Equadiff 9

Abstracts
Enlarged Abstracts

edited by
Z. Došlá, J. Kalas, J. Vosmanský
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Zuzana Došlá
Josef Kalas
Jaromír Vosmanský
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**List of abbreviations:**

- PL - plenary lecture
- L - lecture
- EA - enlarged abstract
- c - communication
- p - poster
PREFACE

The presented publication includes three parts: Abstracts, Enlarged Abstracts and Supplement. Short Abstracts of plenary lectures, invited lectures, communications and posters are given in the first part. Enlarged Abstracts (without direct oral presentation) are also considered as an active participation at the Conference. The Supplement contains Abstracts and Enlarged Abstracts, we received after July 15, 1997, and/or those which have not been available in electronic form.

Some abstracts had to be retyped. We apologize for possible mistakes. Some contributions were not submitted in the corresponding form which might cause that they have not been included in the respective chapter.

We appreciate the valuable help of our PhD. students in preparing this volume. We would like to thank Mrs. Lukešová who made the great part of computer arrangement.

Editors
ABSTRACTS
COMPACTNESS CONDITION FOR
BOUNDARY VALUE PROBLEMS

Ravi P. Agarwal, Singapore

AMS Class.: 34B15 (34C10)

Consider the following \( n (\geq 2) \)th order nonlinear differential equation

\[
y^{(n)} = f(x, y, y', \ldots, y^{(q)}), \quad 0 \leq q \leq n - 1, \text{ but fixed.} \quad (*)
\]

With respect to (*) we shall assume that

(A) \( f(x, u_0, u_1, \ldots, u_q) : (a, b) \times \mathbb{R}^{q+1} \rightarrow \mathbb{R} \) is continuous.

(B) Solutions of initial value problems for (*) are unique.

(C) Solutions of (*) extend to \((a, b)\).

(D) For any \( a < a_1 < a_2 < \cdots < a_n < b \) and any solutions \( y(x) \) and \( z(x) \) of (*), it follows that \( y(a_i) = z(a_i), \ 1 \leq i \leq n \) implies \( y(x) \equiv z(x) \), i.e., the differential equation (*) is \( n \)-point disconjugate on \((a, b)\).

In the study of boundary value problems for the differential equation (*), one of the Propositions which has attracted several Mathematicians and has lead to substantially new mathematics is whether conditions (A) – (D) imply the following compactness condition:

(E) If \([c, d]\) is a compact subinterval of \((a, b)\) and \( \{y_m(x)\} \) is a sequence of solutions of (*) which is uniformly bounded, i.e., \(|y_m(x)| \leq M \) on \([c, d]\) for some \( M > 0 \) and all \( m = 1, 2, \ldots \), then there is a subsequence \( \{y_m(x)\} \) such that \( \{y^{(i)}_{m(j)}(x)\} \) converges uniformly on \([c, d]\) for each \( 0 \leq i \leq n - 1 \).

In this lecture we shall survey most of the known results on this Proposition, and touch on some related topics.

DIFFERENTIAL EQUATIONS OF n-TH ORDER
WITH QUASIDERIVATIVES

Miroslav Bartušek, Brno, Czech Republic

AMS Class.: 34C15

Consider the differential equation

\[
y^{[n]} = a(t) f(y^{[0]}, y^{[1]}, \ldots, y^{[n-1]}), \quad n \geq 3 \quad (1)
\]

where the quasiderivatives are defined by

\[
y^{[i]} = \frac{y}{a_0}, \quad y^{[i]} = \frac{1}{a_i} \left( y^{[i-1]} \right)', \quad i = 1, \ldots, n - 1, \quad y^{[n]} = \left( y^{[n-1]} \right)',
\]

\( a_i \) are positive and continuous on \( \mathbb{R}_+ \), \( a \in L_{\text{loc}}(\mathbb{R}_+) \), \( f \) is continuous on \( \mathbb{R}^n \) and \( f(x_1, \ldots, x_n) x_1 \geq 0 \). There are given sufficient conditions under which (1) has oscillatory proper solutions. Some applications to the equation \( y''' + q(t)y' + r(t)f(y, y', y'') = 0, \ r \geq 0 \) are given.
TRANSFORMATIONS OF DIFFERENTIAL EQUATIONS

J. Chrastina, Brno, Czech Republic

AMS Class.: 34A25 (35A30)

Second order differential equations with one unknown function are mentioned: certain fundamental classical results well-known for the linear subcase \( y'' = q(x)y \) (the dispersion theory), \( u_{xy} = au_x + bu_y + cu \) (the Laplace transform) can be carried over the nonlinear equations \( y'' = f(x,y,y') \) or \( y'' = f(x,y) \), and \( u_{xy} = f(x,y,u,u_x,u_y) \), respectively.


DIRICHLET PROBLEM
FOR FIRST ORDER EQUATIONS

B. Dacorogna, (EPFL)

We consider the following Dirichlet problem

\[
\begin{align*}
F_i (Du (x)) &= 0, \text{ a.e. } x \in \Omega, \ i = 1,...,N \\
u (x) &= \varphi (x), \ x \in \partial \Omega.
\end{align*}
\]  

(1)

where \( \Omega \subset \mathbb{R}^n \) is open, \( u : \Omega \to \mathbb{R}^m \) and therefore \( Du \in \mathbb{R}^{m \times n} \), \( F_i : \mathbb{R}^{m \times n} \to \mathbb{R}, i = 1,...,N \) and \( \varphi \in C^1 (\bar{\Omega}; \mathbb{R}^m) \) (or piecewise \( C^1 \)). We next let \( E = \{ \xi \in \mathbb{R}^{m \times n} : F_i (\xi) = 0, \ i = 1,...,N \} \).

We first show that, in the scalar case \( m = 1 \), the problem (1) has a (dense set of) solution \( u \in W^{1,\infty} (\Omega) \), under the sole compatibility condition \( D\varphi (x) \in E \cup intcoE, x \in \Omega \), where \( intcoE \) stands for the interior of the convex hull of \( E \). We also will briefly discuss the existence and non existence of viscosity solutions under the above compatibility assumption.

In the vectorial case \( m, n > 1 \), the same result holds provided \( intcoE \) is replaced by \( intQcoE \) (i.e. the interior of the quasiconvex hull of \( E \)) and some more involved technical conditions. We then apply this result to problems involving singular values which are encountered in nonlinear elasticity. Finally let us mention that we can also consider problems with \( (x,u) \) dependence. (These results are in a joint work with P. Marcellini).
STRUCTURE OF SOLUTIONS AND EXISTENCE OF
POSITIVE SOLUTIONS OF SOME CLASSES
OF RETARDED FUNCTIONAL DIFFERENTIAL
EQUATIONS

Josef Diblík, Brno, Czech Republic

AMS Class.: 34K15 (34K25)

Some classes of linear as well as nonlinear retarded functional differential equations and systems of the type $\dot{x}(t) = f(t, x_t)$ are considered.

A criterion for convergence of solutions of homogeneous delay linear differential equations is formulated. The structure of solutions in nonconvergent case is given. Moreover, a criterion for existence of positive solutions of systems of retarded functional differential equations is presented together with its application to linear systems. Under our restrictions some results known for one scalar differential equation with delay are generalized.

Comparisons and relativity with known results (obtained e.g. by O. Arino, F.V. Atkinson, T.A. Chanturiya, Y. Domshlak, Á. Elbert, L.H. Erbe, K. Gopalsamy, I. Győri, J.R. Haddock, Q. Kong, R.G. Koplatadze, E. Kozakiewicz, T. Krisztin, G. Ladas, M. Pituk, I.P. Stavrolakis, B.G. Zhang, S.N. Zhang and D. Zhou) are given. A remark concerning simplification of investigations in the case of scalar linear equations with delay, which follows from results by F. Neuman, is discussed.

LINEAR HAMILTONIAN SYSTEMS – DISCRETE
VERSUS CONTINUOUS

Ondřej Došlý, Brno, Czech Republic

AMS Class.: 34C10 (39A10)

We compare oscillatory properties of solutions of continuous linear Hamiltonian systems

$$x' = A(t)x + B(t)u, \quad u' = C(t)x - A^T(t)u$$

with their discrete counterparts

$$\Delta x_k = A_k x_{k+1} + B_k u_k, \quad \Delta u_k = C_k x_{k+1} - A_k^T u_k.$$  (**)

In particular, we point out the cases where full analogy between oscillatory behaviour of (*) and (**) has been found, and, on the other hand, the cases where till now no discrete analogy of “continuous” results is known.
STABILITY OF SOME LINEAR SECOND ORDER DIFFERENTIAL EQUATIONS

Á. Elbert, Budapest, Hungary

In 1934 Milloux proved that at least one (nontrivial) solution of the differential equation
\[ x'' + q(t)x = 0 \quad (t \geq 0) \]
tends to zero when \( q(t) \) is a non-decreasing function and \( \lim_{t \to \infty} q(t) = \infty \). Using Lyapunov method, several authors proved that under some additional regularity conditions on \( q(t) \), all solutions tend to zero.

Following an idea of F. V. Atkinson (1978), we consider the differential equation
\[ x'' + \lambda^2 q(t)x = 0, \quad \lambda \in \mathbb{R} \]
and define the set \( S \) of those \( \lambda \)'s for which (⋆) has a solution \underline{not} tending to zero. Clearly, \( 0 \in S \) and if \( \lambda \in S \), then \( -\lambda \in S \).

In analogy of Atkinson’s result, we can prove that if \( q(t) \) is a non-decreasing step-function, then \( S \) is an additive group, i.e. if \( \lambda_1, \lambda_2 \in S \), then \( \lambda_1 + \lambda_2 \in S \). We give examples for different sets \( S \): \( S = \{0\} \), \( S = \mathbb{Z} \), \( S = \{k/2^n : k, n \in \mathbb{Z}\} \), \( Q \subset S \).

The main tool of these investigations is the technique based on the asymptotic behaviour of the difference equations
\[ \begin{bmatrix} a_n \\ b_n \end{bmatrix} = \begin{bmatrix} p_n & q_n \\ r_n & s_n \end{bmatrix} \begin{bmatrix} a_{n-1} \\ b_{n-1} \end{bmatrix}, \quad n = 1, 2, \ldots \]  
(⋆⋆)
with special structure of the coefficient matrix in (⋆⋆).

BIFURCATION OF PERIODIC AND CHAOTIC SOLUTIONS IN DISCONTINUOUS SYSTEMS

Michal Fečkan, Bratislava, Slovakia

AMS Class.: 34A60 (34C25, 58F13, 58F30)

Chaos generated by the existence of Smale horseshoe is the well–known phenomenon in the theory of dynamical systems. The Poincaré–Andronov–Melnikov periodic and subharmonic bifurcations are also classical results in this theory. The purpose of this talk is to extend those results to ordinary differential equations with multivalued perturbations. We present our recent achievements in this direction. Singly perturbed problems are studied as well. Applications are given to ordinary differential equations with both dry friction and hysteresis terms.
THE NONLINEAR LIMIT-POINT/LIMIT-CIRCLE PROBLEM FOR HIGHER ORDER EQUATIONS

John R. Graef, Mississippi, U.S.A.

AMS Class.: 34C10 (34C15, 34B15)

In this paper, we consider the n-th order nonlinear differential equation

\[ y^{(n)} = r(t)f(y, y', \ldots, y^{(n-1)}) \]  

(E)

where \( r \in L_{\text{loc}}[0, \infty) \), \( f: \mathbb{R}^n \rightarrow \mathbb{R} \) is continuous, and \( f(x_1, \ldots, x_n)x_1 \geq 0 \) on \( \mathbb{R}^n \).

We say that equation (E) is of the nonlinear limit-circle type if every continuable solution \( y \) satisfies

\[ \int_0^\infty y(t)f(y(t), y'(t), \ldots, y^{(n)}(t)) \, dt < \infty; \]

if there is at least one continuable solution \( y \) such that

\[ \int_0^\infty y(t)f(y(t), y'(t), \ldots, y^{(n)}(t)) \, dt = \infty, \]

then equation (E) is said to be of the nonlinear limit-point type.

We give some sufficient conditions for equation (E) to be of the nonlinear limit-point type, and in so doing, give a partial answer to an open conjecture for linear equations. Additional results for special classes of equations and some open problems will also be presented.

