(B)

1.) $D(f) = ?$

$$f(x) = \frac{\arcsin \frac{1-2x}{4}}{7^{x+1} - 3 \cdot 7^x - 28}$$

(i) $t = 7^x \Rightarrow 7t - 3t \neq 28$

$4t \neq 28$

$t \neq 7$

$7^x \neq 7$

$\underline{x \neq 1}$

(ii) $-1 \leq \frac{1-2x}{4} \leq 1$

$-4 \leq 1-2x \leq 4$

$-5 \leq -2x \leq 3$

$\underline{\frac{5}{2} \geq x \geq -\frac{3}{2}}$

$\Rightarrow \underline{D(f) = [-\frac{3}{2}, 1) \cup (1, \frac{5}{2}]}$

URČETE PARITU FUNKCE $g(x) = \frac{|\sin x| \cdot \arcsin x^3}{e^x - e^{-x}}$

$$g(-x) = \frac{|-\sin x| \cdot \arcsin(-x^3)}{e^{-x} - e^x} = \frac{-\sin x \cdot \arcsin x^3}{-(e^x - e^{-x})} = g(x) \Rightarrow \underline{SUDÁ}$$

2.) ROZHODNĚTE A ZDŮVODNĚTE, ZDA JE FUNKCE

$$f(x) = \begin{cases} 3x^4 + 5x^3 - 2x + 3 & x \geq 0 \\ \frac{2x + \sin x}{x}, & x < 0 \end{cases}$$

SPOJITÁ V $x = 0$.

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} (3x^4 + 5x^3 - 2x + 3) = 3$$

$$\lim_{x \to 0^-} f(x) = \lim_{x \to 0^-} \left(\frac{2x}{x} + \frac{\sin x}{x}\right) = 2 + 1 = 3$$

$\Rightarrow \lim_{x \to 0} f(x) = 3 = f(0) \Rightarrow \underline{ANO}$

3.) UVAŽUJME KONVEXNÍ FUNKCE $f, g : \mathbb{R} \to \mathbb{R}$ TAKOVÉ, ŽE EXISTUJÍ JEJICH DERIVACE DO ŘÁDU 2 VČETNĚ. ~~POKUD~~ JESTLIŽE PLATÍ

$f''(g(x)) \geq 0$, $f'(g) > 0$, PAK JE SLOŽENÁ FUNKCE $f(g)$ KONVEXNÍ.

ROZHODNĚTE O PRAVDIVOSTI TVRZENÍ?. JE-LI PRAVDIVÉ, DOKAŽTE, NENÍ-LI PR., UVEĎTE PROTIPŘÍKLAD.

$$\rightarrow (f(g))'' = (f'(g) \cdot g')' = f''(g) \cdot (g')^2 + f'(g) \cdot g'' > 0 \Rightarrow KX$$

4.) $\int_1^2 f(x)dx = 1$, $\int_1^2 f(x)dx = 5$, $f(x) = ?$ ... např. $f(x) = \begin{cases} 1, & x \in \mathbb{Q} \\ 5, & x \notin \mathbb{Q} \end{cases}$

5.) $f(x) = x\cdot e^{-\frac{x^2}{2}}$, $D(f) = \mathbb{R}$, $f'(x) = e^{-\frac{x^2}{2}} + x\cdot e^{-\frac{x^2}{2}}\cdot\left(-\frac{2x}{2}\right) = e^{-\frac{x^2}{2}}\cdot(1-x^2)$

$f'(x) = 0 \iff x = \pm 1$, $D(f') = \mathbb{R}$

		-1		1	
f'	$\ominus$		$\oplus$		$\ominus$
f	$\searrow$		$\nearrow$		$\searrow$
		L.MIN.		L.MAX.	

6.) $$\int \frac{\sin^3 x}{1+4\cdot\cos^2 x+3\cdot\sin^2 x}\,dx = \left|\begin{matrix} t = \cos x \\ dt = -\sin x\,dx\end{matrix}\right| = \int \frac{1-t^2}{1+4\cdot t^2+3\cdot(1-t^2)}\cdot(-1)\,dt =$$

$$= \int \frac{t^2-1}{t^2+4}\,dt = \int \frac{t^2+4}{t^2+4} + \frac{-5}{t^2+4}\,dt = \int 1\,dt - 5\cdot\int\frac{1}{t^2+2^2}\,dt =$$

$$= t - 5\cdot\frac{1}{2}\cdot\operatorname{arctg}\frac{t}{2} + c = \underline{\cos x - \frac{5}{2}\cdot\operatorname{arctg}\frac{\cos x}{2} + c}$$

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7) $$V = \pi\cdot\int_{\frac{\pi}{3}}^{\frac{13}{6}\pi}\left(1+\frac{\sin 3x}{2}\right)^2 dx = \pi\cdot\int 1+\sin 3x+\frac{\sin^2 3x}{4}\,dx =$$

$$= \pi\cdot\int 1+\sin 3x+\frac{1}{4}\cdot\frac{(1-\cos 6x)}{2}\,dx =$$

$$= \pi\cdot\left[x - \frac{\cos 3x}{3} + \frac{1}{8}\cdot\left(x - \frac{\sin 6x}{6}\right)\right]_{\frac{\pi}{3}}^{\frac{13}{6}\pi} =$$

$$= \pi\cdot\left[\frac{13}{6}\pi - \frac{1}{3}\cdot\cos\left(\frac{13}{2}\pi\right) + \frac{1}{8}\cdot\left(\frac{13}{6}\pi - \frac{1}{6}\cdot\sin 13\pi\right)\right] -$$

$$-\pi\cdot\left[\frac{\pi}{3} - \frac{1}{3}\cdot\cos\pi + \frac{1}{8}\cdot\left(\frac{\pi}{3} - \frac{1}{6}\cdot\sin 2\pi\right)\right] = \pi\cdot\left[\frac{13}{6}\pi - 0 + \frac{13}{48}\pi - 0 - \frac{\pi}{3} - \frac{1}{3} - \frac{\pi}{24} - 0\right] =$$

$$= \frac{104+13-16-2}{48}\pi^2 - \frac{\pi}{3} = \frac{99}{48}\pi^2 - \frac{\pi}{3} = \underline{\frac{33}{16}\pi^2 - \frac{\pi}{3}}$$

8.) $y' = -k\cdot(y-T)$, $k > 0$, $T = 20$

$y' + ky = k\cdot 20$, $\mu(x) = e^{\int k\,dx} = e^{kx}$ $\Rightarrow$ $y'\cdot e^{kx} + kye^{kx} = 20k\,e^{kx}$

$y\cdot e^{kx} = \int 20k\cdot e^{kx}\,dx = 20k\cdot\frac{e^{kx}}{k} + c = 20\cdot e^{kx} + c$

$\underline{y(x) = 20 + c\cdot e^{-kx}}$

$y(0) = 90 = 20 + c \Rightarrow c = 70$

$y(10) = 80 = 20 + 70\cdot e^{-k\cdot 10} \Rightarrow 70\cdot e^{-10k} = 60 \Rightarrow -10k = \ln\frac{6}{7} \doteq -0{,}15$

$k = 0{,}015$

$\underline{y(x) = 20 + 70\cdot e^{-0{,}015\cdot x}}$

$y(60) = 20 + 70\cdot e^{-0{,}015\cdot 60} = 20 + 70\cdot e^{-0{,}9} \doteq 20 + 70\cdot 0{,}4 =$ [redacted] $= 48\ [^\circ C]$