_4, \omega_6\}$

$P(A) = \frac{|A|}{|\Omega|} = \frac{3}{6} = \frac{\text{"počet příznivých"}}{\text{"[illegible]"}}$

- 3 podmínky: - výsledky se vzájemně vylučují
- Ω musí být konečná
- všechny el. jevy nastávají se stejnou četí

- vlastnosti: $P(\emptyset)=0$

$$P(A\cup B) = \begin{cases} P(A)+P(B) & \text{, když } A\cap B=\emptyset \\ P(A)+P(B)-P(A\cap B) & \text{, když } A\cap B\neq\emptyset \end{cases}$$

14)

a) $|A|=1 \quad |\Omega|=6^6$

$$P(A)=\frac{1}{6^6}=\cancel{0{,}0154} \doteq 2{,}14\cdot 10^{-5}$$

b) $|A|=6! \qquad P(A)=\frac{6!}{6^6}=\cancel{0{,}0080}\ 0{,}0154$

c) $|A|=\binom{6}{4}\cdot 5^2 \cancel{+1}$

$$P(A)=\frac{\binom{6}{4}\cdot 5^2}{6^6}=0{,}0080$$

d)

$$P(A)=\frac{\binom{6}{4}\cdot 5^2+\binom{6}{5}\cdot 5+1}{6^6}=\cancel{0{,}4}\ 0{,}0087$$

e)

$$P(A)=\frac{3^6}{6^6}=\frac{1}{2^6}=0{,}0156$$

15)

a)

$$P(A)=\frac{1+1+1+1+1}{6^2}=\frac{5}{36}=0{,}1388$$

b)

$$P(A)=\frac{5+4+3+2+1}{6^2}=\frac{15}{36}=\frac{5}{12}$$

16) a)

$$P(A)=1-\frac{c^2}{(b+c)^2}$$

b)

$$P(A)=\frac{b(b-1)}{(b+c)(b+c-1)}=\frac{\binom{b}{2}}{\binom{b+c}{2}}$$

17)

$$P(A)=\cancel{\frac{2}{5}=0{,}4}=\frac{4\cdot 3\cdot 2}{5\cdot 4\cdot 3}=\frac{24}{60}=0{,}4$$

18)

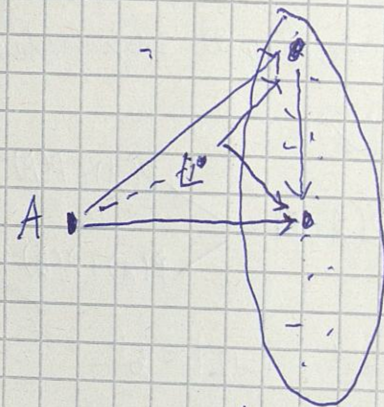
$$P(A)=\frac{365\cdot 364\cdot 363}{365^3}=0{,}9917\ldots$$ (škaredě napsané zadání)

19) a) $P(A)=\frac{8}{1000}=$

b) $P(A)=\frac{64+32}{1000}=\frac{12\cdot 8}{1000}$

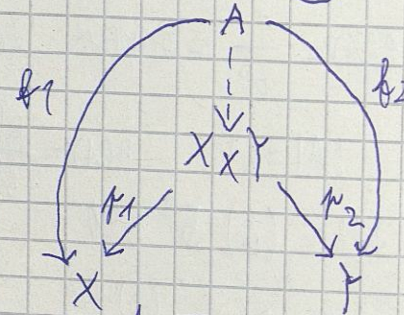
c) $P(A)=\frac{64\cdot 6}{1000}=0{,}384$

d) $P(A)=\frac{8^3}{1000}=0{,}512$



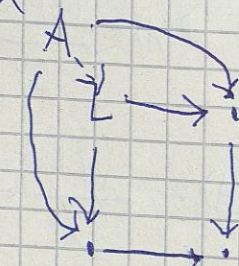
20) a) $P(A)=\frac{4\cdot 3}{32\cdot 31}=0{,}012$

b) $P(A)=\frac{32\cdot 8}{32^2}=\frac{8}{32}=\frac{1}{4}=0{,}25$



21) a) $P(A)=\frac{\binom{5}{1}\cdot\binom{95}{3}}{\binom{100}{4}}=0{,}1765$

b) $P(A)=1-\frac{\binom{95}{4}}{\binom{100}{4}}=0{,}1881$



23)

$P(A)=\frac{1}{\frac{10!}{3!2!2!}}=\frac{3!2!2!}{10!}=6{,}61\cdot 10^{-6}$

24) a) $P(A)=\frac{10}{10^4}=\frac{1}{10^3}=0{,}001$

b) $P(A)=\frac{10\cdot 9\cdot 8\cdot 7}{10^4}=0{,}504$

c) $P(A)=\frac{10\cdot 9\cdot 4}{10^4}=0{,}036$

d) $P(A)=\frac{10\cdot 9\cdot 8\cdot\binom{4}{2}}{10^4}=0{,}432$

25) $P(A)=\frac{8}{120}=\frac{1}{15}$ $\qquad |\Omega|=\frac{10!}{3!\cdot 7!}=\frac{10\cdot 9\cdot 8}{3\cdot 2}=120$

$|A|=\frac{8!}{1!\cdot 7!}=8$

* n losů, m vyhrávají, koupíme k, aspoň 1 výherní

$|\Omega|=\binom{n}{k}$ $\qquad P(A)=1-\frac{\binom{n-m}{k}}{\binom{n}{k}}$

26) a) 2 hráči A, B

$$P(A) = \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5 + \dots = \frac{1}{2} \cdot \frac{1}{1-\frac{1}{4}} = \frac{1}{2} \cdot \frac{4}{3} = \frac{2}{3}$$

$$P(B) = 1 - P(A) = \frac{1}{3}$$

b) 3 hráči A, B, C

$$P(A) = \frac{1}{2} + \left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^7 + \dots = \frac{1}{2} \cdot \frac{1}{1-\frac{1}{8}} = \frac{1}{2} \cdot \frac{8}{7} = \frac{4}{7}$$

$$P(B) = \frac{1}{4} + \left(\frac{1}{2}\right)^5 + \dots = \frac{1}{4} \cdot \frac{1}{1-\frac{1}{8}} = \frac{2}{7}$$

$$P(C) = 1 - \frac{2}{7} - \frac{4}{7} = \frac{1}{7}$$

* n-krát hážíme mincí

a) ~~k-krát~~ vždy padne hlava

$|\Omega| = 2^n$ $\quad P(A) = \frac{1}{2^n}$

$|A| = 1$

b) k-krát líc

$|\Omega| = 2^n$

$|A| = \binom{n}{k}$

$$\underbrace{\_\ \_\ \_}_{k \times \text{líc}}\ \underbrace{\_\ \_\ \dots\ \_}_{(n-k) \times \text{rub}}$$

$\boxed{\text{3. CV.}}$

~~ω_i ... elementární jev, $\omega_1 = \{1\}$~~

~~Ω~~

- sigma algebra $\mathcal{A}$ ($\mathcal{A}$)
  1. $A \in \mathcal{A} \Rightarrow \bar{A} \in \mathcal{A}$
  2. $A_1, A_2 \in \mathcal{A} \Rightarrow \bigcup_{i=1}^{\infty} A_i \in \mathcal{A}$

- jev A je náhodný, když $A \in \mathcal{A}$
- $(\Omega, \mathcal{A})$ je jevové pole

* $\mathcal{A} = \{\emptyset, \{\omega_1\}, \{\omega_2, \omega_3, \omega_4\}, \Omega\}$ $\quad \Omega = \bigcup_{i=1}^{6} \{\omega_i\}$

$\overline{\{\omega_1\}} \notin \mathcal{A}$

28)

* $\Omega = \{\omega_1, \omega_2, \omega_3\}$ najděte σ algebry

$\mathcal{A}_1 = \{\emptyset, \Omega\}$, $\mathcal{A}_2 = \{\emptyset, \Omega, \{\omega_1\}, \{\omega_2, \omega_3\}\}$

$\mathcal{A}_3 = \{\emptyset, \Omega, \{\omega_2\}, \{\omega_3, \omega_1\}\}$ $\mathcal{A}_4 = \{\emptyset, \Omega, \{\omega_3\}, \{\omega_1, \omega_2\}\}$

$\mathcal{A}_5 = 2^{\Omega}$

29) 4 výrobky

A... alespoň 1 zmetek

B... nejvýše 2 zmetky

$\overline{A}$... žádný zmetek

$\overline{B}$... alespoň 3 zmetky

30) 2 paralelní kotle, sériově připojený stroj

A... stroj provozu schopný

B_1... kotel 1 funguje

B_2... kotel 2 funguje

C... strojovna funguje

$C = (B_1 \cup B_2) \cap A$

$\overline{C} = (\overline{B_1} \cap \overline{B_2}) \cup \overline{A}$

31) A = standard

B = použitelné

C = nepoužitelné

a) $A \cup B$ buď S nebo P

b) $\overline{A \cup C}$ použitelné

c) $A \cap C$ $\emptyset$ nemožný jev

d) $(A \cap B) \cup C$ nepoužitelné

e) $A \cup B \cup C$ S+R+N jistý jev

32)

a) A

b) $A \cap \overline{B} \cap \overline{C}$

c) $A \cup B \cup C$

d) $(A \cap \overline{B} \cap \overline{C}) \cup (\overline{A} \cap B \cap \overline{C}) \cup (\overline{A} \cap \overline{B} \cap C)$

e) $A \cap B \cap C$

f) $\overline{A \cap B \cap C}$

- $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

$P(\bigcup_{i=1}^{n} A_i) = \dots$

$P(\bigcup_{i=1}^{4} A_i) = P(A_1) + P(A_2) + P(A_3) + P(A_4) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_1 \cap A_4) - P(A_2 \cap A_3) - P(A_2 \cap A_4)$
$- P(A_3 \cap A_4) + P(A_1 \cap A_2 \cap A_3) + P(A_1 \cap A_3 \cap A_4) + P(A_1 \cap A_2 \cap A_4) + P(A_2 \cap A_3 \cap A_4)$
$- P(A_1 \cap A_2 \cap A_3 \cap A_4)$

33) 4 klobouky, 4 osoby

P, že aspoň 1 osoba si vezme svůj klobouk $\qquad P(A_i)=\frac{3!}{4!}$

A_i ... i-tá osoba si veme svůj klobouk

$$P\left(\bigcup_{i=1}^{4} A_i\right) = 4\cdot\frac{3!}{4!} - 6\cdot\frac{2}{4!} + 4\cdot\frac{1}{4!} - \frac{1}{4!} = 1-\frac{1}{2}+\frac{1}{6}-\frac{1}{24} = \frac{5}{8} = 0{,}625$$

~~6 kostek, 1-6~~

34) A_i ... i-tý hod = číslo i

$$P\left(\bigcup_{i=1}^{6} A_i\right) = 6\cdot\frac{5!}{6!} - \binom{6}{2}\cdot\frac{4!}{6!} + \binom{6}{3}\cdot\frac{3!}{6!} - \binom{6}{4}\frac{2!}{6!} + 6\cdot\frac{1!}{6!} - \frac{1}{6!} = 0{,}6319$$

35) n poschodí, k osob, vystoupí v každém poschodí aspoň 1 os

A_i .. na i-tém poschodí vystoupí aspoň 1 osoba

$$P\left(\bigcap_{i=1}^{n} A_i\right) = 1 - P\left(\bigcup_{i=1}^{n} \overline{A_i}\right) = 1 - \left\{\binom{n}{1}\cdot\frac{(n-1)^k}{n^k} + \binom{n}{2}\cdot\frac{(n-2)^k}{n^k} - \dots + \cancel{(-1)^{n}\cdot\frac{(1)^k}{n^k}}\right.$$

$$+ (-1)^{n-1}\cdot n\cdot\frac{1^k}{n^k} + (-1)^n\cdot\underbrace{\frac{1}{n^k}}_{P(\overline{A_1}\cap\dots\cap\overline{A_n})}$$

nemá to být 0?