ON STABILITY PROPERTIES OF SOLUTIONS OF SECOND ORDER DIFFERENTIAL EQUATIONS

László Hatvani, Szeged, Hungary

AMS Class.: 34D20

We consider the equation

\[ x'' + h(t, x, x')x' + f(x) = 0, \]

where \( xf(x) > 0 \) for \( x \neq 0 \), and there are \( a, b : [0, \infty) \rightarrow [0, \infty) \) and \( \Delta > 0 \) such that \( x^2 + y^2 \leq \Delta \) implies \( a(t) \leq h(t, x, y) \leq b(t) \) (\( t \geq 0 \)). This equation describes the oscillation of a material point around the equilibrium position \( x = 0 \) damped by viscous friction. We give conditions for functions \( a, b \) guaranteeing asymptotic stability of the equilibrium state \( x = x' = 0 \). The cases of small damping \( (0 \leq a(t) \leq b_0 < \infty) \), the large damping \( (0 < a_0 < a(t) \leq b(t) < \infty) \) and the general case are treated separately, because they need different methods. The results on the small damping will be applied to get new Armellini–Tonelli–Sansone–type conditions guaranteeing that all solutions of the equation \( x'' + k(t)f(x) = 0 \), \( (k(t) \uparrow \infty, t \rightarrow \infty) \) tend to zero as \( t \rightarrow \infty \).
SECOND LIAPUNOV’S METHOD FOR STABILITY INVESTIGATION OF DIFFERENTIAL EQUATIONS WITH DEVIATIONS

Denis Khusainov, Kiev, Ukraine

One of the most universal methods for stability investigation of various dynamical systems equations is Second Liapunov’s Method. This method was oriented on stability research of ordinary differential equations. Later it found its application for equations with distributed parameters, systems with delay, stochastic systems and so on.

There exists two alternative approaches for stability investigation of the systems with delay. First one, based on using special functions with additional Razumikhin condition under its derivative sign estimation. In report linear stationary systems of delay and neutral types are considered. Liapunov function of quadratic form is used. Stability conditions for arbitrary delay, and for delay depending on system’s parameters are derived.

The second approaches of Liapunov method is Liapunov–Krasovskij functional method. In the report construction of the functional in quadratic form is proposed. The procedure of the functional construction is reduced to solving optimization task with convex quality criterion on the set of pair of positive definite matrices.

ON SINGULAR BOUNDARY VALUE PROBLEMS FOR HIGHER ORDER ORDINARY DIFFERENTIAL EQUATIONS

I. Kiguradze, Tbilisi, Georgia

AMS Class.: 34B15

Boundary value problems of the kind

\[ u^{(n)} = f(t, u, u', \ldots, u^{(n-1)}), \]

\[ u^{(n_1)}(a) = 0 \ (i = 1, \ldots, m), \quad u^{(n_2)}(b) = 0 \ (j = 0, \ldots, n - m), \]

are considered, where \(-\infty < a < b < +\infty\), \(n \geq 2, n_1, n_2\) and \(n_1, n_2\in\{0, \ldots, n-1\}\). As for the function \(f : [a, b] \times \mathbb{R}^n \to \mathbb{R}\), it satisfies the local Carathéodory conditions but with respect to the first argument it may have nonintegrable singularities at the ends of the interval \([a, b]\).

On the basis of the method of a priori estimates optimal, in a certain sense, criteria of the existence and uniqueness of solution are established.
QUADRATIC FUNCTIONALS: POSITIVITY, OSCILLATION, RAYLEIGH’S PRINCIPLE

W. Kratz, Ulm, Germany

AMS Class.: 34A30 (49K15, 93B60)

In this talk we shall give a survey on the theory of quadratic functionals. Particularly the relationships between positive definiteness and the asymptotic behaviour of Riccati matrix differential equations and between oscillation of linear Hamiltonian systems and Rayleigh’s principle will be discussed. Moreover, we will demonstrate the main tools from control theory (as e.g. strong observability), from the calculus of variations (as e.g. field theory and Picone’s identity), and from matrix analysis (as e.g a l’Hospital’s rule for matrices). Key references are: W. Kratz, Quadratic functionals in variational analysis and control theory, Akademie Verlag, Berlin 1995. M. Morse, Variational analysis: critical extremals and Sturmian theory, Wiley, New York 1973. W.T. Reid, Ordinary differential equations, Wiley, New York 1971.

SCHAUDER ESTIMATES FOR EQUATIONS WITH FRACTIONAL DERIVATIVES

Stig–Olof Londen, Helsinki, Finland

This work is joint with Philippe Clément and Gustaf Gripenberg. The equation

\[ D_t^\alpha (u - h_1) + D_x^\beta (u - h_2) = f, \quad 0 < \alpha, \beta < 1; \quad t, x \geq 0, \]

where \( D_t^\alpha \) and \( D_x^\beta \) are fractional derivatives of order \( \alpha \) and \( \beta \) is studied. It is shown that if \( f = f(t, x), \quad h_1 = h_1(x), \quad h_2(t) \) are Hölder-continuous and \( f(0, 0) = 0 \), then there is a solution such that \( D_t^\alpha u \) and \( D_x^\beta u \) are Hölder-continuous as well. This is proved by first considering an abstract fractional evolution equation and then applying the results obtained to the equation above. Finally the solution of the equation with \( f = 1 \) is studied.
ASPECTS OF REGULARITY THEORY FOR
THE NAVIER-STOKES EQUATIONS
AND THEIR MODIFICATIONS

Josef Málek, Prague, Czech Republic

AMS Class.: 35K55 (35Q30, 76D)

We consider systems describing the steady and unsteady motions of incompressible fluids characterized by the polynomial dependence of the stress tensor on the symmetric velocity gradient of the order $p - 1$, where $p$ is a parameter, generally $\geq 1$. In dependence on $p$ and dimension, we present recent results regarding the existence of weak solutions and their regularity both for stationary and evolutionary models, completed by Dirichlet boundary conditions or the requirement that all functions are periodic (in space).

Second part of the talk is devoted to a particular model of the considered class, the Navier-Stokes equations. We present (i) results on nonexistence of nontrivial singular weak solutions in the selfsimilar form proposed by J. Leray; (ii) remarks to the result of Uchovskii and Yudovich on regularity of weak solution if cylindrical symmetry is assumed.

RELAXATION AND QUASICONVEX ENVELOPE

Jan Malý, Prague, Czech Republic

AMS Class.: 49M20

This contribution presents joint results obtained with Guy Bouchitté (Université de Toulon et du Var) and Irene Fonseca (Carnegie Mellon University, Pittsburgh).

Let $\Omega \subset \mathbb{R}^N$ be an open set and $p, q \in (1, \infty)$. Given a functional

$$\int_{\Omega} F(\nabla u) \, dx$$

defined on the Sobolev space $W^{1,q}(\Omega; \mathbb{R}^d)$, we consider its relaxation with respect to the weak convergence in $W^{1,p}$. We are concentrated on the less standard but important case when $p$ is less than the growth exponent $q$. If $p > q \frac{n-1}{n}$, we show that the relaxation is represented by a measure, and the density of the absolutely continuous part is $QF(\nabla u)$. Here $QF$ denotes the quasiconvex envelope of $F$. 
SEVENTY-FIVE YEARS OF GLOBAL ANALYSIS AROUND THE FORCED PENDULUM EQUATION

Jean Mawhin, Louvain-la-Neuve, Belgium

AMSI Class.: 34C15

The first global results on the forced pendulum equation

\[ x''(t) + cx'(t) + a \sin x(t) = e(t), \]

have been obtained by Georg Hamel in 1922, in a special issue of the *Mathematische Annalen* dedicated to David Hilbert.

In the following seventy-five years, this equation and its variants have inspired, motivated and tested many fundamental new methods in nonlinear and global analysis, and in the topological, variational and dynamical approaches to nonlinear differential equations.

Many efforts have been made in understanding the structure of the set of harmonic, subharmonic, homoclinic and other solutions.

A number of questions still remain open. The lecture will try to describe the state of the art in this active area of research.

THE BOUNDARY-VALUE PROBLEMS FOR LAPLACE EQUATION AND DOMAINS WITH NONSMOOTH BOUNDARY

Dagmar Medková, Prague, Czech Republic

AMSI Class.: 35J05 (35J25, 31B10)

Dirichlet, Neumann and Robin problem for the Laplace equation is investigated on the open set with holes and nonsmooth boundary. The solutions are looked for in the form of a double layer potential and a single layer potential. The measure, the potential of which is a solution of the boundary-value problem, is constructed.
A NEW APPROACH TO AN ANALYSIS OF HENRY’S TYPE INTEGRAL INEQUALITIES AND THEIR BIHARI’S TYPE VERSIONS

Milan Medved, Bratislava, Slovakia

AMS Class.: 34A40, 35K, 45D05

We present a new method how to estimate solutions of nonlinear integral inequalities with singular kernels of various forms, e.g. of the form

\[ u(t) \leq a(t) + \int_0^t (t - s)^{\beta - 1} F(s) \omega(u(s)) ds, \]

where \( \beta > 0 \), \( a(t) \), \( F(t) \), \( \omega(u) \) are nonnegative, continuous functions. Our estimates are of Bihari’s type and in the case \( F(t) \) constant and \( \omega \) linear (the Henry’s inequality), our estimate is exponential. An estimate by D. Henry, frequently used in the geometrical theory of PDEs, is expressed in a more complicated form. The method used by D. Henry is the classical iterative one, not convenient for the nonlinear case.

SOLUTION AND APPLICATION OF THE CURVE EVOLUTION EQUATIONS

Karol Mikula, Bratislava, Slovakia

AMS Class.: 35K65, 65N40, 53C80

Methods of numerical solution of the geometrical equations

\[ v = \beta(\theta, k), \]  

(1)

where \( v \) is the normal velocity of the evolving plane curve, \( k \) its curvature, \( \theta \) the angle of the tangent to the curve and \( \beta \) an increasing function in \( k \), are presented. Such equations are widely used to describe several phenomena in material sciences and computer vision. Anisotropic motion of phase interfaces in free boundary problems with surface tension as well as the image segmentation and shape analysis in image processing are examples of their application. We present approximation schemes and existence and convergence results for the curve evolution leading to the solution of degenerate parabolic problems of slow and fast diffusion type. Approximation of (1) by means of the so called intrinsic heat equations and the numerical solution based on the level set formulation are discussed, too. Computational results from application in echocardiography and phase transition are presented.
MULTIPLE SOLUTIONS, EXISTENCE
AND UNIQUENESS OF NONLINEAR
BOUNDARY VALUE PROBLEMS

Irena Rachůnková, Olomouc, Czech Republic

AMS Class.: 34B15 (34K10)

We present here the application of topological degree methods onto boundary value problems for functional and ordinary differential equations of the second order.

We show the existence and uniqueness results and the results concerning the existence of more solutions for several types of boundary conditions. Besides the classical two-point and some multipoint conditions, we work with the nonlinear conditions of the form \( g(x(a), x'(a)) = 0 \), where \( g \) is a continuous function and with the functional conditions \( \alpha(x) = c \), where \( \alpha: C(J) \to C(J) \) is a linear, bounded and increasing functional and \( c \in \mathbb{R} \).

We give here some new results for the above problems including also the case, where no growth restrictions for \( f \) with respect to \( x' \) are required. For the proper choice of the operators in the functional differential equation we can transfer the results onto the third and fourth order ordinary differential equations.

MODELLING OF MICROSTRUCTURE IN
NON-QUASICONVEX VARIATIONAL PROBLEMS

Tomáš Roubíček, Praha, Czech Republic

AMS Class.: 35M10 (65N30, 73G05)

The contribution addresses a multidimensional vectorial stationary variational problem of the type

\[
\int_{\Omega} \varphi(x, y(x), \nabla y(x)) \, dx + \int_{\Gamma} \phi(x, y(x)) \, dS \to \inf, \quad y \in W^{1,p}(\Omega; \mathbb{R}^m)
\]

with \( \Omega \subset \mathbb{R}^n \) a Lipschitz domain. We have in mind the case when \( \varphi(x, y, \cdot) \) is not quasi-convex so that the minimum of the above problem need not be attained and the problem must be suitably relaxed. Here we will use a continuous extension and assume \( p > 1 \), which, under a coercivity assumption, excludes concentration of energy of minimizing sequences and enables to characterize their faster and faster oscillations in the limit by means of Young measures.

Various numerical approximation techniques applicable to relaxed vectorial variation problems will be surveyed.

Application to a steady-state microstructure of crystalline martensitic materials will be presented. Evolution of microstructure will be touched, too.
COMPUTATIONAL MODELLING OF PROBLEMS WITH MEMORY

Simon Shaw, M. K. Warby and J. R. Whiteman, Uxbridge, U.K.

