# 1. CV

Skripta z mat. analýzy - Zemánek, Zemek, cvič. sešity
- podpisy 2 absence, omluvenky za studijní
- 3 písemky, označeno týdenním předem (20.10., 10.11., 8.12.)
- Integrální počet R - Zemánek, Došlý

Dělení polynomů, Hornerovo schéma, rovn. rozklady

$$\frac{x}{x^3+1} = \frac{-\frac{1}{3}}{x+1} + \frac{\frac{x}{3}+\frac{1}{3}}{x^2-x+1}$$

$$\frac{2x^5+5x^3-x^2+2x-1}{x^6+2x^4+x^2} = \frac{2}{x} - \frac{1}{x^2} + \frac{1}{x^2+1} + \frac{x}{(x^2+1)^2}$$

$$x^4+1 = (x^2+\sqrt{2}x+1)(x^2-\sqrt{2}x+1)$$

# 2. PŘ - funkce reálné proměnné (1.3)

$$f(x) = \ln\left(x+\sqrt{x^2+1}\right)$$

$$f(-x) = \ln\left(\left(-x+\sqrt{x^2+1}\right)\cdot\frac{x+\sqrt{x^2+1}}{x+\sqrt{x^2+1}}\right) = \ln\left(\frac{1}{x+\sqrt{x^2+1}}\right) = -f(x)$$

f je lichá

# 2. CV.

$$\frac{2x^4-x^3+x^2+3x+3}{x^2-1} = 2x^2-x+3+\frac{2x+6}{x^2-1} = 2x^2-x+3+\frac{4}{x-1}-\frac{2}{x+1}$$

$f(x) = \arcsin(\sin x)$

$D_f = \mathbb{R}$

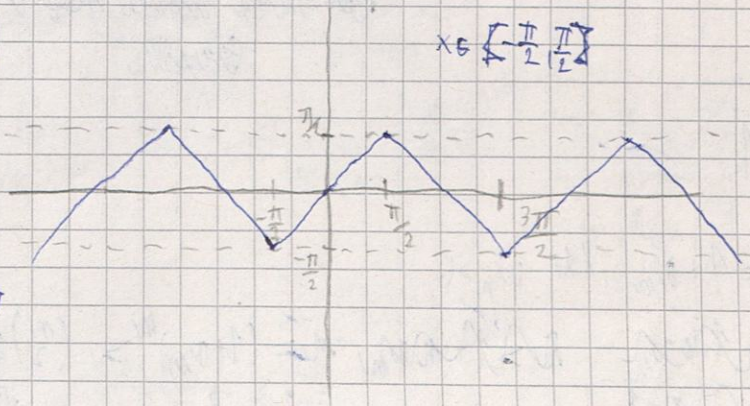
$H_f = \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

nejmenší perioda $2\pi$

$x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$x \in \left[\frac{\pi}{2}, \frac{3\pi}{2}\right]$

$x = \Delta + \pi$



## 3. PŘ

$\lim_{n\to\infty} a_n = 0$ a $(b_n)$ je omezená $\Rightarrow \lim_{n\to\infty}(a_n \cdot b_n) = 0$

– věta 2.12. | ve 1. části (ii) nemůžeme být ostré nerovnosti, viz. $a_n = \frac{1}{n}$, $b_n = 0$

– věta 2.15. – (iv) $b \neq 0 \Rightarrow b_n \neq 0$ pro $n \geq n_0$

## 3. PŘ

– nakreslit graf fce $f(x) = |x^2 - 4|x| + 3|$

$= ||x|^2 - 4|x| + 3|$ (nejdřív odstraním abs.)

$\lim_{n\to\infty} \frac{n}{n+1} = 1$

$\lim_{n\to\infty} \frac{3n+4}{2n^2+6n-5} = 0$

$\lim_{n\to\infty} \frac{a_k n^k + a_{k-1} n^{k-1} + \dots + a_0}{b_l n^l + b_{l-1} n^{l-1} + \dots + b_0} = \begin{cases} \frac{a_k}{b_l} & k = l \\ \operatorname{sgn}\left(\frac{a_k}{b_l}\right) \cdot \infty & k > l \\ 0 & k < l \end{cases}$

$a_k, b_l \neq 0$

$\lim_{n\to\infty} \sqrt[n]{2} = \lim_{n\to\infty} 2^{\frac{1}{n}} = 2^0 = 1$

$\sqrt[n]{2} = 1 + a_n$, kde $a_n > 0$

$2 = 1 + \binom{n}{1} a_n + \binom{n}{2} a_n + \dots + a_n$

$2 > 1 + n a_n$

$a_n < \frac{1}{n}$

$1 + 0 \leq \sqrt[n]{2} \leq 1 + \frac{1}{n}$

důkaz pomocí věty o 3 limitách

Bernoulli

$\lim_{n\to\infty} \sqrt[n]{n} = 1$

$\sqrt[n]{n} = 1 + a_n$, kde $a_n > 0$

$n = (1 + a_n)^n \Rightarrow \binom{n}{2} a_n^2 = \frac{n(n-1)}{2} a_n^2$

$a_n^2 < \frac{2}{n-1}$ $\quad a_n < \frac{\sqrt{2}}{\sqrt{n-1}}$

$$1+0 \le \sqrt[n]{n} \le 1+\sqrt{\frac{2}{n-1}}$$

věta o třech limitách

$$\lim_{n\to\infty} \frac{\sqrt{n^2+1}+\sqrt[3]{n+1}}{n+\sqrt[3]{n^2+3n+1}} = \lim_{n\to\infty} \frac{\sqrt{1+\frac{1}{n^2}}+\sqrt[3]{\frac{1}{n^2}+\frac{1}{n^3}}}{1+\sqrt[3]{\dots}} = 1$$

$$\lim_{n\to\infty} \left(\sqrt{n+1}-\sqrt{n-1}\right) = \lim_{n\to\infty} \frac{2}{\sqrt{n+1}+\sqrt{n-1}} = 0$$

$$\lim_{n\to\infty} \frac{3^n+n^5-4n}{2^n+3^{n+1}+n^2} = \lim_{n\to\infty} \frac{1+\frac{n^5}{3^n}-\frac{4n}{3^n}}{\frac{2^n}{3^n}+3+\frac{n^2}{3^n}} = \frac{1}{3}$$

$$\lim_{n\to\infty} \frac{5n^3+2}{3n^3-2n^2} \cdot 6^{\frac{4n}{2n+2}} = \frac{5}{3}\cdot 36 = 60$$

$$\lim_{n\to\infty} (n^2-n) = \lim_{n\to\infty} n^2\cdot\left(1-\frac{1}{n}\right) = \infty\cdot 1 = \infty$$

$$\lim_{n\to\infty} (n^2-n) = \lim_{n\to\infty} \frac{n^4-n^2}{n^2+n} = \frac{n^2-1}{1+\frac{1}{n}} = \infty$$

$$\lim_{n\to\infty} \left(\frac{n^2+4n+5}{n^2+3n+4}\right)^n = \lim_{n\to\infty} \left(1+\frac{n+1}{n^2+3n+4}\right)^n = \lim_{n\to\infty} \left(1+\frac{1}{\frac{n^2+3n+4}{n+1}}\right)^{\frac{n^2+3n+4}{n+1}}$$

$$= \lim_{n\to\infty} \sqrt{e} = \lim_{n\to\infty} e^{\frac{n}{n+1}} = e$$

$$\lim_{n\to\infty} \left(1+\frac{a}{n}\right)^n = \lim_{n\to\infty} \left(\left(1+\frac{1}{\frac{n}{a}}\right)^{\frac{n}{a}}\right)^a = e^a$$

pro $a>0$

pro $a=0$ triviální

$$\lim_{n\to\infty} \left(1+\frac{a}{n}\right)^n = \lim_{n\to\infty} \left(\left(1-\frac{1}{-\frac{n}{a}}\right)^{-\frac{n}{a}}\right)^{-a} = \left(\frac{1}{e}\right)^{-a} = e^a$$

pro $a<0$

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty} \frac{\left(1+\frac{1}{n}\right)^n}{\left(1+\frac{1}{n}\right)^n} = \frac{e}{e}$$

$$\lim_{n\to\infty} \left(1-\frac{1}{n}\right)^n = \lim_{n\to\infty} \left(\frac{n-1}{n}\right)^n = \lim_{n\to\infty} \left(\frac{n}{n+1}\right)^{n+1} = \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^{n+1}} = \frac{1}{e}$$

$$= \lim_{n\to\infty} \frac{1}{\left(1+\frac{1}{n}\right)^n}\cdot\frac{n}{n+1} = \frac{1}{e}$$

$$\lim_{n\to\infty} \frac{\sin n}{n+\sqrt{n}-1} = 0$$

$$-\frac{1}{n} \le \frac{\sin n}{n+\sqrt{n}-1} \le \frac{1}{n}$$

věta o 3 lim.

1) $\lim_{n\to\infty}\left(\frac{n^2+2n+3}{n^2+3n+4}\right)^n = \frac{1}{e}$

2) $\lim_{n\to\infty}\left(\sqrt[2]{2}\cdot\sqrt[4]{2}\cdot\sqrt[8]{2}\cdots\sqrt[2^n]{2}\right)=2$ } D.Ú.