*) babička rozděluje m tisíci korun do n obálek, peníze rozmístí náhodně, a všechna rozmístění jsou stejně pravděpodobná. Určete pst, že každé vnouče dostane aspoň něco.

A_i ... i-té vnouče dostane aspoň něc $\qquad P(\overline{A_1}\cap\dots\cap\overline{A_n})=0$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = 1 - P\left(\bigcup_{i=1}^{n}\overline{A_i}\right) = 1 + \frac{\binom{n}{1}\cdot\binom{n-1+m-1}{m}}{\binom{n+m-1}{m}} - \frac{\binom{n}{2}\cdot\binom{n-2+m-1}{m}}{\binom{n+m-1}{m}} + \dots$$

$$+ (-1)^{n-1}\cdot\frac{\binom{n}{n-1}\cdot\binom{m}{m}}{\binom{n+m-1}{m}}$$

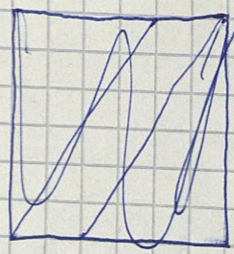
$\boxed{\text{4. CV.}}$

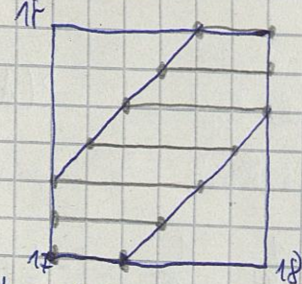
$149\cdot10^6$ km² pevnina

36) $361\cdot10^6$ km² moře

$$P = \frac{\mu(A)}{\mu(\Omega)} = \frac{149\cdot10^6}{510\cdot10^6} = 0{,}292$$

37)





$\mu(\Omega) = 1$

$\mu(A) = 1 - 2 \cdot \dfrac{\left(\frac{2}{3}\right)^2}{2} = \frac{5}{9}$

y přijde v 17 => x v 17:00–17:20

y v 17:10 => x v 17:00–17:30

y v 17:20 => x v 17:00–17:40

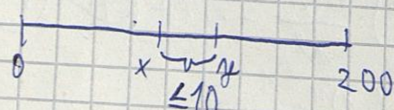
v v 17:30 => x v 17:10–17:50

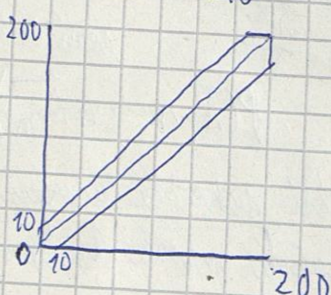
$P(A) = \dfrac{\frac{5}{9}}{1} = \frac{5}{9}$

$\Omega = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x,y \le 1\}$

$A = \{(x,y) \in \mathbb{R}^2 \mid 0 \le x,y \le 1,\ x - \frac{1}{3} \le y \le x + \frac{1}{3}\}$

38)





$P(A) = \dfrac{200^2 - 190^2}{200^2} = 0{,}0975$

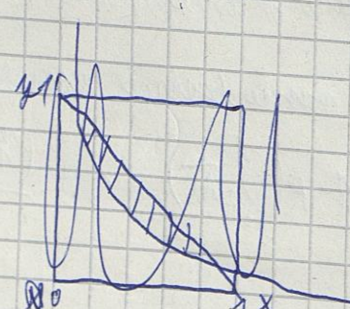
39)

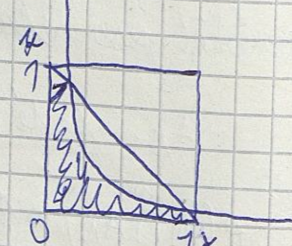
$x + y \le 1$

$xy \le 0{,}09$

$y \le 1 - x$

$y \le \dfrac{0{,}09}{x}$





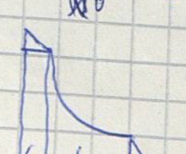
$y = 1 - x$

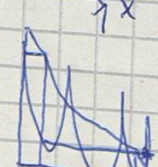
$xy = 0{,}09$

$\dfrac{0{,}09}{x} = 1 - x^2$

$x(1-x) = 0{,}09$

$x^2 - x + 0{,}09$





$S_\square = 0{,}9 \cdot 0{,}1 = 0{,}09$

$x_1 = 0{,}9,\ x_2 = 0{,}1$

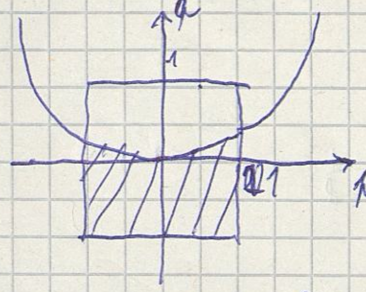
$2 \cdot S_\triangle = 0{,}1^2$

$S = \int_{0,1}^{0,9} \frac{0{,}09}{x}\,dx = 0{,}09[\ln x]_{0,1}^{0,9} = 0{,}09(\ln 0{,}9 - \ln 0{,}1) = 0{,}19775$

$P(A) = \mu(A) = 0{,}01 + 0{,}09 + 0{,}19775 = 0{,}29775 = P(A)$

40) (pozn. $|p| \leq 1, |q| \leq 1$)

a)



$\mu(\Omega) = 4$

$$D = p^2 - 4q \geq 0$$

$$q \leq p^2/4$$

$$S_{\square} = 2 \cdot 1 = 2$$

$$\Omega = \{(p,q) \in \mathbb{R}^2 \mid |p| \leq 1, |q| \leq 1\}$$

$$A = \{(p,q) \in \mathbb{R}^2 \mid |p| \leq 1, |q| \leq 1, q \leq p^2/4\}$$

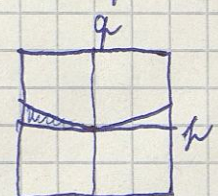
$$\int_0^1 \frac{p^2}{4}\, dp = \left[\frac{p^3}{12}\right]_0^1 = \frac{1}{12}$$

$$P(A) = \frac{\frac{2}{12} + 2}{4} = \frac{\frac{13}{6}}{4} = \frac{13}{24}$$

b)

$$q \leq \frac{p^2}{4} \qquad x_{1,2} = \frac{-p \pm \sqrt{p^2 - 4q}}{2}$$

$$\begin{array}{l} x_1 x_2 = q \\ x_1 + x_2 = -p \end{array} \iff \begin{array}{l} q > 0 \\ p < 0 \end{array}$$



$$P(A) = \frac{\int_{-1}^{0} \frac{x^2}{4}\, dx}{4} = \frac{1}{48}$$

- $A_1, A_2, \ldots, A_n$ jsou stoch. nezávislé, pokud pro lib. skupinu indexů $\{i_1, \ldots, i_k\} \subseteq \{1, \ldots, n\}$ platí:

$$P\left(\bigcap_{j=1}^{k} A_{i_j}\right) = \prod_{j=1}^{k} P(A_{i_j})$$

- 2 jevy $P(A \cap B) = P(A)P(B)$
- 3 jevy

$$P(A \cap B) = P(A)P(B)$$
$$P(B \cap C) = P(B)P(C)$$
$$P(C \cap A) = P(C)P(A)$$
$$P(A \cap B \cap C) = P(A)P(B)P(C)$$

45) 000, 110, 101, 011

A_i ... na i-tém místě jednička

$$P(A_1 \cap A_2) = \frac{1}{4} \qquad P(A_1)P(A_2) = \frac{1}{4}$$

$P(A_1 \cap A_3) = \frac{1}{4}$ $\qquad P(A_1)P(A_3) = \frac{1}{4}$

$P(A_2 \cap A_3) = \frac{1}{4}$ $\qquad P(A_2)P(A_3) = \frac{1}{4}$

$P(A_1 \cap A_2 \cap A_3) = 0$ $\qquad P(A_1)P(A_2)P(A_3) = \frac{1}{8}$

je po dvou, není ~~stoch~~ stochasticky

~~*~~46) 0,4, 7 semínek, aspoň 1 vyklíčí

$P(A) = 1 - 0{,}6^7 = 0{,}972$

~~*~~47) $p_1 = 0{,}4$, $p_2 = 0{,}5$, $p_3 = 0{,}7$

právě 1 zasáhne

$0{,}4 \cdot 0{,}5 \cdot 0{,}3 + 0{,}6 \cdot 0{,}5 \cdot 0{,}3 + 0{,}6 \cdot 0{,}5 \cdot 0{,}7$

aspoň 1 trefí

$1 - 0{,}6 \cdot 0{,}5 \cdot 0{,}3 = 0{,}91$

* $A_1, \dots, A_n$ stoch nezávislé

$P(A_i) = p_i$ $\qquad P(\overline{A_i}) = 1 - p_i$

a) alespoň 1 z jevů

$$P\left(\bigcup_{i=1}^{n} A_i\right) = 1 - P\left(\bigcap_{i=1}^{n} \overline{A_i}\right) = 1 - \prod_{i=1}^{n} (1 - p_i)$$

b) všechny $A_1, \dots, A_n$

$$P\left(\bigcap_{i=1}^{n} A_i\right) = \prod_{i=1}^{n} p_i$$

c) nastane právě jeden

$$p_1 \prod_{i \neq 1} (1 - p_i) + p_2 \prod_{i \neq 2} (1 - p_i) + \dots + p_n \prod_{i \neq n} (1 - p_i)$$

5. CV.

* 2 kostky, 2 × 5, součet ok je dělitelný 5, jsou stoch. nezávislé?
(49)

A ... 2 × 5

$|A| = 1$

B součet ok je dělitelný 5

$$P(A|B) = \frac{\frac{1}{6^2}}{\frac{7}{6^2}} = \frac{1}{7}$$

1+4   4+1

2+3   3+2

5+5

4+6   6+4

$|B| = 7$

$P(A\cap B) \neq P(A)\cdot P(B) \neq \frac{1}{36}\cdot\frac{7}{36}$ $\frac{1}{36} \neq \frac{1}{36}\cdot\frac{7}{36}$

nejsou stoch. nezávislé

*51) $P(A) = 0,3$

$P(B) = 0,4$

$P(A\cup B) = 0,6$

$P(A\cap B) = P(A) + P(B) - P(A\cup B) = 0,1$

$P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{1}{4}$

$P(B|A) = \frac{P(A\cap B)}{P(A)} = \frac{1}{3}$

$0,1 \neq 0,3\cdot 0,4$

nejsou stoch nezávislé

*52) rodiny: 2 děti, předp. 1 z dětí – ♀

pst – 2 dcery?