AMS Class.: 45K05 (73F15)

A discussion is given of how problems with memory can occur when modelling physical problems, and of their mathematical formulation in terms of integro-differential equations.

Focusing on the problem of linear, isothermal quasistatic deformation of an isotropic compressible solid, and assuming a non-ageing fading memory, we present numerical schemes complete with error estimates. The results of computations are given and compared against experimental creep data for a nylon polymer. This comparison demonstrates the limitations of the linear model and so we close with a discussion of a simple but effective method of producing a nonlinear model using reduced time.

A GENERALIZED LINKING THEOREM WITH AN APPLICATION TO NONLINEAR EQUATIONS IN UNBOUNDED DOMAINS

Andrzej Szulkin, Stockholm, Sweden

AMS Class.: 34C15 (34C37, 35J65, 58E05)

Let $E = Y \oplus Z$ be a separable Hilbert space, $\dim Y = \dim Z = \infty$, and $\Phi$ a functional such that $\Phi \geq b > 0$ on $\{z \in Z : \|z\| = \rho\}$ and $\Phi \leq 0$ on $\partial M$, where $M = \{y + \lambda z_0 : y \in Y, \|y + \lambda z_0\| < R, \lambda > 0\}$ and $z_0 \neq 0$ is a fixed element of $Z$. Then, under some additional hypotheses, $\Phi$ has a Palais-Smale sequence $(u_n)$ with $\Phi(u_n) \to c \geq b$. With the aid of this result (which extends the Benci-Rabinowitz linking theorem) we shall consider the following problems:

1) Existence of nontrivial solutions for a semilinear Schrödinger equation in $\mathbb{R}^N$;
2) Existence of time-periodic motions for an infinite chain of particles with nearest neighbour interaction (the so-called Fermi-Pasta-Ulam model);
3) Existence of homoclinics for a first order Hamiltonian system.

In all three problems the underlying functional has a linking structure in the sense described above and is translation-invariant (this implies in particular that the Palais-Smale condition fails).
ASYMPTOTIC ANALYSIS OF THE NAVIER-STOKES AND ATMOSPHERE EQUATIONS

Roger Temam, Indiana, U.S.A.

We study the Navier Stokes equations in a thin layer of fluid around the sphere and show how the primitive equations of the atmosphere can be derived from the Navier stokes equations. Other applications will be also presented.

BRANCHING OF PERIODIC SOLUTIONS IN HAMILTONIAN AND REVERSIBLE SYSTEMS

André Vanderbauwhede, Gent, Belgium

AMS Class.: 58F (34C)

Periodic orbits of Hamiltonian or reversible systems appear generically in one-parameter families; such families may originate at an equilibrium (Liapunov center theorem) or finish in a period blow-up at a homoclinic orbit. Along the one-parameter family branches of subharmonic solutions may bifurcate.

In this lecture we present some general reduction results which can be used to study the bifurcation of branches of periodic orbits from equilibria and the branching of subharmonics. The method does not require any non-resonance conditions, and is based on a combination of normal form and Liapunov-Schmidt reductions. The reduced problems keep the original structure (symplecticity or reversibilty) but they have an additional circle or cyclic symmetry, depending on the case. This additional symmetry can then be exploited to study the actual bifurcations. We illustrate the approach with some results on (i) Hopf bifurcation at $k$-fold resonances, and (ii) basic subharmonic branching in Hamiltonian systems.

The results presented in this lecture are based on joint work with Jan-Cees van der Meer, Jürgen Knobloch and Maria-Cristina Ciocci.

STURM-LIOUVILLE PROBLEMS

Anton Zettl

AMS Class.: 34B24

We discuss the approximation of the discrete and continuous (essential) spectrum of a given singular Sturm-Liouville Problem (SLP) with eigenvalues of regular SLP. This approximation is illustrated with some examples. Eigenvalues of regular and singular SLP can be computed numerically using the FORTRAN code sleign2. It can be downloaded from the web using the address: http://www.math.niu.edu/zettl/SL2/
THE USE OF SEMIREGULAR FINITE ELEMENTS

Alexander Ženíšek, Brno, Czech Republic

AMS Class.: 65N30

The paper is a survey of the author’s results obtained for semiregular finite elements. They concern the interpolation theory, the effect of numerical integration and the approximation of the boundary. This means all basic finite element variational crimes are taken into account. In the case of the interpolation theory the results of other authors are also introduced.

The analysis of both the effect of numerical integration and approximation of the boundary is restricted to triangular elements with linear polynomials and to quadrilateral elements with four–node isoparametric functions. In the case of triangles the effect of numerical integration does not depend on the geometry of elements.

In the case of approximation of the boundary the rate of convergence $O(h)$ in the norm of the space $H^1(\Omega_h)$ is guaranteed under the following conditions:
1. the data are sufficiently smooth; 2. the lengths $b_M$ and $h_M$ of the smallest and largest sides, respectively, of every element $M$ ($M = T, K$) satisfy the relations
$$C_1 h_M^2 \leq b_M \leq C_2 h_M^2$$
where $T$ stands for a triangle and $K$ for a quadrilateral.

RENORMALIZED SOLUTIONS – OF NONLINEAR ELLIPTIC EQUATIONS – WITH MEASURE DATA

François Murat, Paris 6, France

AMS Class.: 35D05 (35J25)

In this lecture I will present results taken from recent joint work with Gianni Dal Maso, Luigi Orsina and Alain Prignet. We consider the nonlinear elliptic problem

$$\begin{cases}
- \text{div} a(x, \nabla u) = \mu \quad \text{in } \Omega \\
\quad u = 0 \quad \text{on } \partial \Omega
\end{cases}
$$

where the operator $u \to \text{div} a(x, \nabla u)$ is a classical monotone operator from $W^{1,p}_0(\Omega)$ into $W^{-1,p'}(\Omega)$, but where $\mu$ is a bounded Radon measure on $\Omega$. We introduce a new definition of solution (the renormalized solution) for this problem. We prove the existence of such a solution, its stability with respect to the right hand side, and partial uniqueness results.
VIBRATIONS AND CHAOTIC MOTIONS OF GIMBAL SUSPENSION GYRO UNDER THE ACTION OF THE PERTURBED MOMENT

S. A. Agafonov, Moscow, Russia, T. V. Muratova, Moscow, Russia

AMS Class.: 34C10, 34C15

The solution of two dynamical problems of the balanced gimbal suspension gyro under the action of the perturbed moments are presented. In the first problem the perturbed moment acts on the external ring and presents the sum of the viscous friction moment and the moment which depends on deviation angle of internal ring. The stability of the stationary motion under which the planes of the rings are orthogonal is analyzed. Under the passing through the critical meaning of the bifurcational parameter the loss of stationary motion stability occurs accompanied by the birth of the limit cycle. The stability condition of this regime is found.

In the second problem the small perturbed moment acts on the internal ring and presents the sum of the viscous friction moment and the moment which is periodically changed by time. Unperturbed system has two types of the stationary motions: first type is stable, but the second one is unstable. In the perturbed system separatrix passing through the hyperbolic points splits and entering and leaving separatrices intersect. The condition of their intersection is found and it is local criterion of the chaotic motion.

CONTROL OF BOUNDARY VALUE PROBLEM FOR WEAKLY NONLINEAR IMPULSIVE SYSTEM

M. U. Akhmetov, L. A. Guseinova, Kazakhstan Zafer A., Turkey

AMS Class.: 34A37, 34B

Let us consider the following system of differential equations with impulsive effects at variable times

\[
\frac{dx}{dt} = A(t)x + C(t)u + \mu f(t, x, u, \mu), \quad t \neq \tau_i + \mu \tau_i(x, \mu),
\]

\[
\Delta x|_{\tau_i + \mu \tau_i(x, \mu)} = B_i x + D_i v_i + \mu I_i(x, v_i, \mu).
\]

(1)

Here \(x \in \mathbb{R}^n, u, v_i \in \mathbb{R}^m; \mu \) is a small real parameter, \(A(t), B_i\) are \(n \times n\) matrices, \(C(t), D_i\) are \(m \times m\) matrices and the function \(f(t, x, u, \mu)\) is continuous in \(t \in [0, 1]\). Moreover, we shall suppose that \(f(t, x, u, \mu)\) and \(I_i(x, v_i, \mu)\) are continuously differentiable in \(x, \mu, v_i, u\). We shall say that control problem for the system (1) is solvable, if for each bounded set \(G \in \mathbb{R}^m\) we can indicate \(\mu_0 > 0\), such that for any \(a, b \in G\) and \(|\mu| < \mu_0\) there exists the controls \(u, v_i\) and solution \(x(t)\) of equation (1) satisfying the conditions \(x(0) = a, x(1) = b\). We determine conditions sufficient for the solvability of the control problem for the system (1) as well as the successive approximations.
ON A PROPERTY OF THE INERTIAL MANIFOLD

Veronica Ion Anca, Bucharest, Romania

AMS Class.: 35B40

The existence of the inertial manifold for dissipative dynamical systems is proved by several methods.

Among them there are the non-constructive method of the Fixed-Point Theorem and the constructive method of the integral manifolds.

In the second theory it is proved a very interesting property of the inertial manifold. Namely, the attractor of the dynamical system is contained in the closure of the inertial manifold and not in its interior.

We prove that this property remains valid in the case of the inertial manifold obtained in the frame of the Fixed-Point Theorem.

WEAK ALMOST–PERIODIC AND BOUNDED SOLUTIONS OF DIFFERENTIAL SYSTEMS

Jan Andres, Olomouc, Czech Republic

AMS Class.: 34C27 (34B15)

We will discuss the existence and uniqueness of weak almost–periodic solutions (in the sense of H. Weyl and A. S. Besicovitch) of the Carathéodory quasi–linear systems of ODEs. For this, the entirely bounded trajectories are treated as the solutions of asymptotic BVPs, at first. The possibility of extending the obtained results to differential inclusions will be mentioned as well.

ON BOUNDARY VALUE PROBLEMS FOR SYSTEMS OF GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS

Malkhaz Ashordia, Tbilisi, Georgia

AMS Class.: 34B10 (34B15)

The sufficient conditions are established for the solvability and unique solvability of the problem

\[
\frac{dx(t)}{dt} = dA(t) \cdot f(t, x(t)), \quad h(x) = 0,
\]

where \( A : [a, b] \to R^{n \times n} \) is a matrix-function with bounded variation components, \( f : [a, b] \times R^n \to R^n \) is a vector-function belonging to the Carathéodory class corresponding of \( A \), and \( h \) is a continuous operator from the space of \( n \)-dimensional vector-functions of bounded variation into \( R^n \).
Moreover, the concept of a strongly isolated solution of the problem (1) is introduced. It is state that the problems with strongly isolated solutions are correct. Sufficient conditions for the correctness of these problems are given.

**ON QUASICONFORMAL MAPPINGS AND 2-D G-CLOSURE PROBLEMS**

K. Astala and M. Miettinen, Jyväskylä, Finland

AMS Class.: 73B27, 35B27, 30C65

We shall discuss a problem of finding the optimal bounds on the conductivity of all possible mixtures of nonisotropic crystals in terms of their volume ratios only. Our study is based on a breakthrough found by V. Nesi (Arch. Rat. Mech. Anal., 134 (1996), pp. 17-51) who obtained optimal bounds by applying the optimal quasiconformal estimates established earlier by K. Astala. Quasiconformal estimates turned out to be a crucial tool in this connection, as optimal conductivity problems can be modelled by elliptic variational integrals. In particular Nesi found optimal estimates only in the case where the mixture of two material (one anisotropic and one isotropic) is isotropic. However, more general quasiconformal methods seem to be appropriate tools to obtain more general conductivity bounds. We shall present some results to this direction.

**ON ASYMPTOTIC BEHAVIOUR OF EMDEN – FOWLER TYPE DIFFERENTIAL EQUATIONS WITH COMPLEX-VALUED COEFFICIENT**

Irina V. Astashova, Moscow, Russia

AMS Class.: 34A34

Consider differential equation $y'' = p|y|^\gamma y$ with $\gamma > 0$ and $p = p_1 + ip_2 \in \mathbb{C}$. For $p_2 \neq 0$ explicit complex-valued solutions are obtained:

$$y(x) = \exp\left[\varphi_0 + \frac{(R - p_1)(\gamma + 4)}{2\gamma p_2} \ln |x - x_0|\right], \quad R = \sqrt{p_1^2 + \frac{8(\gamma + 2)}{(\gamma + 4)^2} p_2^2},$$

with arbitrary real $x_0$, $\varphi_0$ and $x \in (x_0; +\infty)$ or $x \in (-\infty; x_0)$.