3) $\lim_{n\to\infty}\frac{n}{2^n}=0$

1) $\lim_{n\to\infty}\left(1-\frac{n+1}{n^2+3n+4}\right)^n = \lim_{n\to\infty}\left(\left(1-\frac{1}{\frac{n^2+3n+4}{n+1}}\right)^{\frac{n^2+3n+4}{n+1}}\right)^{\frac{n(n+1)}{n^2+3n+4}} = \frac{1}{e}$

2) $\lim_{n\to\infty} 2^{\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots+\frac{1}{2^n}} = \lim_{n\to\infty} 2^{\frac{1}{2}\cdot\left(\frac{\frac{1}{2^n}-1}{\frac{1}{2}-1}\right)} = 2^1 = 2$

3) $\lim_{n\to\infty}\frac{n}{2^n}$

jde to i

$2^n=(1+1)^n$

$n\geq 4$

$0\leq\frac{n}{2^n}\leq\frac{1}{n}$

$0=\lim_{n\to\infty}0\leq\lim_{n\to\infty}\frac{n}{2^n}\leq\lim_{n\to\infty}\frac{1}{n}=0$

$\lim_{n\to\infty}\frac{n}{2^n}=0$

## 4. PŘ

- mezi sup A a A není žádný interval
- věta 2.20.
- věta 2.26.

$\left(1+\frac{1}{n}\right)^{n+1} > \left(1+\frac{1}{n}\right)^n$

$1+\frac{1}{n+1} > \left(1+\frac{1}{n}\right)^{\frac{n}{n+1}}$

$(n+1)\cdot\left(1+\frac{1}{n}\right)^{\frac{n}{n+1}} < 1+\underbrace{\left(1+\frac{1}{n}\right)+\left(1+\frac{1}{n}\right)+\cdots+\left(1+\frac{1}{n}\right)}_{n} = n+2$

$$\left(1+\frac{1}{n}\right)^{n+1} < \left(1+\frac{1}{n-1}\right)^{n}$$

$$\left(1+\frac{1}{n}\right)^{\frac{n+1}{n}} < 1+\frac{1}{n-1}$$

$$n\left(1+\frac{1}{n}\right)^{\frac{n+1}{n}} < \underbrace{\left(1+\frac{1}{n}\right)+\left(1+\frac{1}{n}\right)+\dots+\left(1+\frac{1}{n}\right)}_{n-1} + \left(1+\frac{1}{n}\right)^2 = n+\frac{n+1}{n}+\frac{1}{n^2} < n+\frac{n}{n-1}$$

$$n(n+1)(n-1)+n-1 < n^3$$

$$n^3-n+n-1 < n^3$$

1, 1,2, 1,2,3, 1,2,3,4 ... všechny $n \in \mathbb{N}$ jsou přirozená
jde i pro $\mathbb{Z}$, $\mathbb{Q}$

- věta 2.32.
- lemma 2.36.
- věta 2.39. - Bolzano - Weierstrassova věta
- def 2.40.
  horní limita - limita superior
- Cauchyovská posloupnost
- věta 2.43.

## 5. PŘ

- def 4.1.
- věta 4.7., 4.8., 4.9., 4.10., 4.11., 4.13., 4.15.

## 4. CV

* $a_n = \frac{1}{2}\cdot\left[(a+b)+(-1)^n\cdot(a-b)\right]$ $a,b \in \mathbb{R}$

$a_{2n} = a$
$a_{2n+1} = b$
} 2 hromadné body pokud $a \neq b$
1 pokud $a = b$

$H(a_n) = \{a, b\}$

* $a_n = n^{(-1)^n}$

2|n $a_{2n} = n$

2∤n $a_{2n-1} = \frac{1}{2n}$

$H(a_n) = \{0, \infty\}$

* $a_n = \sqrt[n]{1 + 2^{n(-1)^n}}$

2|n $a_{2n} = \sqrt[n]{1 + 2^n}$ ... $\lim_{n\to\infty} a_n = 2$

2∤n $a_n = \sqrt[n]{1 + \frac{1}{2^n}}$ ... $\lim_{n\to\infty} a_n = 1$

$H(a_n) = \{1, 2\}$

$\liminf_{n\to\infty} a_n = 1$

$\limsup_{n\to\infty} a_n = 2$

$$2 \le \sqrt[n]{1 + 2^n} \le 2^{\frac{n+1}{n}}$$

$$2 = \lim_{n\to\infty} 2 \le \sqrt[n]{1 + 2^n} \le \lim_{n\to\infty} 2^{\frac{n+1}{n}} = 2$$

* $a_n = -1 + \frac{n^2 + \cos n}{n^2 + 3} \cdot \sin\frac{n\pi}{2}$

$$= -1 + \frac{1 + \frac{\cos n}{n^2}}{1 + \frac{3}{n^2}} \cdot \sin\frac{n\pi}{2}$$

$\lim_{n\to\infty} a_n = -1 + \sin\frac{n\pi}{2}$

$H(a_n) = \{-2, -1, 0\}$

$\limsup_{n\to\infty} = 0$ $\liminf_{n\to\infty} = -2$

$$\lim_{n\to\infty} \frac{1 + a + a^2 + \dots + a^n}{1 + b + b^2 + \dots + b^n} = \lim_{n\to\infty} \frac{b-1}{a-1} \qquad |a|<1,\ |b|<1$$

- každá posloupnost má alespoň 1 hromadný bod

$a_n = \{1, \frac{1}{2}, \frac{1}{3}, 1+\frac{1}{3}, \frac{1}{2}+\frac{1}{3}, \frac{1}{4}, 1+\frac{1}{4}, \frac{1}{2}+\frac{1}{4}, \frac{1}{3}+\frac{1}{4}, \dots, \frac{1}{n}, 1+\frac{1}{n}, \frac{1}{2}+\frac{1}{n}, \dots, \frac{1}{n-1}+\frac{1}{n}, \dots\}$

$H \neq \emptyset$ $H(a_n) = \{0, 1, \frac{1}{2}, \frac{1}{3}, \dots\}$

$$\lim_{x\to 1} \frac{x^2-1}{x^3-1} = \lim_{x\to 1} \frac{x+1}{x^2+x+1} = \frac{2}{3}$$

$$\lim_{x\to 2} \frac{x^2}{x^2-3x+2} = \lim_{x\to 2} \frac{x^2}{(x-1)(x-2)} \quad \left(\frac{4}{0}\right)$$

$$\lim_{x\to 2^+} \frac{x^2}{(x-1)(x-2)} = +\infty$$

$$\lim_{x\to 2^-} \frac{x^2}{(x-1)(x-2)} = -\infty$$

(nexistuje)

$$\lim_{x\to 0} \frac{\sqrt{1+x}-\sqrt{1-x}}{x} = \lim_{x\to 0} \frac{2x}{x\left(\sqrt{1+x}+\sqrt{1-x}\right)} = \lim_{x\to 0} \frac{2}{\sqrt{1+x}+\sqrt{1-x}} = 1$$

$$\lim_{x\to 0} \frac{e^x-1}{x} = 1$$

$$\lim_{x\to 0} \frac{\sin(5x)}{\sin(3x)} = \frac{\lim_{x\to 0} \frac{\sin(5x)\cdot 5x}{5x}}{\lim_{x\to 0} \frac{\sin(3x)\cdot 3x}{3x}} = \frac{5}{3}$$

$$\lim_{x\to 0} \frac{\arcsin x}{x} = \lim_{x\to 0} \frac{\arcsin x}{\sin(\arcsin x)} = 1$$

$$\lim_{x\to 0} \frac{\sin^2 x}{x} = \lim_{x\to 0} \sin x \cdot \frac{\sin x}{x} = 0$$

$$\lim_{x\to 0} \frac{x}{\sin x} \cdot \frac{1}{\sin x} = +\infty$$

$$\lim_{x\to\infty} \frac{x+\sin x}{x+\cos x} = \lim_{x\to\infty} \frac{1+\frac{\sin x}{x}}{1+\frac{\cos x}{x}} = 1$$

$$\lim_{x\to\infty} \operatorname{arccos} \frac{1}{1+x} = \frac{\pi}{2}$$

$$\lim_{x\to 0} \frac{e^{5x}-e^{-x}}{\sin 2x} = \lim_{x\to 0} \frac{2x}{\sin 2x} \cdot \frac{1}{2x} \cdot (e^{5x}-e^{-x}+1-1) = \frac{1}{2}\lim_{x\to 0} 5\left(\frac{e^{5x}-1}{5x} + \frac{1}{5}\cdot\frac{e^{-x}-1}{-x}\right) = 3$$

$$\lim_{x\to 0} \frac{1-\cos x}{x^2} = \frac{1}{2} \quad \text{D.Ú.}$$

$$\lim_{x\to 0} \frac{\sin^2 x}{x^2} \cdot \frac{1}{1+\cos x} = \frac{1}{2}$$

## 6. PŘ

- věta 4.16., 4.21., 4.23., 4.24
- lemma 4.18.
- def. 4.19.

## 7. PŘ

- Def. 4.32., 5.1.
- Věta 4.33., 4.35., 4.40.