♂, ♂

$\left.\begin{array}{l} ♂, ♀ \\ ♀, ♂ \\ ♀, ♀ \end{array}\right\} B$

A... ♀, ♀

$P(A|B) = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{3}$

b) DÚ

*54) a)

A... zmetek

B... od 1. dělníka

$P(A\cap B) = P(B)\cdot P(A|B) = 0,6\cdot 0,1$

(bacha, v zadání je a (ne když)

b)

A... zmetek

B... od 2.

$P(A\cap B) = P(B)\cdot P(A|B) = 0,4\cdot 0,05 = 0,02$

*55) $P(\overline{A_1}\cap A_2\cap A_3) = P(\overline{A_1})\cdot P(A_2|\overline{A_1})\cdot P(A_3|(A_2\cap \overline{A_1})) =$

$= \frac{2}{5}\cdot\frac{3}{4}\cdot\frac{2}{3} = \frac{1}{5}$

* n mužů, $2n$ žen, pst., že při výběru tříčlenných skupin bude v každé skupině právě 1 muž

A_i ... v i-té skupině je právě 1 muž

*

$$P\left(\bigcap_{i=1}^{n} A_i\right) = P(A_1)\cdot P(A_2|A_1) \cdots P\left(A_n \middle| \bigcap_{i=1}^{n-1} A_i\right) =$$

$$= \frac{\binom{n}{1}\cdot\binom{2n}{2}}{\binom{3n}{3}} \cdot \frac{\binom{n-1}{1}\cdot\binom{2n-2}{2}}{\binom{3n-3}{3}} \cdots \frac{1}{1} =$$

$$= \frac{n\cdot\frac{2n(2n-1)}{2}}{\quad} \cdots = \frac{\frac{n!\,(2n)!}{2^n}}{\frac{(3n)!}{(3!)^n}} =$$

$$= \frac{3^n \cdot n! \cdot (2n)!}{(3n)!}$$

*5a)

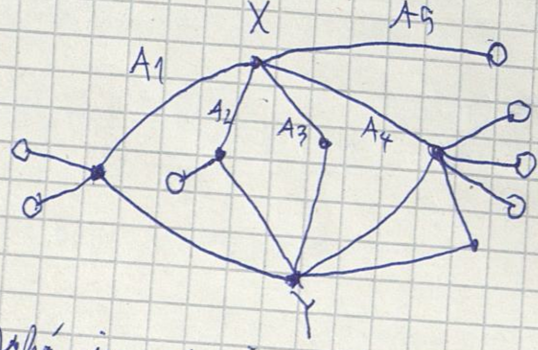
A_i ... zakoupena u i-tého výrobce

B ... žárovka je standardní

$$P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) =$$

$$= 0{,}7\cdot 0{,}45 + 0{,}8\cdot 0{,}4 + 0{,}81\cdot 0{,}15 = 0{,}7565$$

*



(nevracíme se)
(je tu vlastně orient.)

Jaká je pst., že se dostaneme z X do Y?

A_i ... výběr i-té cesty

$$P(B) = \sum_{i=1}^{5} P(B|A_i)\cdot P(A_i) = \frac{1}{3}\cdot\frac{1}{5} + \frac{1}{2}\cdot\frac{1}{5} + \frac{1}{1}\cdot\frac{1}{5} + \frac{2}{5}\cdot\frac{1}{5} + 0\cdot\frac{1}{5} = 0{,}4\overline{46}$$

* 6b) a) 10 zkaž.
400 OK

$P(A) = \frac{10}{400} = \frac{1}{40}$ A ... vytáhl zkažený

$T \dots$ z Turecka | $A_F \dots$ z Afr.

*~~49(60)~~8 alternativně: $P(A) = P(A|T)\cdot P(T) + P(A|A_F)\cdot P(A_F)$

$$= \frac{5}{150}\cdot\frac{150}{400} + \frac{5}{250}\cdot\frac{250}{400} = \frac{10}{400} = \frac{1}{40}$$

b) $ZT \dots$ zvolím Turecko

ZA_F

$$P(A) = P(A|ZT)\cdot P(ZT) + P(A|ZA_F)\cdot P(ZA_F) =$$

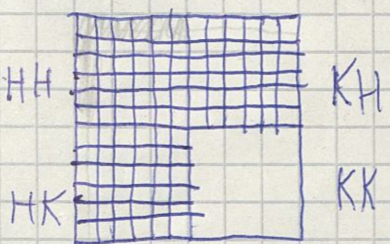
$$= \frac{5}{150}\cdot\frac{1}{2} + \frac{5}{250}\cdot\frac{1}{2} = 0{,}02\overline{6}$$

*61) $P(A) = \frac{8}{23}\cdot 0{,}9 + \frac{12}{23}\cdot 0{,}6 + \frac{3}{23}\cdot 0{,}4 = 0{,}6783$

~~*52) b)~~

$\boxed{6\ \text{CV.}}$

*52) b) předp., že každá 6 holka se změní v kuchařku



$\frac{11}{23}$

každá n-tá je kuchařka: $\frac{2n-1}{4n-1}$ $\lim_{n\to\infty}\frac{2n-1}{4n-1} = \frac{1}{2}$

1. Bayesův vzorec: ~~$P(A_j)$~~

úplný systém

nenulové

$$P(A_j|B) = \frac{P(B|A_j)\cdot P(A_j)}{P(B)} = \frac{P(B|A_j)\cdot P(A_j)}{\sum_{i=1}^{\infty} P(B|A_i)\cdot P(A_i)}$$

*62) test, 100 otázek, 4 možnosti

zná - zvolí

nezná - náhodně vybere

k ze 100 zná

a) pst., že zkoušený odpoví správně

A... zkoušený zná správnou odpověď

B... odpoví správně

$$P(B) = P(A)\cdot P(B|A) + P(\bar{A})\cdot P(B|\bar{A}) = \frac{k}{100}\cdot 1 + \frac{100-k}{100}\cdot\frac{1}{4} = \frac{3k+100}{400}$$

b) odpoví správně, s jakou pst. ~~[illegible]~~ hádal

$$P(\bar{A}|B) = \frac{P(B|\bar{A})\cdot P(\bar{A})}{P(B)} = \cancel{3k+100} = \frac{\frac{1}{4}\cdot\frac{100-k}{100}}{\frac{3k+100}{400}} = \frac{100-k}{100+3k}$$

* 63) 10 krabic, v každé 10 knoflíků

v i-té krabičce i černých

zákazník náhodně zvolí krabičku a z ní náhodně knoflík

jaká je pst., že je černý

B ... černý

~~A_i~~

A_i ... vytáhne i-tou krabičku $i \in \{1,\dots,10\}$ úplný systém

$$P(B) = \sum_{i=1}^{10} P(B|A_i)\cdot P(A_i) = \sum_{i=1}^{10} \frac{i}{10}\cdot\frac{1}{10} = \frac{10\cdot 11}{2\cdot 100} = \frac{11}{20}$$

* 64) A, B, C

N... nehoda

$P(N|A) = 0{,}03$ $\quad P(A) = 0{,}7$

$P(N|B) = 0{,}06$ $\quad P(B) = 0{,}2$

$P(N|C) = 0{,}1$ $\quad P(C) = 0{,}1$

a) $$P(A|N) = \frac{P(N|A)\cdot P(A)}{P(N)} = \frac{0{,}03\cdot 0{,}7}{0{,}043} = 0{,}488$$

$$P(N) = P(A)\cdot P(N|A) + P(B)\cdot P(N|B) + P(C)\cdot P(N|C) = 0{,}021 + 0{,}012 + 0{,}01 = 0{,}043$$

b) $$P(B|N) = \frac{0{,}012}{0{,}043} = 0{,}279$$

c) $$P(C|N) = \frac{0{,}01}{0{,}488} = 0{,}233$$

*65) vada ... 0,1 = P(V)
v záruce se porouchá ... 0,5 když má vadu
... 0,01 když nemá vadu

a) náhodně vybraný výrobek se v záruce porouchá - pst.?

$$P(Z) = P(Z|V) \cdot P(V) + P(Z|\bar{V}) + P(\bar{V}) =$$
$$= 0{,}5 \cdot 0{,}1 + 0{,}01 \cdot 0{,}9 = 0{,}059$$

b) pst., že porouchaný výrobek má vadu?

$$P(V|Z) = \frac{P(Z|V) \cdot P(V)}{P(Z)} = \frac{0{,}5 \cdot 0{,}1}{0{,}059} = 0{,}8475$$

*66) 3 stejné krabice
1: 1b, 2č, 3z
2: 2b, 1č, 1z
3: 4b, 5č, 3z

náhodně 1 krabice: b+z
pst., že byla i-tá krabice

A ... b+z
B_i ... i-tá krabice

a) $P(B_1|A) = \frac{P(A|B_1) \cdot P(B_1)}{P(A)} = \frac{\frac{1}{6} \cdot \frac{3}{5} \cdot \frac{1}{3}}{0{,}1192} = 0{,}2797$

$$P(A) = P(A|B_1) \cdot P(B_1) + P(A|B_2) \cdot P(B_2) + P(A|B_3) \cdot P(B_3) =$$
$$= \frac{1}{6} \cdot \frac{3}{5} \cdot \frac{1}{3} + \frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{3} + \frac{4}{12} \cdot \frac{3}{11} \cdot \frac{1}{3} = \frac{59}{495} = 0{,}1192$$

b) $P(B_2|A) = \frac{\frac{2}{4} \cdot \frac{1}{3} \cdot \frac{1}{3}}{0{,}1192} = 0{,}4661$

c)