It is proved that all other non-trivial solutions are defined on bounded intervals and that their boundary behaviour is equivalent in some sense to this of the above explicit solutions. Some generalizations are obtained for $p$ depending on $x$.

For real $p(x)$ asymptotic behaviour of real-valued solutions was investigated by I. T. Kiguradze (Georgia), V. A. Kondratiev and V. A. Nikishkin (Russia).
ON THE ASYMPTOTICS OF SOLUTIONS FOR SOME DIFFERENCE EQUATIONS

Katalin Balla, Budapest, Hungary

AMS Class.: 39A11 (39A10)

Systems of asymptotically time-invariant linear difference equations

\[ x(n + 1) = A(n)x(n) + b(n), \quad n \in \mathbb{N} \]

may have several solutions exhibiting the same asymptotic behaviour when \( n \to \infty \). From the very beginning, the theory of difference equations is focussed on the description of particular solutions. In opposite to that, here, without explicit construction of individual solutions, the stable sets of solutions with the same asymptotic behaviour will be characterized as a whole under weak convergence assumptions and general eigenstructure of matrix \( A_\infty = \lim_{n \to \infty} A(n) \).

A NEW METHOD OF THE EXACT LINEARIZATION AND SOME CLASSES OF DYNAMIC SYSTEMS

Lev Berkovich, Samara, Russia

AMS Class.: 34A34 (34A05)

In relation with going now de-linearization of Science, in general, and Physics, especially it looks actual to develop the already known and creating new mathematical methods of linearization of the differential equations.

Using the transformation of form

\[ y = v(x,y)z, \quad dt = u_1(x,y)dx + u_2(x,y)dy, \]

where \( v, u_1, u_2 \) are sufficiently differentiable functions in some domain \( G(x,y) \), not annuled in it, had been got the criterion of the exact linearization of the equation of form

\[ F(x, y, y', ..., y^{(n)}) = 0. \]

On consider also the questions:
* A linearization of Liouvillian systems.
* The some systems of hydrodynamic type.
* Semilinear equations, corresponding to nonlinear evolution equations of diffusion type.
DOUBLY ASYMPTOTIC TRAJECTORIES
OF LAGRANGIAN SYSTEMS
AND A PROBLEM BY KIRCHHOFF

M. Letizia Bertotti, Trento, Italy  Sergey V. Bolotin, Moscow, Russia

AMS Class.: 58F05, 58E05, 34C37, 70H05, 34C35

We consider Lagrangian systems with compact \( n \)-dimensional configuration manifold \( M \) and Lagrange function \( L \in C^2(TM \times \mathbb{R}) \) of the form

\[
L(x, \dot{x}, t) = \frac{1}{2} \langle A(x)\dot{x}, \dot{x} \rangle + t \langle w(x), \dot{x} \rangle + t^2 U(x) + V(x),
\]

where \( A(x) \) is a symmetric positive definite operator for all \( x \in M \), \( \langle \cdot, \cdot \rangle \) denotes the pairing between covectors and vectors, \( w \) is a covector field on \( M \) and \( U \) and \( V \) are functions on \( M \). We assume the existence of a nondegenerate unique minimum point \( x_0 \in M \) of the function \( W(x) = U(x) - \frac{1}{2} \langle w(x), A(x)^{-1} w(x) \rangle \). We prove, by means of variational methods, the existence of infinitely many trajectories \( x(t) \) tending, as \( t \to \pm \infty \), to the “equilibrium at infinity” \( x_0 \). We apply this result to the Kirchhoff problem of a heavy rigid body moving through a boundless incompressible ideal fluid, which is at rest at infinity and has zero vorticity.

ON DYNAMIC VISCOELASTIC VON KÁRMÁN’S PLATES

Igor Bock, Bratislava, Slovakia

AMS Class.: 73F15 (45D05)

We shall deal with the system of integro-differential equations describing large deflections of a thin viscoelastic plate acted under the dynamic load. Using the same geometrical assumptions as in the elastic case we obtain the nonlinear system of Volterra integro-differential equation for the Airy stress function and a pseudo-hyperbolic equation for deflections:

\[
\begin{align*}
H_{ijkl}(0) \partial_{ijkl} \Phi(t) + \int_0^t \partial_t H_{ijkl}(t - s) \partial_{ijkl} \Phi(s) ds &= h[\partial_t w, w], \\
\partial_{tt} w - \frac{h^2}{12} \Delta \partial_{tt} w + A^1_{ijkl} \partial_{i} \partial_{ijkl} w + A^0_{ijkl} \partial_{ijkl} w - [\Phi, w] &= f, \\
[\Phi, w] &= \partial_1 \Phi \partial_1 w - 2 \partial_1 \Phi \partial_{12} w + \partial_{22} \Phi \partial_{22} w; \text{ with nonhomogeneous initial and homogeneous boundary conditions.}
\end{align*}
\]
ON THE EXISTENCE AND UNIQUENESS OF SYMMETRIC SOLUTIONS TO A SEMILINEAR PDE

Gabriella Bognár, Miskolc, Hungary

AMS Class.: 35A22

We consider the semilinear partial differential equation
\[ \Delta u + g(u, |\nabla u|) = 0 \quad \text{in} \quad D, \]
\[ u = 0 \quad \text{on} \quad \partial D, \]
where \( u \in C^2(D) \cap C(\overline{D}) \) and \( D \in \mathbb{R}^N \) is an open ball centered at the origin, moreover \( g(x, y) \) is a real valued function in \([0, \infty) \times [0, \infty)\), continuous and non-increasing in \( x \) (or in \( y \)) for all \( y \) (or \( x \)).

The aim of this talk is to establish the existence and uniqueness of this boundary value problem.

QUADRATIC FUNCTIONALS ON TIME SCALES

Martin Bohner, Ulm, Germany  Ravi P. Agarwal, Singapore

AMS Class.: 34C10 (39A10)

We shall consider the self-adjoint second order matrix equation
\[ \left[ R(t)Y^\Delta \right]^\Delta + P(t)Y^\sigma = 0 \]
on an arbitrary time scale so that the special cases of the well-studied corresponding differential equation and the recently developed theory for the related difference equation are unified. This common treatment also shows why the differences between the continuous and the discrete theory occur in the study of this equation.

Motivated by the study of variational problems on time scales we examine quadratic functionals and give versions on time scales for the Wronskian identity, Picone’s identity, Jacobi’s condition, and Sturm’s separation and comparison results.
NONLINEAR BOUNDARY VALUE PROBLEMS
FOR ORDINARY DIFFERENTIAL EQUATIONS
IN RESONANCE

Alexander Boichuk, Kyiv, Ukraine

AMS Class.: 34

The nonlinear boundary value problems
\[ \dot{z} = Z(z, t), \ell z = \varphi(z(\cdot)) \]
in neighborhood of generation solution \( z_0 \) are considered in assumption that the nonlinear \( n \)-dimensional vector-function \( Z(z, t) \) is such that \( Z(z, \cdot) \in C^1([z - z_0 \leq q], Z(z, \cdot) \in C[a, b]; \ell \) and \( \varphi \) - linear and nonlinear \( m \)-dimensional bounded vector-functionals, and \( \varphi \) is continuously differentiable in sens of Frechet.

Applying the Lyapunov-Schmidt reduction and the theory of generalized inverse operators, the sufficient and necessary conditions of the solvability of solutions \( z = z(t) \in C^1[a, b], z : [a, b] \rightarrow \mathbb{R}^n \) and iteration algorithms for constructing these solutions in resonance cases are obtained. General theory of these problems and classification of resonance BVP are received. These schemes for analysis of BVP are applied to investigation of more wide classes of functional differential equations.

ON THE STRUCTURE OF SOLUTION SETS
OF SOME DIFFERENTIAL EQUATIONS
IN BANACH SPACES

Daria Bugajewska, Poznań, Poland

AMS Class.: 34G20

We shall consider the following implicit Darboux problem
\[ z_{xy} = g(x, y, z, z_{xy}), \]
\[ z(x, 0) = 0, \quad 0 \leq x < +\infty, \]
\[ z(0, y) = 0, \quad 0 \leq y < +\infty, \]
in Banach space, where \( z_{xy} \) denotes the second mixed derivative of \( z \). We shall present new results concerning existence and topological structure of the solution sets of the problem (1). The method of our proofs is based on an application of fixed point theorem from [2] or the extension of Vidossich’s theorem from [3]. The main conditions on our theorems will be formulated in terms of the Kuratowski measure of noncompactness.

ON SOME APPLICATIONS OF THEOREMS
ON THE SPECTRAL RADIUS
TO DIFFERENTIAL EQUATIONS

Dariusz Bugajewski, Poznań, Poland

AMS Class.: 34K05

We present new uniqueness theorems for the following Darboux problem of
neutral type in an implicit form:

\[ z_{xy}(x, y) = f(x, y, z(h(x, y)), z_{xy}(H(x, y))) \quad \text{for} \quad (x, y) \in I^2, \]
\[ z(x, 0) = 0 \quad \text{for} \quad x \in I, \]
\[ z(0, y) = 0 \quad \text{for} \quad y \in I, \]

where \( z_{xy} \) denotes the second mixed derivative and \( I = [0, a] \), \( a > 0 \). We shall
consider solutions of (1) in different spaces. Our proofs are essentially based on
Esayan’s or Zima’s theorem concerning the spectral radius of the sum of operators.
For more details see:

[1] D. Bugajewski, On some applications of the theorems on the spectral radius

AN A POSTERIORI ERROR ESTIMATE
FOR THE STOKES FLOW

Pavel Burda, Prague, Czech Republic

AMS Class.: 65N30 (76D07)

We consider the Stokes flow in tubes with abrupt changes of diameter. We
proved in [1] that for nonconvex internal angles the velocities near the corners
possess an expansion of the form

\[ u(\rho, \vartheta) = \rho^\gamma \varphi(\vartheta) + \ldots, \]

where \( \rho, \vartheta \) are local
spherical coordinates. E.g. for the angle \( \alpha = \frac{3}{2} \pi \) we have \( \gamma = 0.5444837 \) (the same
result as for the plane flow). So in this case the regularity is not \( H^2(\Omega) \) but only
\( u \in H^{\gamma+1-\varepsilon}(\Omega) \); here \( H^s \) denotes the standard Sobolev space.

It is well-known that using the standard finite element method on triangles
with polynomials of degree \( p = 1, 2, 3 \) we have the a priori error estimate

\[ \| (u - u_h) \|_{H^1(\Omega)} \leq C \ h^{\gamma-\varepsilon} \| u \|_{H^{\gamma+1-\varepsilon}(\Omega)}, \]

which cannot be improved by increasing the degree of polynomials. In our paper
we present some remedy from this, and give also an a posteriori error estimate.

corner singularities; to appear in Proc. Conf. Finite Element Methods, Jyväskylä,
ASYMPTOTIC EXPANSIONS AND NUMERICAL APPROXIMATION OF NON-LINEAR DEGENERATE BOUNDARY-VALUE PROBLEMS

M. P. Carpentier, Lisboa, Portugal  P. M. Lima, Lisboa, Portugal

AMS Class.: 65L12, 65B05

In the present paper we are concerned with boundary-value problems (BVP) of the form

\[ y''(x) = c x^p y^q, \]
\[ y(0) = 1, \quad y(1) = 0; \]  

(1)  
(2)

where \( c, p \) and \( q \) are real constants, \( p > -2, \quad q > 0 \). Equation (1) is known as the generalized Emden-Fowler equation and many particular cases of it arise in different problems of Physics and Mechanics. Asymptotic expansions of the solution are obtained near the singularities at \( x = 0 \) and \( x = 1 \). Based on these expansions, we introduce new numerical methods, which take into account the behaviour of the solution near the origin. We present numerical results, for some particular cases, and compare them with the results obtained by the methods used before.

THE EXACT SOLUTIONS OF MOTION EQUATIONS OF A LAGRANGE GYROSCOPES SYSTEM

Dmitriy Chebanov, Donetsk, Ukraine

AMS Class.: 34C30 (70E15)

A good many of exact solutions of the Euler-Poisson dynamical equations and their generalizations it is well known. However construction of solutions of motion equations of a system of coupled rigid bodies is difficult because of their awkwardness and a high order of this system of differential equations. In this domain we note a result derived by P.V. Kharlamov. He determined sufficient conditions of existence of a class of solutions of a problem about a motion of \( n \) Lagrange gyroscopes system. This class has the structure: \( \theta_i = \theta(t), \quad \psi_i = \psi(t) + p\pi (p = -1, 0, 1), \quad \varphi_i = \varphi_i(t) \quad (i = 1, n), \) where \( \theta_i, \psi_i, \varphi_i \) are the Euler angles defined the position of the body \( S_i \) with respect to a inertial space.