$C[a,b]$ .. značení

## 5. CV

* $\lim\limits_{x\to\frac{\pi}{4}} \frac{\cos x-\sin x}{\cos 2x} = \lim\limits_{x\to\frac{\pi}{4}} \frac{\cos^2 x-\sin^2 x}{\cos 2x(\cos x+\sin x)} = \lim\limits_{x\to\frac{\pi}{4}} \frac{1}{\cos x+\sin x} = \frac{1}{\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}} = \frac{\sqrt{2}}{2}$

* $\lim\limits_{x\to\infty} \frac{x^5-3x^2+8x-6}{-2x^3+8x-14} = -\infty$

* $\lim\limits_{x\to\infty} \frac{3^x+2^x+x^{157}-12x^2}{\sqrt[5]{x^{12}+3x^2}+3^{x-1}\beta} = \lim\limits_{x\to\infty} \frac{3^x}{3^{x-1}} = 3$

* $\lim\limits_{x\to-\infty} \frac{2^x+\sqrt{x^6-4x^3}+x^5-x^4+7}{3^x+\sqrt[3]{x^2+8x}+(x-3)^5-2x+1} = \lim\limits_{x\to-\infty} \frac{\sqrt{x^6-4x^3}+x^5-x^4+7}{\sqrt[3]{x^2+8x}+(x-3)^5-2x+1} = \lim\limits_{x\to\infty} \frac{x^5}{(x-3)^5} = 1$

* $\lim\limits_{x\to\infty} \frac{\ln x}{x^2} = 0$

* $\lim\limits_{x\to\infty} (x-\ln x) = \lim\limits_{x\to\infty} x\left(1-\frac{\ln x}{x}\right) = \infty$

* $\lim\limits_{x\to-\infty} \frac{\sin x+x\cdot\operatorname{arctg} x}{3+\sqrt{1+x^2}} = \lim\limits_{x\to-\infty} \frac{\frac{\sin x}{x}-\operatorname{arctg} x}{\frac{3}{x}+\sqrt{\frac{1}{x^2}+1}} = \frac{\pi}{2}$

* $f(x) = (x^2-1)\cdot\sin\frac{1}{x-1}$

$D(f) = \mathbb{R}\setminus\{1\}$ spojitě dodefinujte funkci

$\lim\limits_{x\to1} f(x) = 0$ Dodefinování $F(x) = \begin{cases} f(x) & x\neq1 \\ 0 & x=1 \end{cases}$

* $f(x) = \operatorname{arctg} \frac{x-1}{x} = \operatorname{arctg}\left(1 - \frac{1}{x}\right)$

$x = 0$: nespojitost 1. druhu $\lim_{x \to 0^+} f(x) = -\frac{\pi}{2}$

$\lim_{x \to 0^-} f(x) = \frac{\pi}{2}$

* $f(x) = \operatorname{arctg} \frac{x-1}{x^2}$

nespojitost odstranitelná

* $f(x) = e^{x + \frac{1}{x}}$

$\lim_{x \to 0^-} = \left| e^{-\infty} \right| = 0$ nespojitost 2. druhu

$\lim_{x \to 0^+} f(x) = \infty$

D.Ú.: 1) $P(x) = x^{80} - x^{76} + 8x^{25} - 6x^3 + 8x - 1$ ... má kořen polynom kladný reálný kořen

2) $f(x) = \begin{cases} x, & x \in \left\{\frac{1}{n}, n \in \mathbb{N}\right\} \\ 0 & \text{else} \end{cases}$ je $f$ spojitá v 0 ?

8. PŘ

- věta 5.9., 5.12., 5.14.
- lemma 5.11.

$f(f^{-1}(x)) = x$

$f'(f^{-1}(x)) \cdot (f^{-1}(x))' = 1$

$(f^{-1}(x))' = \frac{1}{f'(f^{-1}(x))}$

9. PŘ

- věta 5.18., 5.23., 5.25.
- lemma 5.22.

$f'(x) \neq 0$

$(c)' = 0$

$(\sin x)' = \cos x$

$(\cos x)' = -\sin x$

$(\operatorname{tg} x)' = \frac{1}{\cos^2 x}$

$(\operatorname{cotg} x)' = -\frac{1}{\sin^2 x}$

$(\arcsin x) = \frac{1}{\sqrt{1-x^2}}$

$(\arccos x)' = \frac{-1}{\sqrt{1-x^2}}$

$(\operatorname{arctg} x)' = \frac{1}{1+x^2}$

$(\operatorname{arccotg} x)' = \frac{-1}{1+x^2}$

$(e^x)' = e^x$

$(a^x)' = a^x \cdot \ln a$

$(\ln x)' = \frac{1}{x}$

$(\log_a x)' = \frac{1}{x \cdot \ln a}$

$(x^\mu)' = \mu \cdot x^{\mu-1}$

## 6.CO

$\chi(x) \cdot (x+5) \cdot (x-8) \cdot (x-46)$ ... pro spojitá na množině $\{-5, 8, 46\}$

* $f(x) = x^5 + 4x^3 - 2x^2 + 5$

$f'(x) = 5x^4 + 12x^2 - 4x$

* $g(x) = (2-x^2) \cdot \cos x + 2x \sin x$

$g'(x) = -2x \cdot \cos x + (x^2-2) \sin x + 2 \sin x + 2x \cos x = x^2 \sin x$

* $f(x) = \sin(\ln(x^3+2x))$

$f'(x) = \cos(\ln(x^3+2x)) \cdot \frac{1}{x^3+2x} \cdot (3x^2+2)$

* $g(x) = \frac{1+x-x^2}{1-x+x^2}$

$g'(x) = \frac{(1-2x)(1-x+x^2) - (1+x-x^2)(2x-1)}{(1-x+x^2)^2} = \frac{2(1-2x)}{(1-x+x^2)^2}$

* $f(x) = \sqrt{x\sqrt{x\sqrt{x}}} = x^{\frac{7}{8}}$

$f'(x) = \frac{7}{8} \cdot x^{-\frac{1}{8}}$

* $f(x) = \sqrt{1-x^2} + \arccos x$

$f'(x) = \frac{1}{2} \cdot (1-x^2)^{-\frac{1}{2}} \cdot (-2x) + \frac{-1}{\sqrt{1-x^2}}$

$= -\frac{x+1}{\sqrt{1-x^2}}$

$D(f) = [-1, 1]$

$D(f') = (-1, 1)$

* $f(x) = \ln\left|\operatorname{tg}\frac{x}{2}\right|$

$$(\ln|f|)' = \frac{f'}{f} \quad f \neq 0$$

$D(f) = \mathbb{R} \setminus \{k\pi \mid k \in \mathbb{Z}\}$

$$f'(x) = \frac{\frac{1}{2} \cdot 1/\cos^2\frac{x}{2}}{\operatorname{tg}\frac{x}{2}} = \frac{1}{\sin x}$$

* $f(x) = \dfrac{(x+2)^3\sqrt[4]{x-1}}{e^x \cdot (x+132)^2}$ zadáno $x > 1$

$f' = f \cdot (\ln(f))'$

$$\ln(f(x)) = 3\ln(x+2) + \frac{1}{4}\ln(x-1) - x - 2\ln(x+132)$$

$$(\ln(f(x)))' = \frac{3}{x+2} + \frac{1}{4(x-1)} - 1 - \frac{2}{x+132}$$

* $f(x) = \begin{cases} (x^2-1)\cdot\sin\frac{1}{x-1} & x \neq 1 \\ 0 & x = 1 \end{cases}$

$$\lim_{x\to 1} \frac{(x^2-1)\cdot\sin\frac{1}{x-1} - 0}{x-1} = \lim_{x\to 1}(x+1)\cdot\sin\frac{1}{x-1} \ldots \text{nexistuje}$$

$f$ není diferencovatelná

*

$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\sinh'(x) = \frac{1}{2}\cdot\lim\frac{e^{x+h} - e^{-x+h} - e^x + e^x}{h} = \frac{1}{2}\lim_{h\to 0}\left(e^x\frac{e^h-1}{h} + e^{-x}\frac{e^{-h}-1}{-h}\right) = \cosh(x)$$

* $\operatorname{argsinh}(x)$

$$\operatorname{argsinh}'(x) = \frac{1}{\cosh(\operatorname{argsinh}(x))} = \frac{1}{\sqrt{1+x^2}}$$

* $f(x) = x^{\sin x} = e^{\sin x \cdot \ln x}$

$$f'(x) = e^{\sin x\cdot\ln x}\cdot\left(\cos x\cdot\ln x + \frac{\sin x}{x}\right) = x^{\sin x}\left(\cos x\cdot\ln x + \frac{\sin x}{x}\right)$$

* $f(x) = \dfrac{\ln(2x^3+4x^2-x)}{1+x}$

$$f'(x) = \frac{\frac{1}{2x^3+4x^2-x}\cdot(6x^2+8x-1)\cdot(1+x) - \ln(2x^3+4x^2-x)}{(1+x)^2}$$

$$t(x) = f'(x_0)(x-x_0) + f(x_0)$$

rovnice normály: $y = -\frac{1}{f'(x_0)}(x-x_0) + f(x_0)$

* napište normálu a tečnu $f(x) = x \cdot e^{-\frac{x}{2}}$ v bodě $x_0 = 1$

$$f'(x) = e^{-\frac{x}{2}} + x \cdot e^{-\frac{x}{2}} \cdot \left(-\frac{1}{2}\right) = e^{-\frac{x}{2}}\left(1 - \frac{x}{2}\right)$$

$$f(1) = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

$$f'(1) = \frac{1}{2\sqrt{e}}$$

tečna: $y = \frac{1}{2\sqrt{e}}(x-1) + \frac{1}{\sqrt{e}}$

normála: $y = -2\sqrt{e}(x-1) + \frac{1}{\sqrt{e}}$

## 10. PŘ.

- věta 5.24. i 6.10. i 6.14.
- důkaz 5.27. i 6.4.

$$\lim_{x \to 0^+} x \cdot \ln x = \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}} = \lim_{x \to 0^+} \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x \to 0^+} (-x) = 0$$

$0 \cdot (-\infty)$

$\frac{-\infty}{\infty}$ # 2. Hospital

$$\lim_{x \to 0^+} \left(\frac{1}{x} + \ln x\right) = \lim_{x \to 0^+} \frac{1 + x \cdot \ln x}{x} = +\infty$$

($1 \to 0$, $x \to 0$)

- zápis vlastností monotonie

## 11. PŘ.

- def. 6.16. i 6.24.
- věta 6.26. i 6.28. i 6.30.
- důkaz 6.27.