*67) B ... stihl
A_i ... i-té stanoviště

$P(B|A_1) = 0{,}6$
$P(B|A_2) = 0{,}3$
⋮

$$P(B)=\sum_{i=1}^{5} P(B|A_i)\cdot P(A_i) = \frac{1}{5}(0{,}6+0{,}3+0{,}2+0{,}1+0{,}1) = \frac{1{,}3}{5} = 0{,}26$$

a) $$P(A_1|B) = \frac{P(B|A_1)\cdot P(A_1)}{P(B)} = \frac{0{,}6\cdot 1/5}{0{,}26} = 0{,}4615$$

b) $$P(A_2|B) = \frac{0{,}3\cdot\frac{1}{5}}{0{,}26} = 0{,}2308$$

*69) A_i ... i-tý automat $\quad i\in\{1,2,3\}$

B ... 1. prohraje, 2. vyhraje

a) $$P(B)=\sum_{i=1}^{3} P(B|A_i)\cdot P(A_i) = \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} + \frac{2}{3}\cdot\frac{1}{3}\cdot\frac{1}{3} =$$
$$= \frac{25}{108} = 0{,}2315$$

b) $$P(A_1|B) = \frac{P(B|A_1)\cdot P(A_1)}{P(B)} = \frac{\frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{3}}{0{,}2315} = 0{,}36$$

*71) 8 orig., 2 kopie

expert ~~pozná~~ ... 5/6

rozpozná správně — jestli je to kopie nebo originál

a) expert řekne, že je to orig., s jakou pst. to orig. skutečně je

A_1 ... je to originál

A_2 ... kopie

B ... expert usoudí, že je to originál

$$P(A_1|B) = \frac{P(B|A_1)\cdot P(A_1)}{P(B)} = \frac{\frac{5}{6}\cdot\frac{8}{10}}{\frac{7}{10}} = \frac{5\cdot 8}{6\cdot 7} = 0{,}9524$$

$$P(B) = P(B|A_1)\cdot P(A_1) + P(B|A_2)\cdot P(A_2) = \frac{5}{6}\cdot\frac{8}{10} + \frac{1}{6}\cdot\frac{2}{10} = \frac{7}{10}$$

b) vzorka: 2. Bayesův vzorec: $$P(C|B) = \frac{\sum_{P(B\cap A_i)>0} P(A_i)\cdot P(B|A_i)\cdot P(C|(B\cap A_i))}{\sum_{i=1}^{\infty} P(B|A_i)\cdot P(A_i)}$$

C ... zákazník si vybere originál

$$P(C|\overline{B}) = \frac{\frac{8}{10}\cdot\frac{1}{6}\cdot\frac{7}{9} + \frac{2}{10}\cdot\frac{5}{6}\cdot\frac{8}{9}}{1-\cancel{P(B)}\,\frac{7}{10}} = 0{,}8397$$

8. CV.

- náhodné veličiny

$\omega \in \Omega \qquad X: \Omega \to \mathbb{R} \qquad X: (\Omega, \mathcal{A}) \to (\mathbb{R}, \mathcal{B})$

- Def. $(\Omega, \mathcal{A}, P)$. Reálnou fci. X definovanou na Ω, pro kterou platí $B \in \mathcal{B} \Rightarrow$

$\Rightarrow \{\omega \in \Omega \mid X(\omega) \in B\} = X^{-1}(B) \in \mathcal{A}$ nazýváme náhodná veličina

ω ... vzor

$X(\omega)$ ... obraz

$(-\infty, x) \quad (-\infty, x\rangle$

* Házíme 1. kostkou, $\Omega = \{\omega_1, \dots, \omega_6\}$

$\mathcal{A} = \{\emptyset, \{\omega_1, \omega_2\}, \{\omega_3, \omega_4, \omega_5, \omega_6\}, \Omega\}$

Jsou to náhodné veličiny?

a) $X(\omega_i) = i$

$X^{-1}(1) = \{\omega_1\} \notin \mathcal{A}$

$X^{-1}((-\infty, 1\rangle)$ není

b) $X(\omega_1) = X(\omega_2) = -2$

$X(\omega_3) = X(\omega_4) = X(\omega_5) = X(\omega_6) = 3$

$X^{-1}(-2) = \{\omega_1, \omega_2\} \in \mathcal{A}$

$X^{-1}(3) = \{\omega_3, \omega_4, \omega_5, \omega_6\} \in \mathcal{A}$

(správně se má ověřovat obrazy intervalů $(-\infty, x\rangle$, ale u diskrétních není třeba)

je třeba ověřit i

$X^{-1}((-\infty, 4\rangle) = X^{-1}(\{-2, 3\}) \in \mathcal{A}$

$(X^{-1}(-2) \cup X^{-1}(3))$

* $\Omega = \{\omega_1, \dots, \omega_6\}$

$\mathcal{A} = \{\emptyset, \Omega, \{\omega_1, \omega_2\}, \{\omega_3, \omega_4\}, \{\omega_5, \omega_6\}, \{\omega_1, \omega_2, \omega_3, \omega_4\}, \{\omega_1, \omega_2, \omega_5, \omega_6\}, \{\omega_3, \omega_4, \omega_5, \omega_6\}\}$

najděte náhodnou veličinu $X: \Omega \to \mathbb{R}$

např. $X(\omega_1) = X(\omega_2) = 2$

$X(\omega_3) = X(\omega_4) = 3$

$X(\omega_5) = X(\omega_6) = 4$

- rozdělení psti. $P_X(B) = P(X^{-1}(B))$, $B \in \mathcal{B}$

distribuční funkce. $F_X(x) = P(\{X \leq x\}) = P(\{\omega \in \Omega \mid X(\omega) \leq x\})$

vlastnosti: neklesající, zprava spojitá, $\lim_{x \to \infty} F(x) = 1$, $\lim_{x \to -\infty} F(x) = 0$,

$$P(\{a \leq X < b\}) = F_X(b) - F_X(a)$$

- $p(x_i) = P(X = x_i)$

- $p_i = \{p(x_i)\}_{i=1}^{\infty}$ je řada kladných čísel

$M = \{x_i\}_{i=1}^{\infty}$

(p, M) ... pravděpodobnostní fce

$F_X(x) = \sum_{x_i \leq x} p(x_i)$

* 74) $X = \{0, 1\}$

$P(X=0) = p$

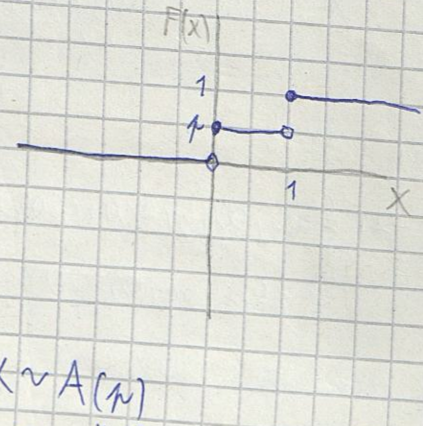
$P(X=1) = p(1) = 1-p$

najít $F(x)$ + graf

$F(-1) = \sum_{x_i \leq -1} p(x_i) = 0$

$F(0) = \cancel{P(X=0)} = \sum_{x_i \leq 0} p(x_i) = p$

$F(1) = \sum_{x_i \leq 1} p(x_i) = P(X=1) + P(X=0) = 1-p+p = 1$



je to tzv. alternativní rozdělení $X \sim A(p)$

* $P(X)$ $F(x)$ třeba psát takto do písemky: $F(x) = \begin{cases} 0 & x < 0 \\ p & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases}$

* 75) $P(X=i) = \frac{1}{6} \quad i \in \{1, \dots, 6\}$ $\qquad p(x) = \begin{cases} \frac{1}{6} & x \in \{1, \dots, 6\} \\ 0 & \text{jinak} \end{cases}$

$F(0) = \sum_{x_i \le 0} p_{x_i} = 0$

$F(1) = \frac{1}{6}$

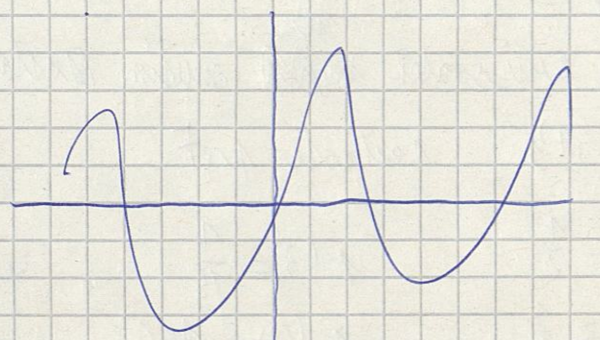
$F(2) = \frac{1}{6} + \frac{1}{6} = \frac{1}{3}$

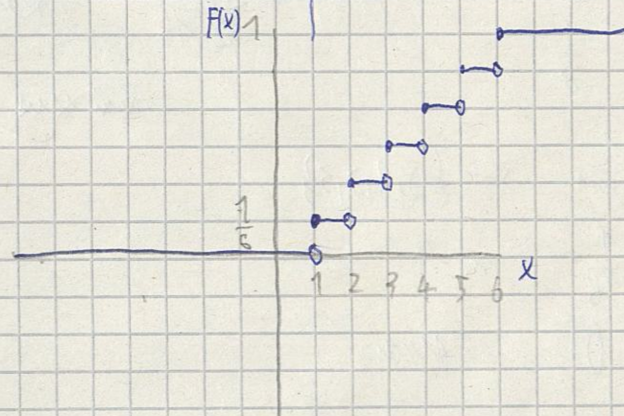
$F(3) = \frac{1}{6} + \frac{1}{6} + \frac{1}{6} = \frac{1}{2}$

$F(4) = \frac{2}{3}$

$F(5) = \frac{5}{6}$

$F(6) = 1$





$F(x) = \begin{cases} 0 & x < 1 \\ \frac{i}{6} & i \le x < i+1 \\ 1 & x \ge 6 \end{cases}$

Tomuhle rozdělení se nějak neříká

* 6) házeme třikrát hrací kostkou

$X = \{0, 1, 2, 3\}$ (kolikrát padne šestka)

$p(x) = P(X=x)$

a) pstní ~~rozdělení~~ funkce

$P(X=0) = \left(\frac{5}{6}\right)^3$

$P(X=1) = \binom{3}{1} \cdot \frac{5^2}{6^2} \cdot \frac{1}{6}$

$P(X=2) = \binom{3}{2} \cdot \frac{5}{6} \cdot \frac{1}{6^2}$

$P(X=3) = \frac{1}{6^3}$

$P(X=10) = 0$

$p(x) = P(X=x) = \begin{cases} \binom{3}{x} \cdot \left(\frac{5}{6}\right)^{3-x} \cdot \left(\frac{1}{6}\right)^{x} & x = 0,1,2,3 \\ 0 & \text{jinak} \end{cases}$

binomické rozdělení $X \sim Bin(n, p)$

b) $P(X>2) = P(X=3) = \frac{1}{6^3}$

$$P(X<2) = \sum_{x_i \le 2} p_{x_i} = \binom{3}{1} \cdot \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6} + \left(\frac{5}{6}\right)^3 \doteq 0{,}9259$$

$$P(0<X<3) = \sum_{0<x_i<3} p_{x_i} = P(X=1) + P(X=2) = \binom{3}{1}\left(\frac{5}{6}\right)^2\left(\frac{1}{6}\right) + \binom{3}{2} \cdot \frac{5}{6} \cdot \frac{1}{6^2} \doteq 0{,}4$$ [illegible]