In this report the way of finding of necessary conditions of existence of the solutions is indicated. They are analyzed. Thanks to the obtained results some dynamical properties of the system of bodies are determined. This way is applied for construction of more general solutions of motion equations of the gyroscopes systems.
Abstracts – Communications, Posters

ANALYTIC SOLUTIONS OF FUNCTIONAL DIFFERENTIAL EQUATIONS LINEAR SYSTEMS

Valery Cherepennikov, Irkutsk, Russia

AMS Class.: 34K15

This paper studies the issues of existence of initial value problems analytic solutions for some functional differential equations (FDE) linear systems in neighbourhoods of ordinary and regular singular points with analytic structure of delay when there is no initial shift of functional argument. Here we have the functions \( \dot{x}(g^i(t)) \) and \( x(h^j(t)) \), where \( g^i(t) = \sum_{n=1}^{\infty} g^i_n t^n \) and \( h^j(t) = \sum_{n=1}^{\infty} h^j_n t^n \); \( 0 \leq g^i(t), h^j(t) < t \), \( t > 0 \). In this case the initial set consists of one point \( t = 0 \).

The conditions of existence of a unique analytic solution for FDE of delay type and also the conditions of existence of an analytic solutions sheaf and separation of the unique analytic solution from the sheaf for FDE of neutral type have been obtained.

NUMERICAL-ANALYTICAL METHOD OF SOLUTION PARTIAL NONLINEAR DIFFERENTIAL EQUATION

V. A. Chiricalov, Kyiv, Ukraine

AMS Class.: 65M60 (65M70, 65Y05)

In this report the parallel numerical-analytical method for nonlinear partial differential equation is considered. For reducing the nonlinear equation to set of linear equation the method of Newton-Cantorovich is using. It is based on peacewise polynomial approximation and collocation method. Solution of partial differential equation of parabolic type in space variable is approximated of peacewise polynomial function with \( N \) points as nodes. In the collocation procedure the residual is equated to zero at \( n \) points on each subinterval of spatial variable \( x \) and a set of coupled ordinary differential equations are obtained. Then all components of the solution of the system differential equations are approximated by interpolation polynomial on finite interval of time variable and collocation procedure is used. After that we have matrix equation for unknown ordinates of solution in collocation point of time and spatial variable. The matrix equation will be reduced to the linear algebraical system with the matrix of special type, that allowed us to construction the parallel numerical algorithm.
RELIABLE SOLUTION FOR TEMPERATURE DISTRIBUTION IN A HOMOGENEOUS AND ISOTROPIC MEDIUM MODELLED BY A NONLINEAR EQUATION

Jan Chleboun, Prague, Czech Republic

AMS Class.: 49J99, 34B15, 65L60

A boundary value problem for the equation \((a(u)u')' = f\) is taken as a state problem. Its coefficient, a function \(a\) dependent on the exact solution \(u\), belongs to a compact set of admissible functions. The coefficient \(a\) is evaluated by a cost functional defined through the local averages of the state solution \(u(a)\). The cost functional is to be maximized over the admissible set. In the words of engineering, the coefficient function determining the highest local temperature is searched for, i.e., the worst case solution an engineer can meet in a reliable model with uncertain data is investigated.

The maximization problem as well as its approximation has at least one solution. In many practical cases, however, the problem is solvable without any calculations as the sensitivity analysis suggests. Though the study is performed in 1D only, its conclusions are applicable to spatial problems too.

GLOBAL ATTRACTOR FOR PARABOLIC PROBLEMS IN BOUNDED OR UNBOUNDED DOMAINS

Jan W. Cholewa, Katowice, Poland

AMS Class.: 35B40, 34G20, 35K22

The existence of global attractors for the semigroups generated by parabolic problems was shown in our recent papers [1], [2] based on the ideas of [3]. In the case of bounded domains this existence result follows from a single \(a\)\_priori estimate of the solutions in the \(W^{k,p}\)-norm. Essential difficulties (like noncompactness of the Sobolev type embeddings and also of the trajectories of the semigroup) appear when the Cauchy problem in the whole of \(\mathbb{R}^n\) is considered. For the existence of a global attractor we then need to have one more \(a\)\_priori estimate of the solutions in the norm of a weighted space \(L^p(\mathbb{R}^n, (1 + |x|^2)^\nu)\).


ON SOME DIFFERENTIAL INCLUSIONS AND THEIR APPLICATIONS

Mieczysław Cichoń, Poznań, Poland

AMS Class.: 34A60, 49J24

Consider the following differential inclusions:

\[(1) \quad x'(t) \in A(t)x(t) + F(t,x(t)), \quad x(0) = x_0\]

in a general Banach space \(E\), where \(\{A(t) : t \in I\}\) is a family of linear operators and \(F\) is a multivalued perturbation.

Our basic idea is to investigate the problem (1) under some assumptions on \(F\) and \(A\) expressed in terms of the weak topology on \(E\). We obtain some existence theorems and we are able to characterize the solution set of (1).

Moreover, some applications of the results mentioned above in the theory of optimal control will be presented.

In this lecture we also discuss the possible generalizations of our results to some functional-differential inclusions of retarded-type.

ON THE DARBOUX PROBLEM FOR PARTIAL FUNCTIONAL DIFFERENTIAL EQUATIONS WITH INFINITE DELAY AT DERIVATIVES

Tomasz Człapiński, Gdańsk, Poland

AMS Class.: 35L15 (35R10)

We consider the Darboux problem for the hyperbolic partial functional differential equation with infinite delay at derivatives

\[
D_{xy}z(x,y) = f(x, y, z(x,y), (D_x z)(x,y), (D_y z)(x,y)), \quad (x, y) \in E := [0, a] \times [0, b],
\]

\[
z(x, y) = \phi(x, y), \quad (x, y) \in E^0 := (-\infty, a] \times (-\infty, b] \setminus (0, a] \times (0, b],
\]

where \(f : E \times B \times B \times B \to \mathbb{R}, \phi : E^0 \to \mathbb{R}\), and \(B\) is an abstract seminormed space satisfying some suitable axioms. Here \(\omega(x,y)\) denotes the Hale operator, which means that \(\omega(x,y) : \mathbb{R}^2_+ \to \mathbb{R}\) is a function defined by \(\omega(x,y)(s,t) = \omega(x+s,y+t)\) for \((s,t) \in \mathbb{R}^2_+\). The axioms that we use are adapted from those introduced by Hale and Kato for ordinary functional differential equations. Using the Schauder or the Banach fixed point theorems we prove, under suitable assumptions, theorems on the existence and on the existence and uniqueness of classical solutions of the above Darboux problem.
FUNCTIONAL EQUATIONS IN
FUNCTIONAL DIFFERENTIAL EQUATIONS

Jan Čermák, Brno, Czech Republic

AMS Class.: 34K25 (39B62)

We discuss the linear functional differential equation

\[ y'(x) = ay(\tau(x)) + by(x), \quad x \in I = [x_0, \infty). \]  \hspace{1cm} (1)

M. L. Heard has described the asymptotic behaviour of all solutions of retarded equation (1) with \( b < 0 \) and delay \( \tau \in C^2(I) \) fulfilling \( 0 < \tau' < 1 \) on \( I \), \( \tau' \) being decreasing on \( I \). His result essentially says that every solution of (1) approaches a solution of the functional nondifferential equation

\[ ay(\tau(x)) + by(x) = 0, \quad x \in I. \]  \hspace{1cm} (2)

We discuss this resemblance between asymptotic behaviour of solutions of (1) and (2). We show that it is preserved also for other equations (1) with unbounded \( r(x) = x - \tau(x) \) and \( b \neq 0 \). If \( \tau(x) < x \), it is possible to discuss the asymptotics of all solutions; if \( \tau(x) > x \), it is shown that, given a specific asymptotic behaviour, there is (a unique) solution of (1) which possess that asymptotic behaviour.

\[ L_{q+\xi}-\text{ESTIMATE } |D^m_x U(x,t)| \]  \hspace{1cm} GENERALIZED
\[ \text{SOLUTION OF NONLINEAR HIGHER ORDER} \]
\[ \text{PARABOLIC EQUATIONS} \]

Grigori Daniljuk, Makeyevka, Ukraine

AMS Class.: 32AXX

The increase of ability for addition of the main derivative from space variables of the bounded high order parabolic equation, having the divergence type, has been found. The key point for obtaining results of the work is Gering’s lemma and its local generalization in the case of parabola.

STABILITY INVESTIGATION OF BIOLOGICAL
SYSTEMS WITH QUADRATIC RIGHT PART

Vladimir Davydov, Kiev University, Ukraine

Many processes studied in biology, physics, chemistry, medicine are quite adequately described by systems of ordinary differential equation with quadratic nonlinearities in right parts.

Specific nature of quadratic systems lays in the fact that they not only describe real processes essentially more precise than systems of first approach, but also better reflect qualitative characteristics of processes.

Constructive sufficient conditions of asymptotic stability of equilibrium position are received for quadratic systems without delay, as well as for systems with
delay: uniformly on delay, and with delay, which value depends on the parameters of the studied systems. Delay is introduced in the models of prey–predator type as an average life length of individuals.

Besides establishment of the fact of stability such estimates of characteristics of dynamic systems, as the degree of monotony tending to equilibrium position and guaranteed region of asymptotic stability are calculated. All received dependences are presented in coefficient form and can be easily calculated for quadratic systems of quite large dimension.

ON NONLINEAR BOUNDARY VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS

Tadeusz Dłotko, Katowice, Poland

AMS Class.: 34K10, 34B15, 34K30

The theory of completely continuous vector fields in Banach spaces enable the demonstration of some existence theorems of solutions to the boundary value problem in vector form

\[
\begin{align*}
    y''(x) &= A(x)y(x) + f(x, y(\alpha(x)), y'(\beta(x))), \\
    g_i(y(.), y'(.) &= 0, \quad i = 1, 2, \quad x \in [0, 1].
\end{align*}
\]

\(\alpha, \beta\) denotes deviations of the argument \(x\), \(g_1, g_2\) are vector functionals define on the solutions of the problem (\(*\)). The results are based on Borsuk’s antipodal theorem and homotopy principle.

ON PROPERTY A AND B FOR THIRD ORDER DIFFERENTIAL EQUATIONS

Z. Došlá, Brno, Czech Republic

AMS Class.: 34C10 (34C15)

We discuss the relations between property A, property B and oscillatory behavior for the third order differential equation in the disconjugate form

\[
\left( \frac{1}{p(t)} \left( \frac{1}{r(t)} x' \right)' \right)' + q(t)f(x) = 0
\]

and in the normal form

\[x''' + a(t)x' + b(t)f(x) = 0 .\]

The functions \(p, r, q, a, b\) are assumed to be continuous on \([a, \infty)\), \(p > 0, r > 0, q\) has a fixed sign and \(f\) is continuous on \(\mathbb{R}, f(u)u \geq 0\).

The presented results have been achieved in joint research with M. Cecchi and M. Marini, University of Florence.
ON BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL-DIFFERENTIAL EQUATIONS

Alexander Domoshnitsky, Haifa, Israel

AMS Class.: 35K15

Existence and uniqueness for the boundary value problem

\[ x''(t) + (Tx)(t) = f(t), \quad t \in [0, \omega], \quad (1) \]
\[ \nu x - \alpha, \quad \mu x = \beta, \quad (2) \]

are investigated. Here \( T : C_{[0, \omega]} \rightarrow L_{[0, \omega]} \) is a linear bounded Volterra operator, \( \nu : C_{[0, \omega]} \rightarrow \mathbb{R}_1, \mu : C_{[\omega, \omega]} \rightarrow \mathbb{R}_1 \), are linear bounded functionals, where \( \omega_\nu < \omega, \omega_\mu < \omega \). A maximum principle for equation (1) is obtained. A scheme of its using in problems of existence and uniqueness for (1), (2) in the case: \( \nu \) is a positive functional and \( \omega_\nu < \omega_\mu \) is proposed.