## 7. CV.

* $f(x) = 2x^3 - 3x^2 - 12x + 1$

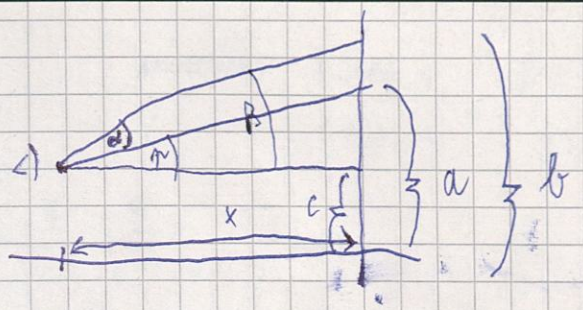
glob. extrémy $I = [-3, 3]$

$$f'(x) = 6x^2 - 6x - 12 = 6(x-2)(x+1) = 0$$

$f(3) = -8$ $\quad f(-1) = 8$ max

$f(2) = -19$ $\quad f(-3) = -44$

*




$a, b, c$ je známe

$A = a - c$

$B = b - c$

$\operatorname{tg} \beta = \frac{B}{x}$

$\operatorname{tg} \gamma = \frac{A}{x}$

$\alpha(x) = \beta - \gamma = \operatorname{arctg} \frac{B}{x} - \operatorname{arctg} \frac{A}{x}$

$\alpha'(x) = \frac{1}{1 + \frac{B^2}{x^2}} \cdot (-1) \cdot \frac{B^2}{x^2} - \frac{1}{1 + \frac{A^2}{x^2}} \cdot (-1) \cdot \frac{A^2}{x^2} = 0$

$x = \sqrt{AB}$ je to maximum

* $y = x^2$

nejbližší bod paraboly, který leží nejblíže $(1/2)$

$(x, x^2)$ ; $(1, 2)$

$D = d^2 = (x-1)^2 + (x^2-2)^2 = x^2 - 2x + 1 + x^4 - 4x^2 + 4 = x^4 - 3x^2 - 2x + 5$

$D' = 4x^3 - 6x - 2 = (x+1)(4x^2 - 4x - 2)$

$x_1 = -1$

$x_2 = \frac{1}{2} + \frac{\sqrt{3}}{2}$

$x_3 = \frac{1}{2} - \frac{\sqrt{3}}{2}$

* DÚ – mezi bodem a přímkou s daným obvodem najděte ten s max. obsahem

$S = \frac{1}{2} \alpha u^2 = \frac{1}{2} \cdot \alpha \left(\frac{o}{4}\right)^2$

$u(2+\alpha) = o$

$u = \frac{o}{4}$

$\alpha = 2$ rad

$x = \varphi(t)$

$y = \psi(t)$

$y = \psi(\varphi^{-1}(x))$

$y' = \psi'(\varphi^{-1}(x)) \cdot \frac{1}{\varphi'(\varphi^{-1}(x))} = \frac{\psi'(t)}{\varphi'(t)}$

$x^2 + y^2 = 1 \quad (y = y(x))$

$2x + 2y \cdot y' = 0$

$y' = -\frac{x}{y} \qquad y' = \frac{\psi'(t)}{\varphi'(t)} = \frac{\cos t}{-\sin t}$

$y''(x) = \frac{\psi'(\varphi^{-1}(x))}{\varphi'(\varphi^{-1}(x))} = \frac{\psi''(t)\cdot\varphi'(t) - \psi'(t)\,\varphi''(t)}{(\varphi'(t))^3}$

$= \frac{(-\sin t)^2 + (\cos t)^2}{(-\sin t)^3} = \frac{1}{-\sin^3 t}$

alt. způsob: $2 + 2\cdot(y'y' + y\,y'') = 0$

$y'' = \frac{-(y')^2 - 1}{y}$

* najděte první a druhou derivaci v bodě $t = \frac{\pi}{2}$

DÚ

$x = t - \sin t = \varphi(t)$

$y = 1 - \cos t = \psi(t) \qquad t \in [0, 2\pi]$

$y' = \frac{\sin t}{1 - \cos t} = 1$

$y'' = \frac{\cos t - \cos^2 t - \sin^2 t}{(1-\cos t)^3} = \frac{a-1}{(1-\cos t)^2} = -1$

$\varphi'(t) = 1 - \cos t$

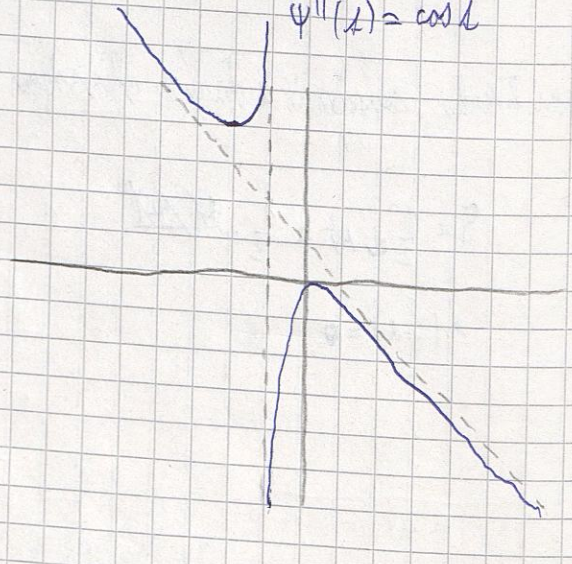
$\psi'(t) = \sin t$

$\varphi''(t) = \sin t$

$\psi''(t) = \cos t$

$y' = 1$

$y'' = -1$



* $f(x) = \frac{-x^2}{x+1}$ průběh funkce

$D(f) = \mathbb{R} - \{-1\}$

není ani sudá ani lichá

$\lim_{x \to -1^+} \frac{-x^2}{x+1} = -\infty$

$\lim_{x \to -1^-} \frac{-x^2}{x+1} = +\infty$

$y = ax + b$

$a = \lim_{x \to +\infty} \frac{f(x)}{x} = -1$

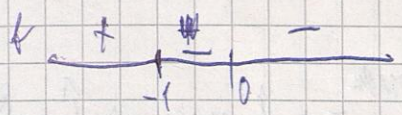
$b = \lim_{x \to +\infty} (f(x) - ax) = 1$

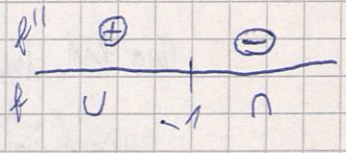
průsečíky s osou y: $y = 0$

s osou x: $0 = \frac{-x^2}{x+1} \quad x = 0$

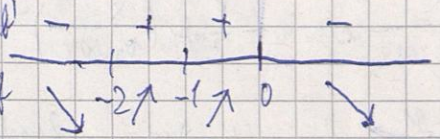
$f''(x) = \frac{-2}{(x+1)^3}$

$D(f'') = \mathbb{R} \setminus \{-1\}$




$f'(x) = \frac{-x^2-2x}{(x+1)^2} = 0$




$x \in \{0; -2\}$ f ↘ −2 ↗ −1 ↗ 0 ↘

* DÚ

$f(x) = \ln \sqrt{\frac{1+\sin x}{1-\sin x}}$ ; proč je lichá?

určete def. obor

$f(-x) = \ln \sqrt{\frac{1-\sin x}{1+\sin x}} = -\ln \sqrt{\frac{1+\sin x}{1-\sin x}} = -f(x)$

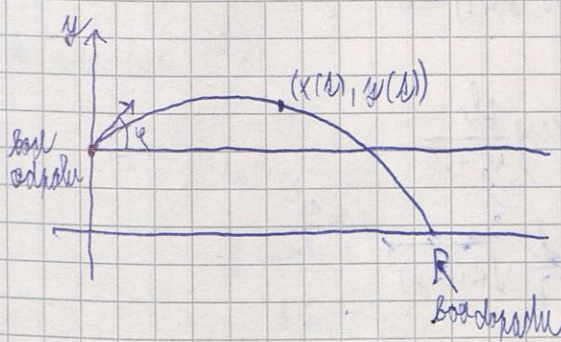
$\sin x \neq \pm 1 \quad x \neq \frac{\pi}{2} + k\pi$

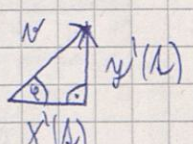
$\frac{1-\sin x}{1+\sin x} \geq 0 \quad \sin x \neq \pm 1$

* $D(f) = \mathbb{R} \setminus \bigcup_{k \in \mathbb{Z}} \{\frac{\pi}{2} + k\pi\}$

## 12. PŘ

- Replika Rozhanova
- Replikovaného problém
- pohyb projektilu v gravitačním poli země
  - projektil je vyražen z bodu o výšce nad zemí s počáteční rychlostí $v_0$ pod úhlem $\varphi$. Projektil dopadne na zem ve vzdálenosti R od místa, které je na zemi kolem pod místem dopadu. Určete elevační úhel $\varphi$ tak, aby byla R maximální