* 4 křižovatky

77) X … počet křižovatek ~~nyní~~ než musím zastavit

im $X = \{0, \dots, 4\}$ sestavte p, F

$p(0) = \frac{1}{2}$

$p(1) = \frac{1}{4}$

$p(2) = \frac{1}{8}$

$p(3) = \frac{1}{16}$

$p(4) = \cancel{\frac{1}{32}}\ \frac{1}{16}$

$\frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}\ \frac{1}{2}$

~~není spíš~~ $\cancel{p(x) = \begin{cases} \frac{\binom{4}{x}}{16} & x \in \{0,1,2,3,4\} \\ 0 & \text{jinak} \end{cases}}$?

$$p(x) = \begin{cases} \frac{1}{2^{x+1}} & x \in \{0,1,2,3\} \\ \frac{1}{16} & x = 4 \\ 0 & \text{jinak} \end{cases}$$

$F(0) = \frac{1}{2}$

$F(1) = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$

$F(2) = \frac{3}{4} + \frac{1}{8} = \frac{7}{8}$

$F(3) = \frac{7}{8} + \frac{1}{16} = \frac{15}{16}$

$F(4) = \frac{15}{16} + \frac{1}{16} = 1$

$$F(x) = \begin{cases} 0 & x \le 0 \\ \frac{1}{2} & 0 \le x < 1 \\ \frac{3}{4} & 1 \le x < 2 \\ \frac{7}{8} & 2 \le x < 3 \\ \frac{15}{16} & 3 \le x < 4 \\ 1 & x \ge 4 \end{cases}$$

* X = počet neúspěšných hodů, než hodím 6

im $X = \{0, 1, 2, \dots\}$

$(\cancel{f\circ g})(\cancel{x}) = \cancel{fgx}$

$p(0) = \frac{1}{6}$

$p(1) = \frac{5}{6}\,\frac{1}{6}$

$p(2) = \left(\frac{5}{6}\right)^2 \cdot \frac{1}{6}$

a) $p(x) = \begin{cases} \left(\frac{5}{6}\right)^x \cdot \left(\frac{1}{6}\right) & \text{pro } x \in \{0, 1, 2, \dots\} \\ 0 & \text{jinak} \end{cases}$

b) $P(X \leq 5) = p(0) + p(1) + \dots + p(5)$

c) $P(X \geq 4) = 1 - P(X < 4) = 1 - (p(0) + \dots + p(3))$

geometrické rozdělení $X \sim Ge(p)$

* $X \sim Po(\lambda)$

$P(X = x) = e^{-\lambda} \cdot \frac{\lambda^x}{x!} \qquad x = 0, 1, 2, \dots$

x ... počet nehod za ~~jeden~~ den

$\lambda = 22$ nehod ... průměr

Jaká je pst., že se za den nestane nehoda

$$P(X=0) = e^{-22} \cdot \frac{22^0}{0!} = e^{-22} \doteq 2{,}8 \cdot 10^{-10}$$

Jaká je pst., že se stanou alespoň 3 nehody?

$P(X \geq 3) = 1 - P(X<3) = 1 - (p(0) + p(1) + p(2))$

* A ... č
  B ... b   nevracíme
  n vytáhnutí
  X ... počet černých z n
  $P(X=x) = ?$
  $X = \{0, 1, \dots, A\}$ (takhle se značí obor hodnot)

$$P(X=0) = \frac{\binom{B}{n}\binom{A}{0}}{\binom{A+B}{n}}$$

$$P(X=x) = \frac{\binom{B}{n-x}\cdot\binom{A}{x}}{\binom{A+B}{n}}$$

19 $X \sim Hg(A+B, A, n)$ hypergeometrické rozdělení

$\boxed{9. \text{CV.}}$

* -||- vracíme

$$X = \{0, 1, \dots, n\}$$

$$\cancel{P(X=x)} \Rightarrow P(X=x) = \left(\frac{A}{A+B}\right)^x \cdot \left(\frac{B}{A+B}\right)^{n-x} \cdot \binom{n}{n-x}$$

$$X \sim Bi\left(n, \frac{A}{A+B}\right)$$

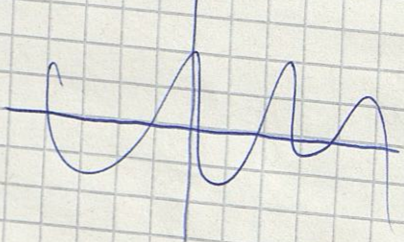
3 ~~*~~ - spojité rozdělení

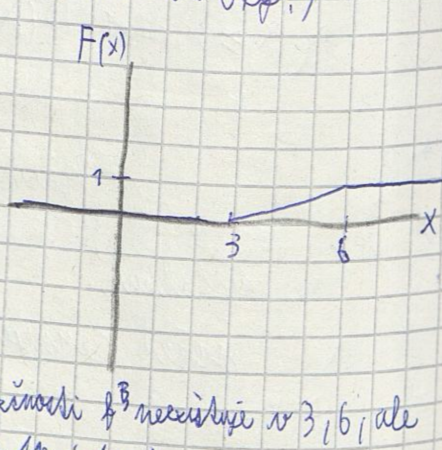
- $F(x) = P(X \leq x)$ - spojitá
- hustota pst. $f(x) = F'(x)$
- $F(x) = \int_{-\infty}^{x} f(u)\,du$
- $P(X=x) = 0$
- $P(a < X \leq b) = F(b) - F(a) = \int_a^b f(x)\,dx$

* 7) $F(x) = \begin{cases} 0 & x < 3 \\ \frac{1}{3}x - 1 & 3 \leq x < 6 \\ 1 & x \geq 6 \end{cases}$

udělejte graf, najděte hustotu

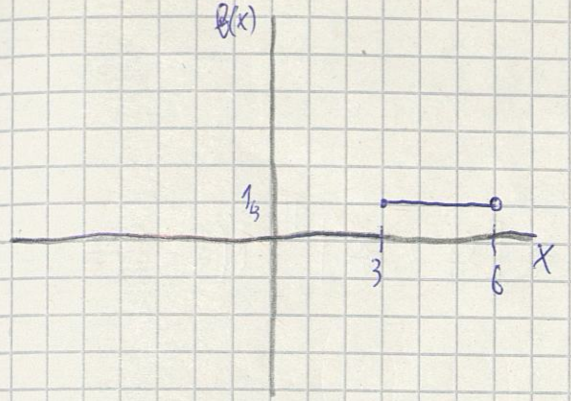
(popisovat osy!)





$$f^*(x) = \begin{cases} 1/3 & 3 \leq x < 6 \\ 0 & \text{jinak} \end{cases}$$

(ve skutečnosti f^B neexistuje v 3, 6, ale je nám to jedno).



*79)

$$F(x) = \begin{cases} 0 & x<0 \\ x^2 & 0 \le x < 1 \\ 1 & x \geq 1 \end{cases}$$

$$f(x) = \begin{cases} 2x & 0 \le x < 1 \\ 0 & \text{jinak} \end{cases}$$

$$P(1<4X<3) = P\left(\frac{1}{4}<X<\frac{3}{4}\right) = F\left(\frac{3}{4}\right)-F\left(\frac{1}{4}\right) = \left(\frac{3}{4}\right)^2-\left(\frac{1}{4}\right)^2 = \frac{1}{2}$$

*80)

$$F(x) = \begin{cases} 0 & x<0 \\ a+b\sin x & 0 \le x < \frac{\pi}{2} \\ 1 & x \geq \frac{\pi}{2} \end{cases}$$

ověřujeme tyto vlastnosti

(i) spojitost

(ii) $\lim\limits_{x\to-\infty} F(x) = 0$

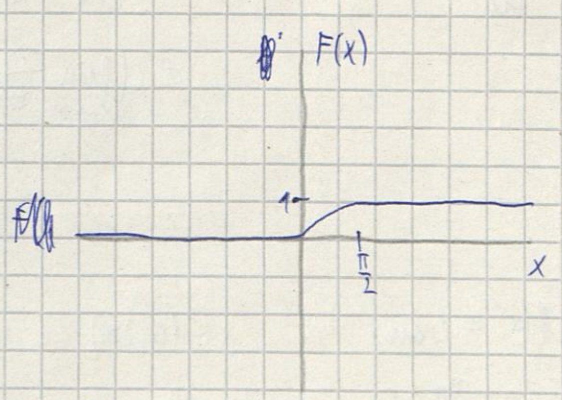
$\lim\limits_{x\to\infty} F(x) = 1$

(iii) F je neklesající

(ii) zřejmě platí

$a + b\sin 0 = 0$

$a + b \sin\frac{\pi}{2} = \frac{\pi}{2}$ $\Rightarrow$ $a=0$, $b=1$



(iii) zřejmě platí

$$-\int_{-\infty}^{\infty} f(x)\,dx = 1$$ (to je přece $\lim\limits_{x\to\infty} F(x) = 1$)

* §1) $f(x) = \begin{cases} 0 & \text{jinak} \\ c(1-x)x & 0 \le x < 1 \end{cases}$

$$\int_{-\infty}^{\infty} f(x)\,dx = \int_{-\infty}^{0} 0\,dx + c\int_0^1 (x-x^2)\,dx + \int_1^{\infty} 0\,dx = \left[\frac{cx^2}{2} - \frac{cx^3}{3}\right]_0^1 = c\left(\frac{1}{2}-\frac{1}{3}\right) = \frac{c}{6} = 1$$

$c = 6$.

$$f(x) = \begin{cases} 0 & \text{jinak} \\ 6x(1-x) & 0 \le x < 1 \end{cases}$$

$$F(x) = \int_{-\infty}^{x} 6y(1-y)\,dy = \int_0^x 6y(1-y)\,dy = \left[3y^2 - 2y^3\right]_0^x = 3x^2-2x^3$$

$$= \int_{-\infty}^{x} f(y)\,dy \quad //$$

$$F(x) = \begin{cases} 0 & x<0 \\ 3x^2-2x^3 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

\- pozn.