INTEGRAL MEANS AND PÓLYA FACTORIZATIONS

Uri Elias, Haifa, Israel

AMS Class.: 26A06 (26D10, 34A30)

We present relations between integral means of functions and integral means of their derivatives. For example, if \( F(t) = \int_a^t w_0 f / \int_a^t w_0 \), then for the \( k \)-th order differential operator \( L_k f = \left( w_1 w_0 \frac{d}{dt} \right)^k f \) with \( w_1 = \int_a^t w_0 \), we have the integral mean \( L_k F(t) = \int_a^t w_0 L_k f / \int_a^t w_0, \quad k = 1, 2, \ldots \). In particular,

\[ F(t) = \frac{\int_0^t s^p f(s) ds}{\int_0^t s^p ds} \quad \text{implies} \quad t^k F^{(k)}(t) = \frac{\int_0^t s^{p+k} f^{(k)}(s) ds}{\int_0^t s^p ds}, \quad k = 1, 2, \ldots \]

The above integral identities may be translated into differential identities. For example, the first pair of equalities implies

\[ \left( \frac{1}{w_0} \frac{d}{dt} w_1 \right) \left( \frac{w_1}{w_0} \frac{d}{dt} \right)^k F = \left( \frac{w_1}{w_0} \frac{d}{dt} \right)^k \left( \frac{1}{w_0} \frac{d}{dt} w_1 \right) F, \]

which represents two different Pólya factorizations of the same linear disconjugate differential operator. We discuss the inverse question, namely, are any two Pólya factorizations of a differential operator a source of integral mean identities.
OSCILLATORY BEHAVIOUR OF SOLUTIONS
OF N TH ORDER DIFFERENTIAL EQUATIONS

Hassan El-Owaidy, Cairo, Egypt

AMS Class.: 34C10 (34C15)

Homogeneous and nonhomogeneous $n$-th order nonlinear differential equations

$$a(t) g(x(t))x^{(n)}(t) + b(t) f(x(q(t))) = e(t)$$

and

$$(a(t) g(x(q(t))))x^{(n-1)}(t) + q(t) f(t, x(q(t))) = 0 \quad n > 2$$

are investigated. New oscillatory results are obtained.

SOME EXAMPLES FOR THE EXTENDED
USE OF THE PARAMETRIC
REPRESENTATION METHOD

Henrik Farkas, Peter L. Simon, Budapest, Hungary

AMS Class.: 58F14 (34C23, 35B32)

The Parametric Representation Method had been applied successfully to construct bifurcation diagrams relating to equilibria of dynamical systems whenever the equilibria are determined from a single equation containing two control parameters linearly. The Discriminant–curve (that is the saddle–node bifurcation curve parametrized by the state variable remained after the elimination) is the base of this method, as it had been shown. The number and even the value of the stationary state variables can be derived from that.

Here we show some possible extensions of the method via three examples.

a.: Nonlinear parameter dependence

b.: Systems of equations for equilibria

c.: Reaction-diffusion equations, condition for multistationarity.

Similarly to the above simple case, the PRM provides us with information about the stationary solutions. Although some features do not remain valid for these extensions.
ASYMPTOTIC CONVERGENCE OF POLYNOMIAL COLLOCATION METHOD FOR PERIODIC PSEUDODIFFERENTIAL EQUATIONS

Alexandr Fedotov, Kazan, Russia

AMS Class.: 65R20

Consider equation $Au + Tu = f$, $A+T : H^{s+\alpha} \rightarrow H^s$, where $H^s$ is a periodic Sobolev space of order $s > 1/2$, $A$ is a pseudodifferential operator of order $\alpha \in \mathbb{R}$ and $T$ is compact operator. We'll seek trigonometric polynomial $u_n(t) \in T_n$, $n \in \mathbb{N}$ as an approximate solution from the collocation method with equidistant nodes. Via proved equivalence between collocation and non-standard Galerkin methods (polynomial analogous of D.Arnold and W.Wendland spline results) and the boundness of Lagrange interpolation operator in $H^s$, $s > 1/2$, it was shown, that collocation method converges for all uniquely solvable elliptic equations with the rate

$$\|u_n - u\|_{s+\alpha} \leq C \inf_{v_n \in T_n} \|v_n - u\|_{s+\alpha}.$$ 

Moreover, the rate of convergence is growing up as a consequence of growing smoothness of $u(t)$ infinitely.

ASYMPTOTIC EXPANSION FOR “PERIODIC” BOUNDARY CONDITION

Ján Filo, Bratislava, Slovakia

AMS Class.: 35K05 (35B27)

In the recent paper [FL] we have determined the first two terms in asymptotic expansion (with respect to a small parameter $\varepsilon$) of solutions of the heat equation with periodic Dirichlet-Neuman condition with period $\varepsilon$ in a smooth domain given by a closed simple curve in $\mathbb{R}^2$.

Some new results to this problem in more space dimensions will be presented.


BASINS OF ATTRACTION AND THEIR STRUCTURE

Jiří Fišer, Olomouc, Czech Republic

AMS Class.: 58F12

Application of the critical surfaces method (due to C. Mira) for detecting the attractors and their basins of attraction will be presented for discrete dynamical systems. The results will be compared with those obtained numerically in the form of pictures.
QUADRATIC INTEGRAL INEQUALITIES
OF THE FIRST ORDER

Bronisław Florkiewicz, Małgorzata Kuchta, Wroclaw, Poland

AMS Class.: 26D10

Some quadratic integral inequalities of the first order of the form
\[ \int_I (r h^2 + 2 s h \dot{h} + u h^2) \, dt \geq 0, \quad h \in H, \]
where \( I = (\alpha, \beta), -\infty \leq \alpha < \beta \leq \infty, \) \( r, s \) and \( u \) are given real functions of the variable \( t, \) \( H \) is a given class of functions \( h \) absolutely continuous and \( \dot{h} \equiv dh/dt \) are derived and examined.

The method used to obtain these integral inequalities is an extension of the uniform method of obtaining various types of integral inequalities involving a function and its derivative. The method makes it possible given a function \( r \) and an auxiliary function \( \phi \) to determine functions \( s \) and \( u \) and next using these functions to determine the class \( H \) of the functions \( h \) for which this integral inequality holds.

In this case \( s \) and \( u \) are solutions of a certain differential inequality which makes it possible to obtain a large set of functions \( s \) and \( u \) for which integral inequality of this form holds.

The positive definiteness of quadratic functionals of this form is a basic problem of the theory of singular quadratic functionals introduced by Morse and Leighton. This problem is of significant importance for the oscillation theory for second order linear differential equations on a non-compact interval.

QUADRATIC INTEGRAL INEQUALITIES
OF THE SECOND ORDER

Bronisław Florkiewicz, Wroclaw, Poland
Katarzyna Wojteczek, Opole, Poland

AMS Class.: 26D10

Some integral inequalities of the second order of the form
\[ \int_I s h^2 \, dt \leq \int_I r \dot{h}^2 \, dt, \]
where \( I = (\alpha, \beta), -\infty \leq \alpha < \beta \leq \infty, \) \( r \) and \( s \) are real functions of the variable \( t, \) \( H \) is a given class of functions \( h \) absolutely continuous on \( I \) and \( \dot{h} = d^2h/dt^2 \) are considered. Some new problems connected with these inequalities are submitted including a new way of deriving them, as well as deriving some new integral inequalities of this form with Chebyshev weight functions.

The method introduced to obtain these inequalities is the uniform method of obtaining various types of integral inequalities involving a function and its derivative extended to the inequalities of the second order. The method makes it possible for a given function \( r \) and auxiliary function \( \phi \) to determine directly the function \( s \) as a solution of a certain differential equation, and next use these functions to determine the class \( H \) of functions \( h \) for which the inequality holds.

These inequalities are strongly connected with the theory of singular quadratic functionals introduced by Morse and Leighton and are important in the oscillation theory for the fourth order linear differential equations on a non-compact interval.
HOMOGENIZATION OF SCALAR HYSTERESIS OPERATORS

Jan Franců, Brno, Czech Republic

AMS Class.: 35B27, 73B27

The contribution deals with homogenization problem for parabolic and hyperbolic equations with scalar hysteresis operator. For $\varepsilon \to 0$ we study a limit behaviour of the sequence of equations with Ishlinskii hysteresis operator

$$F^\varepsilon(\sigma) = \int_0^\infty \eta_h\left(\frac{x}{\varepsilon}\right) f_h(\sigma) dh,$$

where $f_h$ is the stop operator and the material “stiffness” function $\eta_h(y)$ is periodic in $y$. Form of the limit homogenized operator $F^0$ is derived.

THE FIRST LYAPUNOV METHOD FOR STRONGLY NONLINEAR SYSTEMS OF DIFFERENTIAL EQUATIONS

Stanislav D. Furta, Moscow, Russia

The report is devoted to the problem of construction of some classes of solutions to ODE. For that purpose a procedure of the construction of solutions in a form of series is developed. The above series are analogous to those used in the Lyapunov first method. The so-called asymptotic solutions going to equilibrium positions as the time infinitely increases or decreases, play a special role in the report. The author developed a theory for such solutions for the so-called "strongly non–linear case" when the existence of whole–parametric families of those solutions cannot be deduced only from the linear theory. The author considers a lot of concrete examples where the existence of various classes of solutions drives at some peculiarities in the dynamical behaviour of the system under consideration.

GLOBAL QUALITATIVE INVESTIGATION AND LIMIT CYCLE BIFURCATIONS

Valery Gaiko, Minsk, Belarus

AMS Class.: 34C05 (34C23)

Two-dimensional polynomial dynamical systems are considered. By means of Erugin’s two isoclines method we carry out the global qualitative investigation of such systems, construct canonical systems with field-rotation parameters and study limit cycle bifurcations.
In particular, we consider a quadratic system with three field-rotation parameters and show that it is a generic one for the study of limit cycle bifurcations. We use this canonical system for the investigation of multiple limit cycles.

As is known, bifurcation surfaces of multiplicity-two and three limit cycles are the familiar saddle-node and cusp bifurcation surfaces, respectively. We apply Perko’s termination principle that the maximal one-parameter family of multiple limit cycles terminates either at a critical point, which is typically a fine focus of the same multiplicity, or on a separatrix cycle, which is also typically of the same multiplicity, to prove the non-existence of swallow-tail bifurcation surface of multiplicity-four limit cycles for quadratic systems (by contradiction).

HYDROELASTICITY OF THIN CYLINDRICAL SHELLS WITH ELASTIC INCLUSIONS

Elena Gavrilova, Sofia, Bulgaria

AMS Class.: 73K70

A circular cylindrical rigid shell is fulfilled with compressible, irrotational and inviscid fluid. An elastic element (a flexible membrane or an elastic plate) is a part of the cover of the cylindrical shell as the centres of the elastic element and the cover can be coincided or not. Free coupled vibrations of the received hydroelastic system are investigated and the frequency equation is obtained using the Galerkin method. To illustrate the analytical results, some numerical examples are given and discussed.

DEVELOPMENT OF THE INTEGRAL RELATION METHOD IN CONFORMITY TO SOME PROBLEMS ON STABILITY

Dimitri Georgievskii, Moscow, Russia

AMS Class.: 76E05 (35P05)

The well-known integral relations method (IRM) or Joseph’s method is applied for a long time in hydrodynamical problems on stability of flows. It makes possible to obtain sufficient integral estimates of stability in the certain real- or complex-valued function spaces. But a class of flows that appropriate for IRM procedure is pretty limited: the steady 1D shear flows of either ideal liquid (the Rayleigh problem) or viscous incompressible fluid (the Orr—Sommerfeld problem) or stratified ideal liquid (the Taylor—Goldstein problem) or stratified viscous fluid (the Drazin problem).

It is suggested a generalization of classical IRM procedure. This extension means the following: a) not only steady but unsteady processes are considered too; b) not only 1D shear but flows with arbitrary incompressible kinematics in
plane domain with simply connected boundary (may be, infinite) are appropriate; c) not only ideal or viscous fluid but non-Newtonian, viscoplastic as well as flows with complex rheology may be analyzed by this new method.

**DIFFERENTIAL EQUATIONS WITH PROPERTY A, B**

Milan Gera, Valter Šeda, Bratislava, Slovakia

AMS Class.: 34C10 (34C15)

In the talk the properties A, B of differential equations

\[ x^{(n)} + \sum_{k=1}^{n} p_k(t)x^{(n-k)} = 0, \quad (L) \]

\[ x^{(n)} + \sum_{k=1}^{n} P_k(t,x,x',\ldots,x^{(n-1)})x^{(n-k)} = Q(t,x,x',\ldots,x^{(n-1)}), \quad (N) \]

where \( p_k \in C([a, \infty)), P_k, Q \in C([a, \infty) \times \mathbb{R}^n), k = 1, 2, \ldots, n, \) will be investigated.