- je-li $\varphi$ počáteční úhel mezi $v_0$ a zemí, pak je

$\frac{dx}{dt} = x'(t) = v_0 \cos\varphi$

$\frac{dy}{dt} = y'(t) = v_0 \sin\varphi - gt$

počáteční podmínky $t=0 \Rightarrow x(0)=y(0)=0$ platí

$$x(t)=v_0 t\cos\varphi$$

$$y(t)=v_0 t\sin\varphi-\frac{1}{2}gt^2$$

Pro bod dopadu platí: $x=R,\ y=-h$ $\quad R=x(\hat{t})$ (čas dopadu) $= v_0\hat{t}\cdot\cos\varphi \Rightarrow \hat{t}=\frac{R}{v_0\cos\varphi}$

$$y=v_0\cdot\frac{x}{v_0\cos\varphi}\sin\varphi-\frac{1}{2}g\left(\frac{x}{v_0\cos\varphi}\right)^2$$

$$y=-\frac{1}{2}\frac{g}{v_0^2\cos^2\varphi}x^2+(\operatorname{tg}\varphi)x \quad \ldots \text{kvadratická rovnice} \Rightarrow \text{dráha je parabola}$$

Pro čas $\hat{t}=\frac{R}{v_0\cos\varphi}$ je $x=R,\ y=-h$

$$-h=-\frac{1}{2}\cdot\frac{g}{v_0^2\cos^2\varphi}\cdot R^2+(\operatorname{tg}\varphi)R$$

$$R\cdot v_0^2(\sin 2\varphi)-gR^2+2hv_0^2\cos^2\varphi=0 \qquad (*)$$

Hledáme $\varphi$ takové, že $R$ bude maximální. $R=R(\varphi)$

$R\cdot v_0^2(\sin 2\varphi)$ derivujeme

$$v_0^2\left(R'\sin 2\varphi+2R\cos 2\varphi\right)-2gRR'-2hv_0^2\cos\varphi\sin\varphi=0$$

$$R'=2v_0^2\cdot\frac{h\sin 2\varphi-R\cos 2\varphi}{v_0^2\sin 2\varphi-2gR}$$

pro max je $R'=0 \Rightarrow h\sin 2\varphi=R\cos 2\varphi$

$$R_{max}=h\operatorname{tg}2\varphi$$

dosadíme do $(*)$

$$h(\operatorname{tg}2\varphi)v_0^2\sin 2\varphi-gh^2(\operatorname{tg}^2 2\varphi)+2hv_0^2\cos^2\varphi=0$$

$$(1+\cos 2\varphi)\left(gh+\cos(2\varphi)(v_0^2+gh)\right)=0$$

$\cos$

$$\varphi_{max}=\frac{1}{2}\arccos\frac{gh}{v_0^2+gh}$$

$$R_{max}=h\cdot\operatorname{tg}2\varphi_{max}=\frac{h}{g}\sqrt{2(2+g)} \qquad 2=\frac{v_0^2}{h}$$

- Úloha z basketbalu (dělostřelecká úloha, Edmond Halley, Halleyova kometa, Halleyův kalibrační granát)

basketbal limitní případ kdy $h\to 0$.

# 13. PŘ

- def. 7.2.
- věta 7.3.

# 8 CV.

* $\lim_{x\to 0} \frac{e^x - e^{-x} - 2x}{x - \sin x} = \lim_{x\to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} = \lim_{x\to 0} \frac{e^x - e^{-x}}{\sin x} = \lim_{x\to 0} \frac{e^x + e^{-x}}{\cos x} = 2$

* $\lim_{x\to\infty} \frac{x + \sin x}{x + \cos x} = \left|\frac{\infty}{\infty}\right| = \lim_{x\to\infty} \frac{1 + \cos x}{1 - \sin x}$ neexistuje

* $\lim_{x\to 0^+} x \cdot \ln x = \left|0 \cdot (-\infty)\right| = \lim_{x\to 0^+} \frac{\ln x}{1/x} = \lim_{x\to 0^+} \frac{1/x}{-1/x^2} = \lim_{x\to 0^+} (-x) = 0$

* $\lim_{x\to 0} \left(\frac{1}{x} - \operatorname{cotg} x\right) = \lim_{x\to 0} \frac{\sin x - \cos x}{\sin x \cdot \cos x} = 0$ ✓ D.Ú.

* $\lim_{x\to\infty} \left(\sqrt{x^3 + x^2} - x\right) = \lim_{x\to\infty} \frac{x^2}{\sqrt[3]{(x^3+x^2)^2} + x\sqrt[3]{x^3+x^2} + x^2} = \frac{1}{3}$

* $\lim_{x\to 0^+} x^{\sin x} \overset{\text{věta}}{=} \lim_{x\to 0^+} e^{\sin x \cdot \ln x} = \dots = 1$ ✓

* $\lim_{x\to 0} \left(\frac{\sin x}{x}\right)^{\frac{1}{x^2}} = |1^\infty| = \lim_{x\to 0} e^{\frac{1}{x^2} \ln \frac{\sin x}{x}} = \dots = e^{-\frac{1}{6}}$ ✓

* $\lim_{x\to \frac{\pi}{2}^-} (\cos x)^{\frac{\pi}{2} - x} = \dots = 1$ ✓

* $\lim_{x\to 0} \left(\frac{1}{x} - \frac{1}{\sin x}\right) = \dots = 0$ ✓

* $\lim_{x\to 0} \frac{a^{x^2} - b^{x^2}}{(a^x - b^x)^2} = \frac{1}{\ln a - \ln b}$ ✓ $(a, b \in \mathbb{R}^+ \setminus \{1\},\ a \neq b)$

* odhadněte $\sqrt[3]{70}$

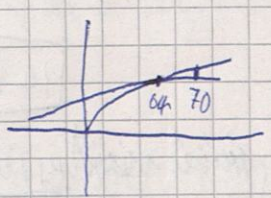
$f(x) = \sqrt[3]{x}$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$

$f(64) = 4$

$x_0 = 64$ $\quad y = y_0 + k \cdot (x - x_0)$

$\sqrt[3]{70} \simeq 4 + \frac{1}{48}(70 - 64) = 4{,}125$




* určete 0,99

* $f(x) = \ln x^2$ Taylor, stupeň 4, $x_0 = 1$

$f'(x) = \frac{2}{x}$ $\quad f(1) = 0$

$f''(x) = \frac{-2}{x^2}$ $\quad f'(1) = 2$

$f'''(x) = \frac{4}{x^3}$ $\quad f''(1) = -2$

$f''''(x) = \frac{-12}{x^4}$ $\quad f'''(1) = 4$

$f''''(1) = -12$

$T(x) = 2(x-1) - (x-1)^2 + \frac{2}{3}(x-1)^3 - \frac{1}{2}(x-1)^4$

*

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots$ Maclaurin.

$\lim_{x \to 0} \frac{\sin x - x}{x^3} = \lim_{x \to 0} \left( -\frac{1}{3!} + \frac{x^2}{5!} - \dots \right) = -\frac{1}{6}$

↳ metoda ochlazování

~~$2048 + 1 = n^3$~~

~~$2049 = (n-1)(n^2+n+1)$~~

*

$\sin 1°$ s chybou menší než $10^{-16}$

$\sin 1° = \sin \frac{\pi}{180}$

$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \dots + (-1)^{n-1} \cdot \frac{x^{2n-1}}{(2n-1)!} + 0 + (-1)^n \cdot \frac{\cos \alpha}{(2n+1)!} \cdot x^{2n+1}$ $\quad \alpha \in (0, \frac{\pi}{180})$

$\left| (-1)^n \frac{\cos \alpha}{(2n+1)!} \cdot \left(\frac{\pi}{180}\right)^{2n+1} \right| < 10^{-16}$

$\frac{1}{(2n+1)!} \cdot \left(\frac{\pi}{180}\right)^{2n+1} < 10^{-16}$

$\sin \frac{\pi}{180} \overset{n=2}{\approx} \frac{\pi}{180} - \frac{1}{3!} \cdot \left(\frac{\pi}{180}\right)^3 + \frac{1}{5!} \cdot \left(\frac{\pi}{180}\right)^5$

14. PŘ

- chybět přím (integrály, různé výpočty, pro maturitu)

15. PŘ

- metoda substituce
- učebnice 1.28. | ~~1.30~~ 1.32 1.33

16. PŘ.

- Eulerovy substituce
- univerzální substituce

17. PŘ.

- chyběl jsem

9. CV.

* $\int \sqrt{x\sqrt{x\sqrt{x}}}\,dx = \int x^{\frac{7}{8}}\,dx = \frac{x^{\frac{15}{8}}}{15/8} + C$

* $\int (2^x+3^x)^2\,dx = \int (4^x + 2\cdot 6^x + 9^x)\,dx = \frac{4^x}{\ln 4} + \frac{2\cdot 6^x}{\ln 6} + \frac{9^x}{\ln 9}\,dx$

* $\int \operatorname{cotg}^2 x\,dx = \int \left(\frac{1}{\sin^2 x} - 1\right)dx = -\operatorname{cotg} x - x + C$

* $\int \frac{1}{\sin^2 x \cos^2 x}\,dx = \int \left(\frac{1}{\sin^2 x} + \frac{1}{\cos^2 x}\right)dx = -\operatorname{cotg} x + \operatorname{tg} x + C$

* $\int \frac{1}{\sqrt{4-4x^2}}\,dx = \frac{1}{2}\cdot\int \frac{1}{\sqrt{1-x^2}}\,dx = \frac{1}{2}\cdot \arcsin x + C$

* $\int \frac{\cos x}{1+\sin x}\,dx = \ln|1+\sin x| + C$

$x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$

* $\int x^2 e^x\,dx = x^2\cdot e^x - \int 2x e^x\,dx = x^2\cdot e^x - 2xe^x + \int 2e^x\,dx = x^2e^x - 2xe^x + 2e^x + C$

* $\int \operatorname{arctg} x\,dx = \int 1\cdot \operatorname{arctg} x\,dx = x\operatorname{arctg} x - \int x\cdot\frac{1}{x^2+1}\,dx = x\cdot\operatorname{arctg} x - \frac{\ln|x^2+1|}{2} + C$

* $\int \frac{\ln x}{x}\,dx = \int \frac{1}{x}\cdot \ln x\,dx = \ln^2 x - \int \ln x\cdot\frac{1}{x}\,dx = \frac{\ln^2 x}{2} + C$

$\int \ln x$

D.Ú.