$$P(X > 0{,}2) = 1 - P(X \le 0{,}2) = 1 - \int_0^{0{,}2} f(x)\,dx$$

$$\left(= 1 - F(0{,}2)\right)$$

* $f(x) = cxe^{-x}$ $\quad x \in (0,\infty)$

$$\int_0^{\infty} cxe^{-x}\,dx = \begin{vmatrix} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{vmatrix} = c\left(\left[-xe^{-x}\right]_0^{\infty} + \int_0^{\infty} e^{-x}\,dx\right) = c\left[-xe^{-x} - e^{-x}\right]_0^{\infty} =$$

$$= c\left(\lim_{x\to\infty} \frac{-x}{e^x} - \lim_{x\to\infty} e^{-x} + 1\right) = c = 1$$

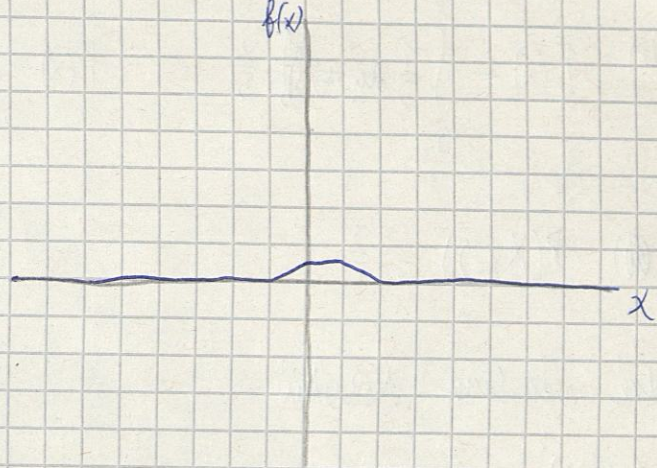
$\Rightarrow c = 1$

* $f(x) = c\sin x$ $\quad x \in (0, 2\pi)$

$$\int_0^{2\pi} c\cdot\sin x\,dx = c\int_0^{2\pi} \sin x\,dx = c\cdot\left[-\cos x\right]_0^{2\pi} = c(-\cos 2\pi) + c\cdot\cos 0 = -c + c = 0 \neq 1$$

f nemůže být hustota

* $f(x) = \begin{cases} \frac{x}{2}+\frac{1}{2} & -1<x\leq 0 \\ \frac{1}{2} & 0<x\leq 1 \\ -\frac{x}{2}+1 & 1<x\leq 2 \\ 0 & \text{jinak} \end{cases}$



$F(x) = \begin{cases} 0 & x\leq -1 \\ \frac{x^2}{4}+\frac{x}{2}+\frac{1}{4} & -1<x\leq 0 \\ \frac{x}{2}+\frac{1}{4} & 0<x\leq 1 \\ -\frac{x^2}{4}+x & 1<x\leq 2 \\ 1 & x>2 \end{cases}$

$\int_{-1}^{x}\left(\frac{t}{2}+\frac{1}{2}\right)dt = \left[\frac{t^2}{4}+\frac{t}{2}\right]_{-1}^{x} = \frac{x^2}{4}+\frac{x}{2}-\frac{1}{4}+\frac{1}{2}$
$= \frac{x^2}{4}+\frac{x}{2}+\frac{1}{4}$

$\int_{-\infty}^{x} f(x)\,dx = \int_{-\infty}^{-1} 0\,dx + \int_{-1}^{0}\left(\frac{x}{2}+\frac{1}{2}\right)dx + \int_{0}^{x}\frac{1}{2}\,dt = \frac{1}{4}+\left[\frac{t}{2}\right]_0^x = \frac{x}{2}+\frac{1}{4}$

$\int_{-\infty}^{-1} 0\,dx + \int_{-1}^{0}\left(\frac{x}{2}+\frac{1}{2}\right) + \int_{0}^{1}\frac{1}{2}dx + \int_{1}^{x}\left(-\frac{t}{2}+1\right)dt = \frac{3}{4}+\left[-\frac{t^2}{4}+t\right]_1^x = -\frac{x^2}{4}+x$

a) $P\left(-\frac{1}{2}<X\leq\frac{3}{2}\right) = F(3/2)-F\left(-\frac{1}{2}\right) = \frac{-(3/2)^2}{4}+3/2-\left(\frac{(-1/2)^2}{4}-\frac{1/2}{2}+\frac{1}{4}\right) = \frac{14}{16}$

$\left(=\int_{-\frac{1}{2}}^{3/2} f(x)\,dx\right)$

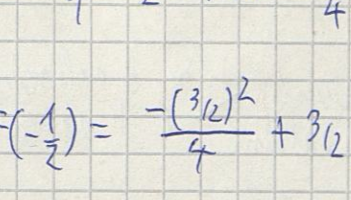
b) $P\left(X\geq\frac{1}{2}\right) = 1-P\left(X<\frac{1}{2}\right) = 1-F(1/2) = 1-\left(\frac{1}{4}+\frac{1}{4}\right) = \frac{1}{2}$

$\left(=\int_{-\infty}^{1/2} f(x)\,dx\right)$

10. CV.

* 87) 1. hr 5 min

X ... $(0,5)$ a)



rovnoměrné rozdělení

$f(x) = \begin{cases} \frac{1}{5} & x\in\langle 0,5\rangle \\ 0 & \text{jinak} \end{cases}$

b) $F(x) = \int_0^x \frac{1}{5}\,du = \frac{x}{5}$ $\quad F(x) = \begin{cases} 0 & x<0 \\ \frac{x}{5} & x \in \langle 0;5\rangle \\ 1 & x>5 \end{cases}$

c) $P(X \leq 2) = \frac{2}{5}$

– náhodné vektory diskrétní

(X,Y)

$F(x,y) = P(X \leq x, Y \leq y)$

$p(x,y) = P(X=x, Y=y)$ … simultánní pravděp. funkce

$p_X(x) = \sum_y p(x,y)$

$p_Y(y) = \sum_x p(x,y)$ $\Big\}$ marginální

$F(x,y) = \sum_{(x_i,y_i) \leq (x,y)} p(x_i,y_i)$

nezávislost: $p(x,y) = p(x) \cdot p(y)$

* 93) 10 výrobků $<$ 8 kvalitní $<$ 5 I. jakost, 3 II. jakost; 2 nekvalitní

X … počet kvalitních
Y … počet I. jakosti

$p(0,0) = \frac{\binom{2}{2}}{\binom{10}{2}} = \frac{2\cdot 1}{10\cdot 9} = \frac{1}{45}$

$p(1,0) = \frac{\binom{5}{0}\cdot\binom{3}{1}\cdot\binom{2}{1}}{\binom{10}{2}} = \frac{3\cdot 2}{45} = \frac{6}{45}$

$p(1,1) = \frac{\binom{5}{1}\cdot\binom{2}{1}}{\binom{10}{2}} = \frac{10}{45}$

$p(2,0) = \frac{\binom{3}{2}}{\binom{10}{2}} = \frac{3}{45}$

$p(2,1) = \frac{\binom{5}{1}\binom{3}{1}}{\binom{10}{2}} = \frac{15}{45}$

$p(2,2) = \frac{\binom{5}{2}}{\binom{10}{2}} = \frac{10}{45}$

$p(0,1) = 0$
$p(0,2) = 0$
$p(1,2) = 0$

$x \backslash y$	0	1	2	$p_X(x)$
0	1/45	0	0	1/45
1	6/45	10/45	0	16/45
2	3/45	15/45	10/45	28/45
$p_Y(y)$	10/45	25/45	10/45	1

(sčítáme řádky a sloupce)

opravdu $\frac{1}{45}+\frac{16}{45}+\frac{28}{45} = \frac{10}{45}+\frac{25}{45}+\frac{10}{45} = 1$

opravdu $\frac{1}{45}+\frac{6}{45}+\frac{3}{45}+\frac{10}{45}+\frac{15}{45}+\frac{10}{45} = 1$ (to je dobrá kontrola)

$p(0,0) = \frac{1}{45} \neq \frac{10}{45} = p_X(0)\cdot p_Y(0) \Rightarrow$ nejsou nezávislé

spočtěme $P(X<2, Y>0) = \frac{10}{45}+0+0+0 = \frac{10}{45}$

$F(0,0) = P(X\leq 0, Y\leq 0) = \frac{1}{45}$

$F(1,0) = \frac{1}{45}+\frac{6}{45} = \frac{7}{45}$

$F(2,0) = \frac{10}{45}$

$F(0,1) = \frac{1}{45}$

$F(0,2) = \frac{1}{45}$

$F(1,1) = \frac{17}{45}$

$F(1,2) = \frac{17}{45}$

$F(2,1) = \frac{35}{45}$

$F(2,2) = 1$

místo rozepisování $F(x,y) = \begin{cases} \frac{1}{45} & x,y \in \langle 0,1) \\ \frac{7}{45} & x \in \langle 1,2), y \in \langle 0,1) \\ \dots \end{cases}$

lze použít (i na písemce) ~~graf~~ grafický zápis

	1/45	17/45	45/45	1
2	$\frac{1}{45}$	$\frac{17}{45}$	$\frac{45}{45}$	45/45
1	$\frac{1}{45}$	$\frac{17}{45}$	$\frac{35}{45}$	35/45
0	$\frac{1}{45}$	$\frac{7}{45}$	$\frac{10}{45}$	10/45
y / x	0	1	2	

0

0

0

$F_X(x) = P(X \leq x)$

$F_X(\cancel{0}) = \frac{1}{45}$

$F_X(1) = \frac{17}{45}$

$F_X(2) = 1$

$$F_X(x) = \begin{cases} 0 & \text{jinak} \\ \frac{1}{45} & x \in [0,1) \\ \frac{17}{45} & x \in [1,2) \\ 1 & x \geq 2 \end{cases}$$

$$F_Y(y) = \begin{cases} \frac{10}{45} & y \in [0,1) \\ \frac{35}{45} & y \in [1,2) \\ 1 & y \geq 2 \\ 0 & \text{jinak} \end{cases}$$

* 92) 9 Č, 8 Z, 3 Ž 6 vytáhneme

$X \ldots$ počet Č

$Y \ldots$ počet Z

$p(x,y) = P(X=x, Y=y)$

$$p(x,y) = \begin{cases} \dfrac{\binom{9}{x} \cdot \binom{8}{y} \cdot \binom{3}{6-x-y}}{\binom{20}{6}} & \cancel{x+y}\ 3 \leq x+y \leq 6,\ x \in \{0,1,\ldots,6\},\ y \in \{0,1,\ldots,6\} \\ 0 & \text{jinak} \end{cases}$$

- náhodné vektory spojité

(X,Y)