Sufficient conditions for the property A and B will be given in the case when the functions \( p_k, P_k \) are of constant sign. According to the behaviour of nonoscillatory solutions of (L) or of (N) the property A, the strict and the strong property A will be distinguished. Similarly the property B can be dealt with.

**NONLINEAR ORDINARY DIFFERENTIAL EQUATION WITH THE NONLINEARITY IN THE DERIVATIVE - THE EXISTENCE OF PERIODIC SOLUTIONS**

Petr Girg, Plzeň, Czech Republic

AMS Class.: 34B15 (34C15, 34C25, 34C99)

In the communication we want to answer some questions about the existence of the periodic solutions of the equation of the type

\[ m\ddot{x}(t) + g_1(\dot{x}(t)) + g_0(x(t)) = f(t). \quad (1) \]

We suppose overall that \( m > 0, g_0 \) is a real continuous function, \( g_1 \) is a continuously differentiable real function, \( f \) is a continuous periodic real function. Then we consider additional assumptions on functions \( g_0, g_1 \) and relations between specific cases are discussed. We formulate very sharp necessary conditions of the existence of the periodic solution of the equation (1) with \( g_0 \equiv 0 \). Then we will discuss some qualitative properties of the set of the right-hand side for which the equation (1) has periodic solution. Consequently we will show some connections between the cases, when \( g_0 \equiv 0 \) and when the function \( g_0 \) is \( 2\pi \)-periodic Lipschitz continuous and \( \int_0^{2\pi} g_0(s)ds = 0 \).
SIMULATION OF ECONOMIC EVOLUTION  
OF TWO AREAS IN TENERIFE ISLAND  
WHICH COMPITE FOR THE  
EUROPEAN TOURISTIC DEMAND  

José Manuel González, La Laguna, Canary Islands, Spain

AMS Class.: 34 – 90

The simulation with logistic curves allows us to modelize the economic evolution of two Areas of Localization of Touristic Offer in Tenerife island, in Canary Islands, Spain. These two regions compete to receive the European demand of visitors such that the economic growth of each one of them depends on the increase of the tourist places in the other. Then, by simulating the evolution of the populations, urban soils and touristic demands as variables which compete by the same resources, we have found a differential system of logistic equations which reproduces the real scenario of economic growth of these areas. This differential system is solved with techniques of System Dynamics, being the mean operational tool in this resolution the identification of the parameters which appear in the equations.

ON SPECTRAL FUNCTIONS OF A SECOND-ORDER  
OPERATOR DIFFERENTIAL EQUATION  

Valentina Gorbachuk, Kyiv, Ukraine

AMS Class.: 34G10

We consider a differential equation of the form

\[ y''(t) + (A + q(t))y(t) = \lambda y(t), \quad t \in [0, b], \quad b < \infty, \]  

where \( A \) is a lower semibounded selfadjoint operator in a Hilbert space \( \mathcal{H} \), \( q(t) \) is a continuous function whose values are continuous operators on \( \mathcal{H} \). It is established that the operator generated in \( L_2([0, b], \mathcal{H}) \) by the expression on the left hand side in (1) is entire operator with infinite defect numbers. Using this fact, we give a constructive description of all spectral functions of the equation (1).
ON ASYMPTOTICS OF SOLUTIONS
OF A SECOND–ORDER DIFFERENTIAL EQUATION
IN A BANACH SPACE

Volodymyr Gorbachuk, Kyiv, Ukraine

AMS Class.: 47D06, 34G10

We consider a differential equation of the form

\[ y''(t) = B^2 y(t) + f(t), \quad t \in [0, \infty), \]

where \( B \) is the generator of a bounded \( C_0 \) group of linear operators on a Banach space \( \mathcal{B} \). It is established that if \( f \in C^1([0, \infty), \mathcal{B}) \) and the function \( f(t) \) has a Cesaro asymptotics \( t^\alpha f_0 \) \((\alpha \geq 0, f_0 \in \mathcal{B})\) when \( t \to \infty \), then any solution \( y(t) \) of the equation (1) has that of the form

\[ Py(0) + tPy'(0) + \frac{t^{\alpha+2} Pf_0}{(\alpha+1)(\alpha+2)}, \]

where \( P \) is the projector from \( \mathcal{B} \) onto \( \ker B \) in the decomposition \( \mathcal{B} = \mathcal{R}(B) + \ker B \) \((\mathcal{R}(B) \) is the range of \( B \)).

ON THE ASYMPTOTIC BEHAVIOR OF
NONLINEAR OSCILLATORY SYSTEMS
UNDER IMPULSE PERTURBATIONS

János Karsai, Szeged, Hungary    John R. Graef, Mississippi, U.S.A.

Systems with impulse effects are important models of several physical and biological phenomena. Some impulse effects are analogous to the continuously distributed ones, but because of the instantaneous nature, impulses can result in new unexpected properties. In this talk, we present some results on the asymptotic and oscillatory behavior of nonlinear second order systems of form

\[ \ddot{x} + f(x) = 0, \quad (t \neq t_n); \quad \dot{x}(t_n + 0) = b_n h(\dot{x}(t_n)), \quad (t_n \nearrow \infty, \text{ as } n \to \infty) \]

For example, if \( 0 \leq b_n \leq 1 \) and \( h(y) = y \), then the behavior of the solutions of this system is analogous to that of the nonlinear damped oscillator

\[ \ddot{x} + g(t) \dot{x} + f(x) = 0 \quad (g(t) \geq 0), \]

while if \( h(y) = |y^\beta| \text{sign } y \) \((0 < \beta < 1)\), then the system shows analogies to

\[ \ddot{x} + G(\dot{x}) + f(x) = 0 \]

with an unstable origin. Finally, in all the above cases, the beating impulses \((b_n < 0)\) describe some new and physically important phenomena.
NONLINEAR THIRD ORDER DIFFERENTIAL EQUATIONS OF THE CLASS B

Michal Greguš, Bratislava, Slovakia

AMS Class.: 34C15

In the last two years, I myself and my colleagues John R. Graef and Milan Gera, we have investigated the oscillatory and asymptotic properties of solutions of the nonlinear third order differential equation of the form

\[ y''' + p(t, y, y', y'')y'' + q(t, y, y', y'')y' + r(t, y, y', y'')y = f(t, y, y', y'') \]  \( (N) \)

where \( p, q, r \) and \( f \) are continuous functions on \( I \times \mathbb{R}^3, I = (a, \infty), -\infty < a < \infty, \mathbb{R} = (-\infty, \infty), q, r \) are nonnegative functions, and \( f \) has the sign property.

In this short communication I introduce some results in the case \( q, r \) are nonpositive functions, that is, sufficient conditions for the solutions \( y \) of \( (N) \) to be either oscillatory or \( y(t) = (1) y^{(i)}(t) > 0, i = 1, 2, \) for sufficiently large \( t \), i.e. equation \( (N) \) has the property B.

MODELS FOR LIQUID-GAS PHASE TRANSITIONS

M. Grinfeld, University of Strathclyde, UK

In this talk we present three approaches to modelling liquid-gas phase transitions: van der Waals fluids, a coagulation fragmentation discrete velocity model, and a “minimal” model derived from a cellular automaton. Ability of these models to reproduce benchmark experiments of Dettleff and Thompson is discussed.

SOME RESEARCHES OF THE THEORY OF AN AUTHENTICITY AND SECURITY OF COMPUTER SYSTEMS

Andriy Gundar

AMS Class.: 35, 33

About the mathematical bases to the decision of the problem of choice of the optimum concept of information protection in computer systems.

Until now, despite large amount of an accumulated practical material, mathematically reasonable, completed and generally accepted theory of an authenticity and security of computer systems has not created. As a result the existing tools, approaches and techniques have number, sometimes not visible, disadvantages, lowering expected efficiency of protection from unauthorized access. The offered
solution of the problem consists in development of the common theory of protection of any object from unauthorized access in the terms of $t_1$ – life time of protection object (PO); $P_x$ – probability of protection system (PS) detour; $t_x$ – time, necessary for overcoming of the barrier; $T_c$ – periodicity of the “perimeter” control. Then determine “information” as PO.

Using two major properties of PO – ability to attract of the intruder and PO life time, we have appropriate integral equations on “intruder–PS–PO”–structure.

Offered principles, allow precisely and clearly estimate a reliability of the PS in computer systems and networks, and as a result – to construct the optimal strategy of protection of the information from an unauthorized access.

GLOBAL ATTRACTIVITY AND OSCILLATION OF DELAY EQUATION IN POPULATION MODELS

István Győri, Veszprém, Hungary

In this lecture we study the global attractivity properties of equilibriums in some scalar delay differential equations arising in population models, medicine, ecology and environmental sciences. New sufficient conditions for the global stability of the unique positive steady state are obtained. We also give sufficient conditions for the persistence and rapidly oscillatory properties of the solutions.

QUADRATIC RELATIONS FOR CONFLUENT HYPERGEOMETRIC FUNCTIONS

Yoshishige Haraoka, Dep. Math., Kumamoto Univ., Japan

AMS Class.: 33C15, 33C65, 33C70

Stimulated by the reformulation of the theory of hypergeometric functions by Aomoto and Gel’fand, Yoshida et al established the intersection theories of twisted homologies and of twisted cohomologies, which lead to the discovery of quadratic relations for hypergeometric functions.

Gel’fand’s reformulation also yielded the definition of the generalized confluent hypergeometric functions (CHGF, for short). We have studied the structures of CHGF’s as a whole; among them the confluence is one of the most useful structures. We apply the confluence to the quadratic relations for hypergeometric functions, and then obtain quadratic relations for CHGF’s including classical CHGF (Kummer, Bessel, Whittaker, Hermite-Weber, Airy) and Horn’s CHGF in 2 variables.

Example

$$aF(c-a-1,2-c;x)F(c-a-1,c;-x) + (c-a-1)F(a,c;x)F(-a,2-c;-x) = c-1,$$

where $F(a,c;x) = \sum_{n=0}^{\infty}[(a)_n/(c)_n n!]x^n$, Kummer’s CHGF.
ON THE RELATIONSHIP BETWEEN THE MULTIPOINT BOUNDARY VALUE PROBLEMS FOR N-TH ORDER LINEAR DIFFERENTIAL EQUATIONS OF NEUTRAL TYPE

Alexander Haščák, Košice, Slovakia

AMS Class.: 34K10

We shall give some relationship between the boundary value problems for differential equation:

\[ x^{(n)}(t) + a(t)x^{(n)}(t - \Delta_0(t)) + \sum_{i=1}^{n} \sum_{j=1}^{m} b_{ij}(t) x^{(n-i)}(t - \Delta_{ij}(t)) = 0, \quad n \geq 1 \] (1)

with continuous coefficients \( a(t), b_{ij}(t), (i = 1, 2, \ldots, n); j = 1, 2, \ldots, m \) and delays \( \Delta_0(t) > 0, \Delta_{ij}(t) \geq 0 \) on the interval \((a, b)\).

ON STABILITY PROPERTIES OF SOLUTIONS OF SECOND ORDER DIFFERENTIAL EQUATIONS

László Hatvani, Szeged, Hungary

AMS Class.: 34D20

We consider the equation

\[ x'' + h(t, x, x')x' + f(x) = 0, \]

where \( xf(x) > 0 \) for \( x \neq 0 \), and there are \( a, b : [0, \infty) \rightarrow [0, \infty) \) and \( \Delta > 0 \) such that \( x^2 + y^2 \leq \Delta \) implies \( a(t) \leq h(t, x, y) \leq b(t) \) (\( t \geq 0 \)). This equation describes the oscillation of a material point around the equilibrium position \( x = 0 \) damped by viscous friction. We give conditions for functions \( a, b \) guaranteeing asymptotic stability of the equilibrium state \( x = x' = 0 \). The cases of small damping \( (0 \leq a(t) \leq b_0 < \infty) \), the large damping \( (0 < a_0 < a(t) \leq b(t) < \infty) \) and the general case are treated separately, because they need different methods. The results on the small damping will be applied to get new Armellini–Tonelli–Sansone–type conditions guaranteeing that all solutions of the equation \( x'' + k(t)f(x) = 0, \) \((k(t) \uparrow \infty, t \rightarrow \infty)\) tend to zero as \( t \rightarrow \infty \).
ON THE SOLUTIONS OF A BVP FOR A CLASS OF NONLINEAR ELLIPTIC EQUATIONS  

Jenő Hegedűs, Szeged, Hungary  

AMS Class.: 35B05  

Nonlinear second order PDE-s of the form  

$$\triangle u(x) + f(\rho, u, |\nabla u|) = 0 \quad x \in B \equiv B^0_1 := \{x | |x| \equiv \rho < 1\}$$ (1)  

will be considered in the class of the functions $u(x)$:  

$$u \in C^2(B) \cap C(\overline{B}), u|_B > 0, \exists v \in C^2[0, 1] \cap C[0, 1] : u(x) = v(\rho) \quad x \in \overline{B}$$ (2)  

with boundary condition  

$$u|_{\Gamma \equiv \partial B} = a \geq 0 \quad a = \text{const.}$$ (3)  

The existence and uniqueness of the solution to the problem (1)-(3) will be proven. Some qualitative characteristics of the solutions will be given, too. The method of the paper: GABRIELLA BOGNÁR, On the radially symmetric solutions to a nonlinear PDE, Publ. Univ. of Miskolc, Series D. Natural Sciences. Vol. 36. No. 2. Mathematics (1996) - is used and modified.