* $\int e^x\cdot\sin x\,dx \left|\begin{matrix} u=\sin x & u'=\cos x \\ v'=e^x & v=e^x \end{matrix}\right| = e^x\sin x - \int \cos x\,e^x\,dx \left|\begin{matrix} u=\cos x & u'=-\sin x \\ v'=e^x & v=e^x \end{matrix}\right| = e^x\sin x - e^x\cos x - \int e^x\sin x\,dx$

* $\int e^{2x}\cdot\sin 5x\,dx$

* $\int \frac{\ln x}{x}\,dx = \left|\begin{matrix} \ln x = t \\ \frac{1}{x}dx = dt \end{matrix}\right| = \int t\,dt = \frac{t^2}{2} + C = \frac{\ln^2 x}{2} + C$

$$* \int \frac{1}{x \ln^2 x}\,dx = \left| \begin{matrix} t = \ln x \\ dt = \frac{1}{x}dx \end{matrix} \right| = \int \frac{1}{t^2}\,dt = -\frac{1}{t} + C = -\frac{1}{\ln x} + C$$

$$* \int \frac{x^2}{e^{x^3}}\,dx = \left| \begin{matrix} x^3 = t \\ x^2 dx = \frac{dt}{3} \end{matrix} \right| = \frac{1}{3}\int \frac{dt}{e^t} = \left| \begin{matrix} -t = u \\ -dt = du \end{matrix} \right| = -\frac{1}{3}\int e^u\,du = -\frac{1}{3}e^{x^3} + C$$

$$* \int \frac{\operatorname{arctg}(\sqrt{x})}{\sqrt{x}(1+x)}\,dx = \left| \begin{matrix} t = \operatorname{arctg}(\sqrt{x}) \\ dt = \frac{1}{2\sqrt{x}(x+1)}dx \end{matrix} \right| = 2\int t\,dt = t^2 + C = (\operatorname{arctg}(\sqrt{x}))^2 + C$$

$$* \int \frac{e^x}{e^{2x}+3}\,dx = \left| \begin{matrix} t = e^x \\ dt = e^x dx \end{matrix} \right| = \int \frac{dt}{t^2+3} = \frac{1}{3}\int \frac{dt}{\frac{t^2}{3}+1}\,dt = \frac{1}{3}\int \frac{dt}{(\frac{t}{\sqrt{3}})^2+1} = \left| \begin{matrix} u = t/\sqrt{3} \\ du = \frac{dt}{\sqrt{3}} \end{matrix} \right| = \frac{\sqrt{3}}{3}\operatorname{arctg}\frac{t}{\sqrt{3}}+C$$

$$= \frac{\sqrt{3}}{3} \cdot \operatorname{arctg}\frac{e^x}{\sqrt{3}} + C$$

$$* \int \sin\sqrt{x}\,dx = -2\sqrt{x}\cos\sqrt{x} + 2\sin\sqrt{x} + C \quad \text{DÚ.}$$

$$\int \sin\sqrt{x}\,dx \left| \begin{matrix} t = \sin\sqrt{x} \\ dt = \frac{\cos\sqrt{x}}{2\sqrt{x}}dx \end{matrix} \right| = \int \ldots \quad x = t^2$$

## 18. PŘ.

## 19. PŘ.

- chybět příp
- důkazek 2.36.
- věta 2.38. | 2.39.; 2.42.
- důkazek 2.40.
- příp. 2.39. | 2.42.

## 10. CV.

$$* \quad \int \frac{1}{x^2+4x+8}\,dx = \int \frac{1}{(x+2)^2+4}\,dx = \frac{1}{2}\operatorname{arctg}\left(\frac{x+2}{2}\right)+C$$

$$* \quad \int \frac{3x+5}{x^2+4x+8}\,dx = 3\int \frac{x+\frac{5}{3}}{x^2+4x+8}\,dx = \frac{3}{2}\int \frac{2x+\frac{10}{3}}{x^2+4x+8}\,dx = \frac{3}{2}\int \frac{2x+4}{x^2+4x+8}\,dx + \frac{3}{2}\int \frac{-\frac{2}{3}}{x^2+4x+8}\,dx =$$

$$= \frac{3}{2}\ln(x^2+4x+8) - \frac{3}{2}\cdot\frac{2}{3}\cdot\frac{1}{2}\operatorname{arctg}\left(\frac{x+2}{2}\right)+C$$

$$* \quad \int \frac{4x+8}{(x^2+2x+5)^3}\,dx = 2\underbrace{\int \frac{2x+2}{(x^2+2x+5)^3}\,dx}_{I_1} + 4\underbrace{\int \frac{1}{(x^2+2x+5)^3}\,dx}_{}$$

$$I_1 = \left|\begin{matrix} x^2+2x+5=t \\ (2x+2)dx=dt \end{matrix}\right| = \int \frac{dt}{t^3} = t^{-2}\cdot\frac{1}{-2}+C = \frac{1}{-2(x^2+2x+5)^2}+C$$

$$I_2 = \frac{1}{4}\int \frac{1}{\left(\left(\frac{x+1}{2}\right)^2+1\right)^3}\,dx = \left|\begin{matrix} u=\frac{x+1}{2} \\ du=\frac{1}{2}dx \end{matrix}\right| = \frac{1}{32}\underbrace{\int \frac{1}{(u^2+1)^3}\,du}_{I_3}$$

$$I_3 = \int \frac{1+u^2-u^2}{(u^2+1)^3}\,du = \int \frac{1}{(u^2+1)^2}\,du - \int \frac{u^2}{(u^2+1)^3}\,du = \frac{u}{4(u^2+1)^2} + \frac{3}{4}\cdot\int \frac{1}{(u^2+1)^2}\,du$$

(dopočítat domov)

$$\int \frac{u^2}{(u^2+1)^3}\,du = \left|\begin{matrix} f=u & f'=1 \\ g'=\frac{u}{(u^2+1)^3} & g=\frac{-1}{4(u^2+1)^2} \end{matrix}\right| = -\frac{u}{4(u^2+1)^2} + \frac{1}{4}\int \frac{1}{(u^2+1)^2}\,du$$

pomocí substituce $t=u^2+1$

* pomocí rekurentní formule předchozí úloha

$$I_3=\int\frac{1}{(u^2+1)^3}du=\frac{u}{2\cdot(3-1)\cdot(1+u^2)^{3-1}}+\frac{2\cdot3-3}{2\cdot3-2}\int\frac{1}{(u^2+1)^{3-1}}=$$

$$=\frac{u}{4(1+u^2)^2}+\frac{3}{4}\left(\frac{u}{\cdot}\right.$$

$$=\frac{u}{4(1+u^2)^2}+\frac{3u}{8(1+u^2)}+\frac{3}{8}\operatorname{arctg}u+C$$

* $\int\frac{\sin^5x}{\cos^4x}dx=\left|\begin{array}{l}t=\cos x\\ dt=-\sin x\,dx\end{array}\right|=\int\frac{-\sin^4x}{\cos^4x\,t^4}dt=\int\frac{t^4-1}{t^4}dt=t+\int t^{-4}dt$

$$=t+\frac{1}{3t^3}+C=\cos x+\frac{1}{3\cos^3x}+C=\int\frac{(1-t^2)^2}{t^4}dt=\dots$$

* $\int\frac{\sin x}{\sin x-\cos x}dx=\left|\begin{array}{l}t=\operatorname{tg}\frac{x}{2}\\ dx=\frac{2}{t^2+1}dt\end{array}\right|=$

$$\int\frac{\frac{2t}{t^2+1}}{\frac{2t}{t^2+1}-1}\cdot\frac{2}{t^2+1}dt$$

$$\int\frac{\sin x}{\cos x\operatorname{tg}x-1}$$

$$\sin x=\frac{t}{\sqrt{t^2+1}}$$

$$\cos x=\frac{1}{\sqrt{t^2+1}}$$

$$\int\frac{\frac{t}{\sqrt{t^2+1}}}{\frac{t-1}{\sqrt{t^2+1}}}\cdot\frac{1}{t^2+1}dt$$

* $\int\frac{1}{\sin x}dx=\left|\begin{array}{l}t=\operatorname{tg}\frac{x}{2}\\ dx=\frac{2}{t^2+1}dt\end{array}\right|=\int\left(2\cdot\frac{t}{t^2+1}\right)^{-1}\cdot\frac{2}{t^2+1}dt=\dots$

* $\int\cos5x\cdot\cos7x\,dx=\int\frac{1}{2}\left(\cos(12x)+\cos(-2x)\right)dx=\frac{1}{2}\left(\frac{\sin12x}{12}+\frac{\sin2x}{2}\right)dx$

* $\int\frac{3+2x-\sqrt[3]{x^2}}{\sqrt[5]{x^3}+2\sqrt[3]{x}}dx=\left|\begin{array}{l}x=t^6\\ dx=6t^5dt\end{array}\right|=\frac{2+2t^6-t^4}{t^3+2t^2}\cdot6t^5dt=\dots$

* $\int\frac{x+1}{\sqrt[3]{3x+1}}dx=\left|\begin{array}{l}3x+1=t^3\\ x=\frac{t^3-1}{3}\\ dx=t^2\end{array}\right|=\frac{\frac{t^3-1}{3}+1}{t}\cdot t^2\,dt=\dots$