$F(x,y) = P(X \leq x, Y \leq y)$

$f(x,y) = \dfrac{\partial F(x,y)}{\partial x \partial y}$

$F(x,y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u,v)\, du\, dv$

$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\, dy$

$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\, dx$

$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y)\, dx\, dy = 1$

$* \; 100)$ $f(x,y) = \begin{cases} c\,(x/2 + y/3) & 0<x<2,\ 0<y<3 \\ 0 & \text{jinak} \end{cases}$

a) najděte c

$$1 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)\,dx\,dy = \int_0^3\int_0^2 c\left(\frac{x}{2}+\frac{y}{3}\right)dx\,dy = c\int_0^3 \left[\frac{x^2}{4}+\frac{xy}{3}\right]_0^2 dy =$$

$$= c\int_0^3 \left(1+\frac{2y}{3}\right)dy = c\left[y+\frac{y^2}{3}\right]_0^3 = c\cdot\left(3+\frac{9}{3}\right) = c\cdot 6$$

$$c = \frac{1}{6}$$

$$f(x,y) = \begin{cases} \frac{1}{6}\,(x/2+y/3) & 0<x<2,\ 0<y<3 \\ 0 & \text{jinak} \end{cases}$$

b) $$f_X(x) = \int_0^3 \frac{1}{6}\left(\frac{x}{2}+\frac{y}{3}\right)dy = \frac{1}{6}\left[\frac{xy}{2}+\frac{y^2}{6}\right]_0^3 = \frac{1}{6}\left(\frac{3x}{2}+\frac{9}{6}\right) = \frac{x}{4}+\frac{1}{4}$$

$$f_Y(y) = \int_0^2 \frac{1}{6}\left(\frac{x}{2}+\frac{y}{3}\right)dx = \frac{1}{6}\left[\frac{x^2}{4}+\frac{xy}{3}\right]_0^2 = \frac{1}{6}\left(1+\frac{2y}{3}\right) = \frac{y}{9}+\frac{1}{6}$$

c) $$P(0<X\leq 1,\ 2<Y\leq 3) = \int_0^1\int_2^3 \frac{1}{6}\left(\frac{x}{2}+\frac{y}{3}\right)dy\,dx = \frac{1}{6}\int_0^1 \left[\frac{xy}{2}+\frac{y^2}{6}\right]_2^3 dx =$$

$$= \frac{1}{6}\int_0^1 \left(\frac{3x}{2}+\frac{3}{2}-x-\frac{2}{3}\right)dx = \frac{1}{6}\int_0^1 \left(\frac{x}{2}+\frac{5}{6}\right)dx = \frac{1}{6}\left[\frac{x^2}{4}+\frac{5x}{6}\right]_0^1 =$$

$$= \frac{1}{6}\left(\frac{1}{4}+\frac{5}{6}\right) = \frac{13}{72}$$

$\boxed{11.\text{CV.}}$

- $f(x,y) = \dfrac{\partial F(x,y)}{\partial x\,\partial y}$

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f(s_1,s_2)\,ds_1\,ds_2$$

$$f_X(x) = \int_{-\infty}^{\infty} f(x,y)\,dy$$

$$f_Y(y) = \int_{-\infty}^{\infty} f(x,y)\,dx$$

$$F_X(x) = \lim_{y\to\infty} F(x,y) \qquad F_Y(y) = \lim_{x\to\infty} F(x,y)$$

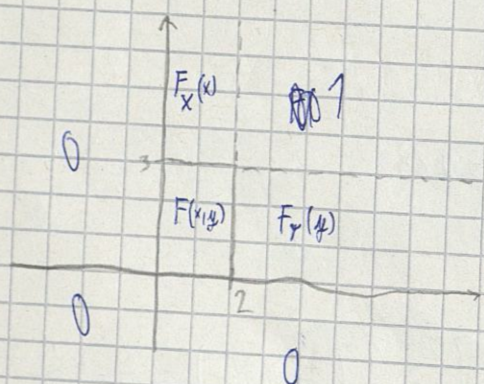
– nezávislost $F(x,y) = F_X(x) F_Y(y)$, $f(x,y) = f_X(x)\cdot f_Y(y)$
(jiná ekviv.)

* navazujeme na minulou úlohu

$$F(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} \frac{1}{6}\left(\frac{s_1}{2}+\frac{s_2}{3}\right) ds_2\, ds_1 = \frac{1}{6}\int_0^x\int_0^y \left(\frac{s_1}{2}+\frac{s_2}{3}\right) ds_2\, ds_1 = \frac{1}{6}\int_0^x \left[\frac{s_1 s_2}{2}+\frac{s_2^2}{6}\right]_0^y ds_1 =$$

$$= \frac{1}{6}\int_0^x \left(\frac{s_1 y}{2}+\frac{y^2}{6}\right) ds_1 = \frac{1}{12}\int_0^x \left(s_1 y + \frac{y^2}{3}\right) ds_1 = \frac{1}{12}\left[\frac{s_1^2 y}{2}+\frac{s_1 y^2}{3}\right]_0^x = \frac{1}{12}\left(\frac{x^2 y}{2}+\frac{x y^2}{3}\right)$$

$$= \frac{xy}{12}\left(\frac{x}{2}+\frac{y}{3}\right) \quad \text{pro} \quad (x,y) \in (0,2)\times(0,3)$$



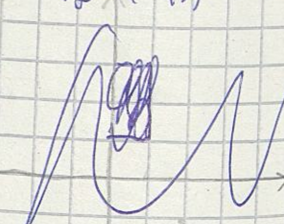
(toto vždy nakreslit)

$$F_X(x) = \int_{-\infty}^{x} \frac{1}{4}(s+1)\, ds = \int_0^x \frac{1}{4}(s+1)\, ds = \frac{1}{4}\left[\frac{s^2}{2}+s\right]_0^x = \frac{1}{4}\left(\frac{x^2}{2}+x\right) \quad x\in(0,2)$$

jiný způsob:

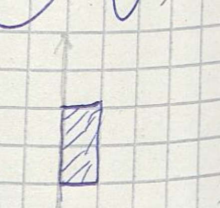
$$F_X(x) = \lim_{y\to 3^-} \frac{xy}{12}\left(\frac{x}{2}+\frac{y}{3}\right) = \frac{x}{4}\left(1+\frac{x}{2}\right) \quad x\in(0,2)$$

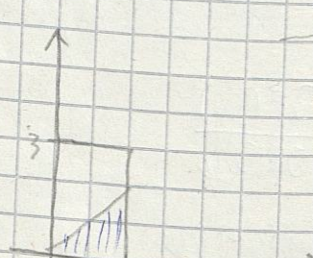
$$F_Y(y) = \lim_{x\to 2^-} \frac{xy}{12}\left(\frac{x}{2}+\frac{y}{3}\right) = \frac{y}{6}\left(1+\frac{y}{3}\right) \quad y\in(0,3)$$



$$P(0<X\le 1, 1<Y\le 3) = F(1,3) - \cancel{F(0,3)}\, F(1,1) - F(0,3) + F(0,1) =$$

$$= \frac{3}{12}\left(\frac{3}{3}+\frac{1}{2}\right) - \frac{1}{12}\left(\frac{1}{3}+\frac{1}{2}\right) = \dots = \frac{11}{36}$$





$$P(X\ge Y) = \cancel{\int_0^2\int_0^x \frac{1}{6}\left(\frac{s_1}{2}+\frac{s_2}{3}\right) ds_2\, ds_1 - \frac{1}{6}\int_0^2 [} =$$

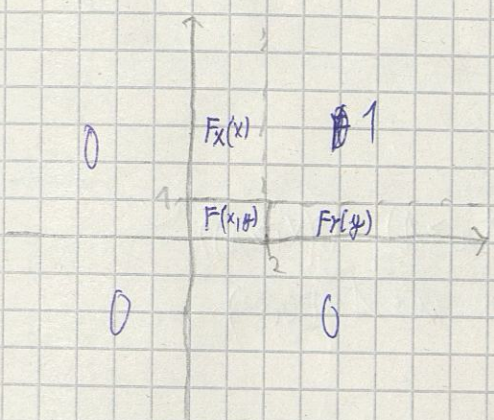
$$= \int_0^2\int_0^x \frac{1}{6}\left(\frac{x}{2}+\frac{y}{3}\right) dy\, dx = \frac{1}{6}\int_0^2 \left[\frac{xy}{2}+\frac{y^2}{6}\right]_0^x dx =$$

$= \frac{1}{6}\int_0^2 \left(\frac{x^2}{2} + \frac{x^2}{6}\right)dx = \frac{1}{6}\left[\frac{4x^3}{18}\right]_0^2 = \frac{1}{6}\cdot\frac{4\cdot 8}{18} = \dots = \frac{8}{27}$

$\frac{1}{4}(x+1)\cdot\frac{1}{6}(1+2y/3) = \frac{x}{24} + \frac{xy}{36} + \frac{1}{24} + \frac{y}{36} \neq \cancel{\frac{1}{12}\left(\frac{x^2 y}{2}\right.}\ \frac{1}{6}\left(\frac{x}{2} + \frac{y}{3}\right)$

nejsou nezávislé

* $F(x,y) = \frac{1}{4}x^2y^2 \qquad x \in [0,2],\ y \in [0,1]$



$F_X(x) = \lim_{y\to 1^-} \frac{1}{4}x^2y^2 = \frac{1}{4}x^2$

$F_Y(y) = \lim_{x\to 2^-} \frac{1}{4}x^2y^2 = y^2$

$\frac{1}{4}x^2y^2 = \frac{1}{4}x^2\cdot y^2$ … jsou nezávislé

* ~~lahvové~~ litrové láhve $0{,}98\,l - 1{,}02\,l$

3 láhve, pst., že vyjmenně naplněná ~~ma~~ má $\geq 1\,l$

$(X_1, X_2, X_3) \qquad X_i$ … objem i-té láhve

$P(\min\{X_1, X_2, X_3\} \geq 1) = P(X_1 \geq 1 \wedge X_2 \geq 1 \wedge X_3 \geq 1) = P(X_1 \geq 1)\cdot P(X_2 \geq 1)\cdot P(X_3 \geq 1)$
$= \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8}$

$F_{X_1}(x_1) = \int_{0,98}^{x_1} \frac{1}{0{,}04}\,dt = \frac{x_1}{0{,}04} - \frac{0{,}98}{0{,}04} = 25x_1 - 24{,}5$

$P(X_1 \geq 1) = 1 - P(X_1 < 1) = 1 - F_{X_1}(1) = 1 - 0{,}5 = \frac{1}{2}$

nejvíce naplněná láhev $\leq 1{,}01\,l$

$P(\max\{X_1, X_2, X_3\} \leq 1{,}01) = P(X_1 \leq 1{,}01)\cdot P(X_2 \leq 1{,}01)\cdot P(X_3 \leq 1{,}01) = (25\cdot 1{,}01 - 24{,}5)^3 =$
$= \frac{27}{64}$

* $X \sim Ex(\lambda)$

$$f(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{jinak} \end{cases}$$

$$F_X(x) = \int_0^x \lambda \cdot e^{-\lambda t} \, dt = \left[-e^{-\lambda t}\right]_0^x = -e^{-\lambda x} + 1 \quad \text{pro } x \geq 0.$$

104)

* $X_1, X_2, \dots, X_n \sim Ex(\lambda)$

X_i ... čekání i-tého zákazníka ve frontě

nejkratší čekání $\geq t$

$$P(\min\{X_1, \dots, X_n\} \geq t) = P(X_1 \geq t) \cdot \dots \cdot P(X_n \geq t) = (P(X_1 \geq t))^n =$$

$$= (1 - F(t))^n = \begin{cases} (1 + e^{-\lambda t} - 1)^n = e^{-\lambda t n} & \text{pro } t \in [0, \infty) \\ (1-0)^n = 1 \end{cases}$$

nejdelší čekání $\leq t$

$$P(\max\{X_1, \dots, X_n\} \leq t) = P(X_1 \leq t) \cdots P(X_n \leq t) = (P(X \leq t))^n =$$

$$= \begin{cases} (-e^{-\lambda t} + 1)^n & \text{pro } t \in \langle 0, \infty) \\ 0 & \text{pro } t \in (-\infty, 0) \end{cases}$$

105)

* $X_1 \sim Ex(\lambda_1) \quad t \geq 0$ ... čas ud.