* $\int\left(x+2\cdot\left(\frac{2x+1}{x-3}\right)^{\frac{3}{2}}\right)dx=\left|\begin{array}{l}\frac{2x+1}{x-3}=t^2\\ x=\frac{-3t^2-1}{2-t^2}\\ dx=\frac{-14t}{(t^2-2)^2}\end{array}\right|=\left(-\frac{3t^2-1}{2-t^2}+2t^3\right)\cdot\frac{-14t}{(t^2-2)^2}dt=\dots$

* $\int x\sqrt[3]{8-7x^3}dx=\left|\begin{array}{l}m=1\quad\frac{m+1}{n}+p=\frac{2}{3}-1\\ n=3\\ p=1/3\\ 8x^3-7=t^3\\ x=\frac{1}{(t^3+7)^{1/3}}\end{array}\right|$

$$dx=-\frac{2}{3}(t^3+7)^{-\frac{4}{3}}\cdot3t^2dt$$

$$=2\left(t^3+7\right)^{-\frac{1}{3}}t\cdot2\cdot(t^3+7)^{-\frac{1}{3}}\cdot\left(-\frac{2}{3}\right)\cdot(t^3+7)^{-\frac{4}{3}}\cdot3t^2dt$$

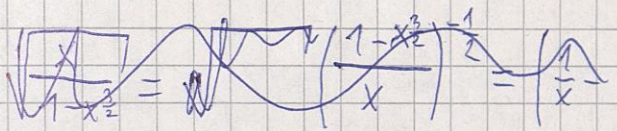
$$=-8\cdot\int\frac{t^3}{(t^3+7)^2}dt=\dots$$

* $\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx = \dots = -\frac{4}{3}\sqrt{1-x\sqrt{x}}$ D.Ú.

* $\int \frac{dx}{(x-1)\sqrt{x^2+x+1}} = \left| \begin{array}{l} \sqrt{x^2+x+1} = x+t \\ x+1 = x^2+2xt+t^2 \\ x = \frac{t^2-1}{1-2t} \\ dx = \frac{-2t^2+2t-2}{(1-2t)^2}\,dt \end{array} \right| = \int \frac{\frac{-2t^2+2t-2}{(1-2t)^2}\,dt}{\left(\frac{t^2-1}{1-2t}-1\right)\cdot\left(\frac{t^2-1}{1-2t}+t\right)} = 2\cdot\int \frac{1}{t^2+2t-2}\,dt$

## 20. PŘ

- věta 2.45., 2.47., 2.49.
- def. 3.1



$$\int \sqrt{\frac{x}{1-x\sqrt{x}}}\,dx = \left| \begin{array}{l} x = t^2 \\ dx = 2t\,dt \end{array} \right| = \int \frac{t^2}{\sqrt{1-t^3}}\,dt = \left| \begin{array}{l} m=2 \\ n=3 \\ r=-\frac{1}{2} \end{array} \right| \begin{array}{l} 1-t^3 = z^2 \\ t^3 = 1-z^2 \\ 3t^2\,dt = -2z\,dz \end{array} \Bigg| =$$

$$= -\frac{4}{3}\int \frac{z}{z}\,dz = -\frac{4}{3}z = -\frac{4}{3}\sqrt{1-t^3} = -\frac{4}{3}\sqrt{1-x^{\frac{3}{2}}}$$

## 21. PŘ.

chybělí jsem

## 11. CV.

* $\int_{-2}^{5} (3x^2-5x+7)\,dx = \left[x^3 - 5\frac{x^2}{2} + 7x\right]_{-2}^{5} = (125 - \frac{125}{2} + 35) - (-8 - 10 - 14)$

* $\int_0^{\frac{\pi}{2}} x\cdot\sin x\,dx = \left| \begin{array}{ll} u = x & u' = 1 \\ v' = \sin x & v = -\cos x \end{array} \right| = \left[-x\cdot\cos x\right]_0^{\frac{\pi}{2}} + \int_0^{\frac{\pi}{2}} \cos x\,dx = \left[-x\cdot\cos x + \sin x\right]_0^{\frac{\pi}{2}} = 1$

* $\int_1^5 \sqrt{2x-1}\,dx = \left| \begin{array}{l} 2x-1 = t^2 \\ dx = t\,dt \end{array} \right| = \int_1^3 t^2\,dt = \left[\frac{t^3}{3}\right]_1^3 = \frac{27}{3} - \frac{1}{3} = \frac{26}{3}$

* $\int_0^1 x^2(5-2x^3)^4\,dx = \left| \begin{array}{l} t = 5-2x^3 \\ dt = -6x^2\,dx \end{array} \right| = \int_5^3 -\frac{1}{6}t^4\,dt = \frac{1}{6}\int_3^5 t^4\,dt = \frac{1}{6}\left[\frac{t^5}{5}\right]_3^5 = \frac{1}{6}\left(\frac{5^5}{5} - \frac{3^5}{5}\right) = \frac{1441}{15}$

* $\int_0^{\pi/2} \frac{\cos x}{1+\sin x}\,dx = \left[\ln|\sin x+1|\right]_0^{\pi/2} = \ln 2 - \ln 1 = \ln 2$

* $\int_{-1/2}^{1/2} \frac{dx}{\sqrt{4-4x^2}} = \frac{1}{2}\left[\arcsin x\right]_{-\frac{1}{2}}^{\frac{1}{2}} = \frac{1}{2}\left(\frac{\pi}{6}+\frac{\pi}{6}\right) = \frac{\pi}{6}$

* $\int_1^{e^3} \frac{dx}{x\sqrt{1+\ln x}} = \left|\begin{array}{l} t^2 = 1+\ln x \\ 2t\,dt = \frac{1}{x}dx \end{array}\right| = \int_1^3 2\,dt = \left[2t\right]_1^3 = 6-2 = 4$

* $\int_0^{\ln 2} \frac{x}{e^x}dx = \left|\begin{array}{ll} u = x & u' = 1 \\ v' = e^{-x} & v = -e^{-x} \end{array}\right| = \left[-xe^{-x}\right]_0^{\ln 2} + \int_0^{\ln 2} e^{-x}dx = \left[-xe^{-x} - e^{-x}\right]_0^{\ln 2} = e^{-\ln 2}(-\ln 2 - 1) + 1$
$= \frac{1-\ln 2}{2}$

* $\int_1^{\infty} \sin x\,dx = \left[-\cos x\right]_1^{\infty} = \lim_{a\to\infty}\left[-\cos x\right]_1^a = \lim_{a\to\infty}(-\cos a + \cos 1)$ neexistuje

* $\int_{-\infty}^{\infty} \frac{dx}{4+x^2} = \left[\frac{1}{2}\operatorname{arctg}\frac{x}{2}\right]_{-\infty}^{\infty} = \lim_{a\to\infty}\frac{1}{2}\operatorname{arctg}\frac{a}{2} - \lim_{b\to-\infty}\frac{1}{2}\operatorname{arctg}\frac{b}{2} = \frac{\pi}{4} - \left(-\frac{\pi}{4}\right) = \frac{\pi}{2}$

* $\int_0^{\frac{3}{2}\pi} \frac{\cos x}{1+\sin x}\,dx = \left[\ln|1+\sin x|\right]_0^{\frac{3}{2}\pi} = \lim_{a\to\frac{3}{2}\pi}\ln|1+\sin a| = -\infty$

* $\int_0^{\infty} \frac{1}{x}dx = \left[\ln|x|\right]_0^{\infty} = \lim_{a\to\infty}\ln|a| - \lim_{b\to 0^+}\ln|b| = \infty - (-\infty) = \infty$

* $\int_0^4 \frac{2x^2+\sqrt{x}}{x}dx = \int_0^4 \left(2x + x^{-\frac{1}{2}}\right)dx = \left[x^2\right]_0^4 + \left[2\sqrt{x}\right]_0^4 = 16+4 = 20$

## 21. PŘ.

- nepravé Riemannovy integrály
- aplikace určitého integrálu

## 23. PŘ.

- kvadratické
- speciálních kvadratik

- statický moment
- těžiště

## 12. CV.

* $\int_{-1}^{1} \frac{dx}{x^2} = \int_{-1}^{0} \frac{dx}{x^2} + \int_{0}^{1} \frac{dx}{x^2} = \left[-\frac{1}{x}\right]_{-1}^{0} + \left[-\frac{1}{x}\right]_{0}^{1} = \lim_{x\to 0^-}\left(-\frac{1}{x}\right) - 1 - 1 - \lim_{x\to 0^+}\left(-\frac{1}{x}\right) = +\infty - 2 + \infty = +\infty$

* $\int_{-1}^{1} \frac{dx}{x} = \int_{-1}^{0} \frac{dx}{x} + \int_{0}^{1} \frac{dx}{x} = \left[\ln|x|\right]_{-1}^{0} + \left[\ln|x|\right]_{0}^{1} = \lim_{x\to 0^-} \ln|x| - \ln 1 + \ln 1 - \lim_{x\to 0^+} \ln|x| = -\infty + \infty$ nelze určit, ale ze symetrie je to plocha 0.

* $\int_{-\infty}^{0} x e^{-x^2} dx = \left|\begin{matrix} -x^2 = t \\ -2x\,dx = dt \end{matrix}\right| = -\frac{1}{2}\int_{-\infty}^{0} e^t dt = -\frac{1}{2}\left[e^t\right]_{-\infty}^{0} = -\frac{1}{2}\left(1 - \lim_{t\to-\infty} e^t\right) = -\frac{1}{2}$

* $\int_{2}^{\infty} \frac{\sqrt[3]{x^2+x+5}}{x^5+x^3+6} dx$ konv. $\Leftrightarrow$ $\int_{1}^{\infty} \frac{x^{2/3}}{x^5} dx = \int_{1}^{\infty} \frac{1}{x^{13/3}} dx$ konv., tedy konverguje

* $\int_{0}^{1} \frac{dx}{\sqrt[3]{1-x}} = \left|\begin{matrix} 1-x = t \\ -dx = dt \end{matrix}\right| = \int_{0}^{1} \frac{dt}{\sqrt[3]{t}}$ … konverguje

* $\int_{-\infty}^{\infty} \frac{dx}{x^2-1} = \int_{-\infty}^{\infty} \frac{1}{(x-1)(x+1)} dx$ nekonverguje

* $\int_{1}^{\infty} \frac{\operatorname{arctg} x}{\sqrt[3]{x^5+x^3}} \le \frac{\pi}{2}\int_{1}^{\infty} \frac{1}{x^{5/3}} dx$ konverguje

* $\int_{1}^{\infty} \frac{\operatorname{arctg} x}{\sqrt{x}} dx \ge \operatorname{arctg} 1 \cdot \frac{\pi}{4}\int_{1}^{\infty} \frac{1}{\sqrt{x}} dx = \infty$ … diverguje

* $\int_{1}^{\infty} \left|\frac{2\cos x + 1}{x\sqrt{x}}\right| dx = \int_{1}^{\infty} \frac{|2\cos x + 1|}{x^{3/2}} dx \le \int_{1}^{\infty} \frac{3}{x^{3/2}} dx$ abs. konv.

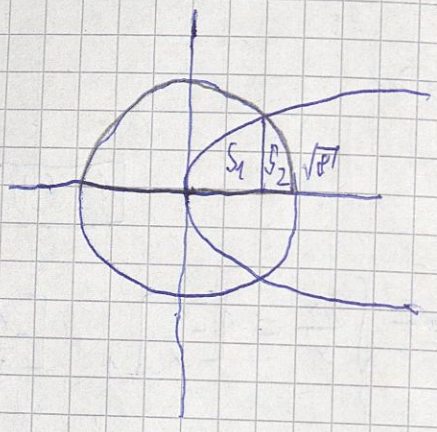
* $\int_{1}^{\infty} \left|\frac{\sin x}{x}\right| dx \ge \int_{1}^{\infty} \frac{\sin^2 x}{x} dx = \frac{1}{2}\int_{1}^{\infty} \frac{1}{x} dx - \frac{1}{2}\int_{1}^{\infty} \frac{\cos 2x}{x}$ (první: diverguje; druhý: konv. dle Dirichleta)

  diverguje

* $\int_{1}^{\infty} \frac{\sin x}{x} dx$ konverguje (Dirichlet)

* v jakém poměru dělí parabola $y^2 = 2x$ kruh $x^2 + y^2 \leq 8$

$$S_\Delta = \frac{\pi r^2}{2} = 4\pi$$




$x^2 + 2x = 8$

$x^2 + 2x + 1 = 9$

$(x+1)^2 = 9$

$x + 1 = \pm 3$

$x = \pm 3 - 1$

$x \in \{-4, 2\}$

$x = 2$

$$S_2 = \int_2^{\sqrt{8}} \sqrt{8 - x^2}\, dx = \left| \begin{array}{l} x = \sqrt{8}\sin t \\ dx = \sqrt{8}\cos t\, dt \end{array} \right| = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \sqrt{8 - 8\sin^2 t}\, \sqrt{8}\cos t\, dt = 8\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 t\, dt$$

$$= 4\left(\frac{\pi}{2} - \frac{\pi}{4}\right) - 2$$

$$S_1 = \int_0^2 \sqrt{2x}\, dx = \dots = \frac{8}{3}$$

$$\frac{S_\sigma}{S_\Delta} = \frac{4\pi - \frac{8}{3} - (\pi - 2)}{\frac{8}{3} + \pi - 2} = \frac{(3\pi + 2)}{9\pi - 2}$$

* určete délku křivky $\ln x$ od $\sqrt{3}$ do $\sqrt{15}$

$$\int_{\sqrt{3}}^{\sqrt{15}} \sqrt{1 + [(\ln x)']^2}\, dx = \int_{\sqrt{3}}^{\sqrt{15}} \sqrt{1 + \frac{1}{x^2}}\, dx = \dots = 2 + \ln\left(\frac{3}{\sqrt{15}}\right)$$

* určete délku 1 oblouku cykloidy — $[0, 2\pi]$

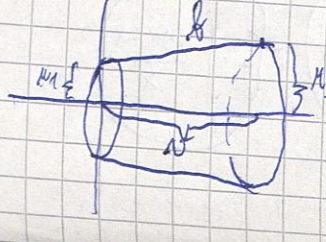
$x = t - \sin t$

$y = 1 - \cos t$

$$\int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt = \sqrt{2}\int_0^{2\pi} \sqrt{1 - \cos t}\, dt = \sqrt{2}\int_0^{2\pi} \sqrt{2\sin^2 \frac{t}{2}}\, dt =$$

$$= 2\int_0^{2\pi} \sin\frac{t}{2}\, dt = 8$$

* určete objem komolého kužele




$[0, r_1]$

$[v, r_2]$

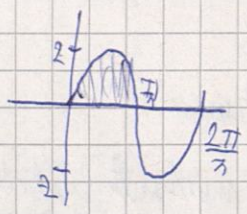
$f(x) = \frac{r_2 - r_1}{v}x + r_1$

$$\pi \int_0^v \left(\frac{r_2 - r_1}{v} x + r_1\right)^2 dx = \cdots = \frac{\pi v}{3}(r_1^2 + r_1 r_2 + r_2^2)$$

* $y = 2\sin 3x$ $\quad x = 0$ $\quad y = 0$ $\quad x = \frac{\pi}{3}$ $\quad$ hrubé těleso

$\sigma(x) = 1 \ldots$ homogenní plocha

$\sigma(x) = 1 \ldots$ spec. hmotnost




$$m = \int_a^b \sigma(x) f(x)\,dx = 2\int_0^{\frac{\pi}{3}} \sin 3x\,dx = 4/3$$

$$S_x = \frac{1}{2}\int_a^b \sigma(x) f^2(x)\,dx = \frac{1}{2}\cdot 4 \int_0^{\pi/3} \sin^2 3x\,dx = \frac{\pi}{3}$$

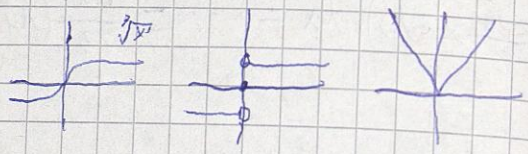
$$S_y = \int_a^b \sigma(x)\cdot f(x)\cdot x\,dx = 2\int_0^{\pi/3} x \sin 3x\,dx = \frac{2}{9}\pi$$

$$T = \left[\frac{S_y}{m}, \frac{S_x}{m}\right] = \left[\frac{\pi}{6}, \frac{\pi}{4}\right]$$

## 24. PŘ.

- axiom supremy
- hustota racionálních čísel a další vlastnosti
- reálná čísla a vlastnosti funkcí
- limita posloupnosti
- Eulerovo číslo
- vybraná posloupnost (podposloupnost), hromadný bod
- každá posloupnost má hromadný bod
- z div. posloupnosti lze vybrat konv. podposloupnost
- lim sup, lim inf
- Cauchyovská posloupnost
- limita funkce, vlastnosti limit
- jednostranné limity
- limita složené funkce
- spojitost v bodě
- spojitá funkce
- Weierstrassova věta
- Bolzanova věta
- inverze spojité funkce
- druhy nespojitosti
- derivace

- nevlastní derivace
- fyzikální význam derivace
- vlastnosti derivace - vztah se spojitostí



- pravidla pro derivování
- ~~Cauchyho~~ Lagrangeovo lemma
- derivace složených a inverzních funkcí
- derivace elementárních funkcí
- Rolleova věta
- Lagrangeova věta
- l'Hospitalova pravidla
- funkce se stejnou derivací se liší o konst.
- rostoucí, klesající fce a vztah k derivaci
- lokální extrémy - vztah na lokální extrém
- stacionární body
- konvexní / konkávní funkce a vztah k druhé derivaci a první derivaci
- inflexní body
- asymptoty funkce
- diferenciál !
- aproximace funkcí
- Taylorův ~~xxx~~ polynom
- Taylorova věta, Taylorův zbytek
- primitivní funkce, neurčitý integrál
- věta o existenci primitivní funkce
- per-partes, substituce
- integrace racionálních lomených funkcí
- Riemannův integrál
- dolní / horní součet
- dělení, norma dělení, zjemnocení dělení
- horní / dolní integrál
- integrální součet
- každá spojitá fce je integrovatelná (každá monotónní fce je integrovatelná)
- věta o dělení $< \varepsilon$
- vlastnosti ~~určitého~~ Riemannova integrálu
- pravidlo návaznosti
- ~~Průměr fce~~ střední hodnota fce
- vlastnosti střední hodnoty

- plocha (v prostoru Lebesgueova míra) je spojitá v nule při integrálu se spojité fce diferencovatelné
- Newtonův - Leibnizův vzorec
- per-partes / substituce pro určité integrál
- nevlastní integrály
- singulární bod
- srovnávací kritéria (prosté a limitní), Dirichletovo kritériem, Abelovo kritériem
- absolutní konvergence integrálu, relativní konvergence integrálu
- aplikace Riemannova integrálu (plocha, délka křivky, objem a povrch rotačního tělesa, těžiště)