$X_2 \sim Ex(\lambda_2)$

a) 1. součástka přežije dobu t

$$P(X_1 > t) = 1 - P(X_1 \leq t) = 1 - (e^{-\lambda t} + 1) = e^{-\lambda t}$$

b) $P(X_1 > t \wedge X_2 > t) = P(X_1 > t) \cdot P(X_2 > t) = e^{-\lambda_1 t} \cdot e^{-\lambda_2 t} = e^{-t(\lambda_1 + \lambda_2)}$

c) $P(X_1 > t \wedge X_2 \leq t) = P(X_1 > t) \cdot P(X_2 \leq t) = e^{-\lambda_1 t} \cdot (1 - e^{-\lambda_2 t})$

d) (e) DÚ $P(X_1 \leq t \wedge X_2 > t) = e^{-\lambda_2 t} \cdot (1 - e^{-\lambda_1 t})$

d, (e) DÚ

$\boxed{12.\text{CV.}}$

- 2 zápočet - od náhodné veličiny (naučit se rozdělení z paměti)

- $X \rightsquigarrow Y$ pomocí transformace

$Y = h(X)$ např. $Y = X^2$

$$F_Y(y) = P(Y \leq y) = P(h(X) \leq y) = P_X(X \leq \dots)$$

Pokud $\exists h^{-1}$ (spojitý případ)

$$f_Y(y) = f_X(h^{-1}(y)) \left| \frac{\partial h^{-1}(y)}{y} \right|$$

diskrétní případ

$$p_Y(y) = P(Y = y) = P(h(X) = y) = P_X(X = \dots)$$

* $p(x) = \begin{cases} \frac{\lambda^x}{x!} e^{-\lambda} & x = 0, 1, \dots \quad \lambda > 0 \\ 0 & \text{jinak} \end{cases}$

$p(x) = P(X = x)$

$Y = 4X$

$$p_Y(y) = P(Y = y) = P(4X = y) = P(X = \tfrac{y}{4}) = p(\tfrac{y}{4})$$

$$p(y) = \begin{cases} \frac{\lambda^{(\frac{y}{4})}}{(\frac{y}{4})!} e^{-\lambda} & y = 0, 4, 8, \dots \\ 0 & \text{jinak} \end{cases}$$

* $Y = 2X + 1$

$p(x) = \begin{cases} 1/3 & x = 1, 2, 3, \dots \\ 0 & \text{jinak} \end{cases}$

$$p_Y(y) = P(Y = y) = P(2X + 1 = y) = P(X = \tfrac{y - 1}{2})$$

$$p_Y(y) = \begin{cases} 1/3 & y = 3,5,7,\dots \\ 0 & \text{jinak} \end{cases}$$

* $Y = X^2 - 1$

X	-2	-1	0	1	2
$p(x)$	0,1	0,25	0,15	0,3	0,2

... 0 jinak

$Y + 1 = X^2$

$X = \pm\sqrt{Y+1}$

x	± 2	± 1	0
y	3	0	-1
$p(y)$	0,3	0,55	0,15

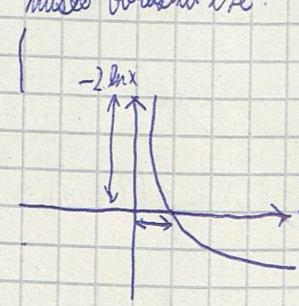
alternativně: $p_Y(y) = P(Y=y) = P(X^2 - 1 = y) = \begin{cases} P(X = \sqrt{y+1} \vee X = -\sqrt{y+1}) & y = 0 \dots \\ P(X = \sqrt{y+1}) & y = \dots \\ 0 & \text{jinak} \end{cases}$

* $Y = -2\ln X$

$$f_X(x) = \begin{cases} 1 & x \in (0,1) \\ 0 & \text{jinak} \end{cases}$$

$x = 0$ "$y = -2\ln 0 = \infty$"
$x = 1$ "$y = -2\ln 1 = 0$"

místo obrázku lze:



vzorečkem: $-\frac{Y}{2} = \ln X$

$$X = e^{-\frac{Y}{2}}$$

$$f_Y(y) = \underbrace{f_X\left(e^{-\frac{y}{2}}\right)}_{1} \cdot \underbrace{\left|e^{-\frac{y}{2}} \cdot \left(-\frac{1}{2}\right)\right|}_{e^{-\frac{y}{2}} \cdot \frac{1}{2}} = \begin{cases} e^{-\frac{y}{2}} \cdot \frac{1}{2} & y \in (0,\infty) \\ 0 & \text{jinak} \end{cases}$$

bez vzorečku: $F_Y(y) = P(Y \le y) = P(-2\ln X \le y) = P_X\left(X \ge e^{-\frac{y}{2}}\right) =$

$= 1 - P_X\left(X \le e^{-\frac{y}{2}}\right) = 1 - F_X\left(e^{-\frac{y}{2}}\right)$

$$F_X(x) = \int_{-\infty}^{x} f(t)\,dt = \int_0^x 1\,dt = x$$

$$F_X(x) = \begin{cases} 0 & x \le 0 \\ x & 0 < x < 1 \\ 1 & x \ge 1 \end{cases}$$

$$F_Y(y) = 1 - F_X(e^{-y/2}) = \begin{cases} 1 - e^{-\frac{y}{2}} & y \in (0,\infty) \\ 0 & \text{jinak} \end{cases}$$

$$f_Y(y) = \begin{cases} -e^{-\frac{y}{2}} \cdot (-1/2) = e^{-\frac{y}{2}} \cdot \frac{1}{2} \ \dots & y \in (0,\infty) \\ 0 & \text{jinak} \end{cases}$$

* $Y = \frac{1}{2} X^2$ $\quad f_X(x) = \begin{cases} \frac{3}{4}(1-x^2) & x \in (-1,1) \\ 0 & \text{jinak} \end{cases}$ $\qquad \frac{1}{2}X^2 \le y$

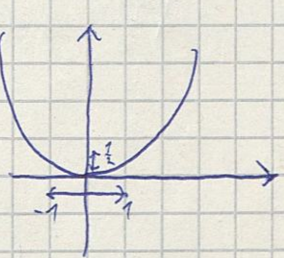
$X = \pm\sqrt{2Y}$ $\qquad |X| \le \sqrt{2y}$

$$F_Y(y) = P(Y \le y) = P(-\sqrt{2y} \le X \le \sqrt{2y}) = F(\sqrt{2y}) - F(-\sqrt{2y})$$

$$\int_{-1}^{x} \frac{3}{4}(1-t^2)\,dt = \frac{3}{4}\left[t - \frac{t^3}{3}\right]_{-1}^{x} = \frac{3}{4}\left(x - \frac{x^3}{3} + \frac{2}{3}\right)$$

$$F_X(x) = \begin{cases} 0 & x \le -1 \\ \frac{3}{4}\left(x - \frac{x^3}{3} + \frac{2}{3}\right) & x \in (-1,1) \\ 1 & x \ge 1 \end{cases}$$

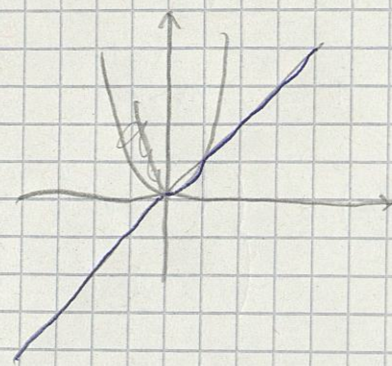
$$F_Y(y) = F(\sqrt{2y}) - F(-\sqrt{2y}) = \frac{1}{4}\left(3\sqrt{2y} - (\sqrt{2y})^3 + 2\right) - \frac{1}{4}\left(-3\sqrt{2y} + (\sqrt{2y})^3 + 2\right) = \frac{3}{2}\sqrt{2y} - \frac{1}{2}(2y)^{\frac{3}{2}}$$



$$f_Y(y) = \begin{cases} \frac{3}{2} \cdot \frac{1}{\sqrt{2y}} - \frac{1}{2} \cdot \frac{3}{2} \cdot 2 \cdot \sqrt{2y} = \frac{3}{2\sqrt{2y}} - \frac{3}{2}\sqrt{2y} & y \in (0, \frac{1}{2}) \\ 0 & \text{jinak} \end{cases}$$

* $Y = \min\{X, X^2\}$

$$f_X(x) = \begin{cases} 1/2 & x \in (0,2) \\ 0 & \end{cases}$$



rozdělme na případy: $x \in (0,1)$

$x \in (0,1)$: $F_Y(y) = P(Y \leq y) = P(X^2 \leq y) = P(X \leq \sqrt{y})$

$$F_X(x) = \int_0^x \frac{1}{2}\,dt = \left[\frac{1}{2}t\right]_0^x = \begin{cases} \frac{x}{2} & x \in (0,2) \\ 0 & x \leq 0 \\ 1 & x \geq 2 \end{cases}$$

$$F_Y(y) = \begin{cases} \frac{\sqrt{y}}{2} & y \in (0,1) \\ 0 & y \leq 0 \\ 1 & y \geq 2 \\ ? & y \in (1,2) \end{cases}$$

$x \in \langle 1,2)$: $F_Y(y) = P(Y \leq y) = P(X \leq y) = \begin{cases} \frac{y}{2} & y \in (1,2) \\ 1 & y \geq 2 \\ 0 & y \leq 0 \\ \frac{\sqrt{y}}{2} & y \in (0,1) \end{cases}$

$$f_Y(y) = \begin{cases} \frac{1}{4\sqrt{y}} & y \in (0,1) \\ \frac{1}{2} & y \in \langle 1,2) \\ 0 & \text{jinak} \end{cases}$$