NUMERICAL SOLUTION OF DIFFERENTIAL EQUATIONS WITH HYSTERESIS  

Miroslav Hlavička, Brno, Czech Republic  

AMS Class.: 65M99 (73E50)  

Hysteresis effects are represented by means of continuous causal (Volterra) and rate independent operator. The contribution deals with Ishlinskii operator defining constitutive stress–strain relation in continuum mechanics. Implicit time discretization scheme is used for solving an ordinary differential equation with this hysteresis operator. Generalization of this scheme is used for solving a system of such equations which arises by space discretization of space one–dimensional hyperbolic equation describing longitudinal vibrations of a rod.
ON EXISTENCE OF BROKEN LIMIT CYCLES
FOR A SYSTEM OF IMPULSIVE
DIFFERENTIAL EQUATIONS

Tamara Horbachuk, Kiev, Ukraine    Mykola Perestyuk, Kiev, Ukraine

AMS Class.: 34A37 (34A45)

We consider a weakly nonlinear autonomy impulsive system of differential equations of the form

\[
\begin{align*}
\frac{dx}{dt} &= \omega y + \varepsilon f(x, y) y, \\
\frac{dy}{dt} &= -\omega x + \varepsilon g(x, y), y \neq 0 \\
\Delta y |_{y=0} &= \varepsilon I(x).
\end{align*}
\]

For this system, we find the conditions, under which any positive root of the equation

\[
\int_{0}^{2\pi} [f(a \cos \varphi, a \sin \varphi) a \cos \varphi + g(a \cos \varphi, a \sin \varphi)] \sin \varphi \, d\varphi = 0
\]

(it doesn’t depend on \(I(x)\)), generates a broken limit cycle. The functions \(f(x, y)\), \(g(x, y)\), \(I(x)\) are assumed to be continuous and satisfying the Lipschitz condition in a certain domain. The improved first approximations are constructed.

ON THE MAXIMAL DEGREE OF THE STABLE
MULTISTEP MULTIDERIVATIVE METHODS

Vagif Ibrahimov, Baku, Azerbaijan

AMS Class.: 65L

By G. Dahlquist result \(p \leq 2\lfloor \frac{k}{2} \rfloor + 2\) we receive that the degree (exactness) of stable \(k\)-step method with the constant coefficients is bounded. This classical result was the base for the construction the stable multistep methods with higher degrees. G. Dahlquist himself for the construction the stable \(k\)-step method with the degree \(p = 2k + 2\) suggested to use the second derivative of the solution of the problem: \(y' = f(x, y), y(x_0) = y_0\). Many authors by generalization this way constructed multistep multiderivative method. Using higher derivatives of the solution of the above mentioned problem for construction numerical method with the higher degree was suggested by L. Euler. But author’s, which have researched this problem, didn’t completely investigated multistep multiderivative methods. Therefore we considered determination of the maximal degree of the stable \(k\)-step multiderivative method.
Let us consider the next method 

\[ y(x+h) = y(x) + h(f(x, y(x)) + f(x+h, y(x+h))) + h \cdot h(g(x, y(x)) - g(x+h, y(x+h))) + z(x+h, y(x+h)) \] 

\[ /120, \] here \( \frac{dy}{dx} = g(x, y) \) and \( \frac{dz}{dx} = z(x, y) \).

This method has the maximal degree \( p = 6 \) and stable \( k = 1 \). Note that if we don’t use the third derivative of the function \( y(x) \), then maximal degree for the stable \( k \)-step second derivative method will be equal to 4 \( (p_{\text{max}} = 4) \) in the case \( k = 1 \). By this way we receive effectiveness in constructed \( k \)-step multiderivative method and prove that if \( k \)-step multiderivative method stable and it has the degree \( p \) (with the exception forward–jumping methods), then \( p \leq r(k+1) + 1 \) for even \( k \) and odd \( r \); \( r(k+1) \) other cases, here \( r \) is the maximal order of the multiderivative, used in the method.

**ON THE STABILITY OF LINEAR OSCILLATOR**

Alexander O. Ignatyev, Donetsk, Ukraine

AMS Class.: 34D20

Consider the linear oscillator which may be described by means of differential equation

\[ \frac{d^2 x}{dt^2} + f(t) \frac{dx}{dt} + g(t)x = 0 \]

where \( f(t) \) and \( g(t) \) are certain bounded continious functions. The sufficient conditions, under which the solution \( x = 0, \dot{x} = 0 \) is asymptotically stable, are stated. It is shown that these conditions are close to the necessary and sufficient ones.

**POLYHEDRAL HARMONICS**

Katsunori Iwasaki, Fukuoka, Japan

AMS Class.: 35N05 (05E05, 31B05, 52B11)

A well-known theorem of Gauss (1840) states that the harmonic functions are precisely those continuous functions which satisfy the mean value property with respect to a ball. What happens if the ball is replaced by a polytope? This very simple question is the theme of the present talk.

Let \( P \) be any polytope in \( \mathbb{R}^n \), and \( P(k) \) be its \( k \)-skeleton for \( k = 0, \ldots, n \). Let \( \mathcal{H}_{P(k)} \) be the set of all continuous functions satisfying the mean value property with respect to \( P(k) \). Then our problem is to characterize the linear space \( \mathcal{H}_{P(k)} \).

Concerning this problem, Friedman and Littman (1962) posed the following question: Is \( \mathcal{H}_{P(k)} \) finite-dimensional? (Recall that the classical harmonic functions form an infinite-dimensional linear space.) This rather surprising question had been open until recently when the speaker settled it affirmatively.

**Main Theorem.** For any polytope \( P \) in \( \mathbb{R}^n \) and any \( k = 0, \ldots, n \), the function space \( \mathcal{H}_{P(k)} \) is a finite-dimensional linear space of polynomials. Moreover, if the symmetry group of \( P \) is irreducible, then \( \mathcal{H}_{P(k)} \) is a finite-dimensional linear space of harmonic polynomials.
NONUNIQUENESS THEOREMS FOR ORDINARY DIFFERENTIAL EQUATIONS

Josef Kalas, Brno, Czech Republic

AMS Class.: 34A12 (34A34)

In contradistinction to the uniqueness, only few nonuniqueness criteria for ordinary differential equations are known till now. We shall present several general results for the nonuniqueness of the solutions of an initial value problem

\[ x' = f(t, x), \quad x(t_0) = x_0, \]

where \( x, f \) are \( n \)-dimensional vectors. Our results contain the most of known results as a special case. The Lyapunov function method together with the modified method of I. T. Kiguradze are the main tool for our considerations. The absolute values of Lyapunov functions \( V(t, x) \) appearing in our results need not be positive definite in \( x \) at an initial point \((t_0, x_0)\), but only in some components of \( x \). This allows us to suppose that \( V \) is dependent only on several components of \( x \) and thus we need the estimations only of several components of \( f \) for the nonuniqueness. Hence our results are applicable to \( n \)-th order differential equations.

INVERSE PROBLEMS OF DETERMINING NONLINEAR TERMS

Yutaka Kamimura, Tokyo, Japan

AMS Class.: 34A55, 34B15, 45G05, 45E10

We deal with some inverse problems of determining unknown nonlinear terms appearing in boundary value problems from their spectral information.

First we consider the problem: \textit{given functions} \( a(\lambda), b(\lambda) \) \textit{on the interval} \([0, \Lambda]\), \textit{determine a nonlinear term} \( g \), \textit{with which the (overdetermined) boundary value problem}

\[
\begin{cases}
    u'' = \lambda g(u), & 0 < x < 1, \quad ' = \frac{d}{dx} \\
    u(0) = 1, u'(0) = a(\lambda), u(1) = b(\lambda),
\end{cases}
\]

\textit{admits a solution} \( u \) \textit{for each} \( \lambda \in [0, \Lambda] \). Existence, uniqueness theorems of the nonlinear term \( g \) will be given.

Second an existence theorem of the nonlinear term of a nonlinear Sturm-Liouville problem realizing a given first bifurcating branch will be given.
STABILITY OF PERMANENT ROTATIONS
FOR THE GENERALIZED PROBLEM OF THE MOTION
OF A HEAVY RIGID BODY ABOUT A FIXED POINT

Natalija N. Kashta, Donetsk, Ukraine

AMS Class.: 34D20 (70E15)

Under analysis of realizable mechanical model reflecting characteristics of a heavy rotating elastic drum A.Ya.Savchenko obtained the equations, which generalize the known Euler-Poisson equations for the motion of a heavy rigid body about a fixed point. The main singularity of the obtained equations is the fact, that in these equations generalized inertia tensor depends upon components of vertical vector, i.e. it is not constant. These equations accept three general integrals: energy, area and geometric. And under certain conditions for mechanical parameters they accept new fourth general integral, generalized integral of Euler in classical case.

In the present paper stability of permanent rotations in this problem are considered. Using the method of construction of the Lyapunov function from integrals of the equations for the perturbations the sufficient conditions of stability are established. The necessary conditions are obtained in ordinary way starting from linearized system in neighbourhood of the solution.

BOUNDED IN THE PLANE SOLUTIONS OF LINEAR
HYPERBOLIC SYSTEMS

Tariel Kiguradze, Tbilisi, Georgia

AMS Class.: 35L55

The linear hyperbolic system

\[ \frac{\partial^2 u}{\partial x \partial y} = P_0(x, y)u + P_1(x, y)\frac{\partial u}{\partial x} + P_2(x, y)\frac{\partial u}{\partial y} + q(x, y) \]

is considered, where \( P_k = \mathbb{R}^2 \rightarrow \mathbb{R}^{n \times n} \) \( (k = 0, 1, 2) \) and \( q : \mathbb{R}^2 \rightarrow \mathbb{R}^n \) are measurable and essentially bounded matrix and vector functions.

Optimal in a sense sufficient conditions of existence and uniqueness of an absolutely continuous solution bounded in \( \mathbb{R}^2 \) together with its partial derivatives of the first order are established.
AN ASYMPTOTIC ANALYSIS OF DIFFERENTIAL EQUATIONS

Julka Knežević–Miljanović, Beograd, Yugoslavia

The purpose of this article is to determinate the asymptotic form of the nonoscillatory solutions and the existence of positive, monotonic, unbounded solutions of the equations

\[(r(x)y^{(n)})^{(n)} = \pm y f(x, y)\]

\[(r(t)y')' = y(t) F(t, y(t)).\]

We obtain necessary and sufficient conditions for the existence of different classes of these solutions.

DYNAMIC NEAR HOMOCLINIC ORBITS

Jürgen Knobloch, Ilmenau, Germany

AMS Class.: 34C, 58F

We consider periodic pertubations \(\dot{x} = f(x, \lambda) + \epsilon g(t, x, \lambda, \epsilon)\) of a family of autonomous vector fields, where \(\dot{x} = f(x, 0)\) has a degenerate homoclinic orbit asymptotic to a hyperbolic fixed point.

We determine all homoclinic points and characterize them by the dimension of the intersection of the tangent spaces of stable and unstable manifolds of the associated Poincaré map.

Furthermore we are interested in subharmonic solutions nearby the primary homoclinic orbit. For this end we develop a discrete version of Lin’s method.

ALMOST SHARP EXISTENCE CONDITIONS FOR SMOOTH INERTIAL MANIFOLDS

Norbert Koksch, Dresden, Germany

AMS Class.: 34C30 (35K22, 34G20, 47H20)

Let \(\mathbb{H}\) be a separable, real Hilbert space. We consider the nonlinear evolutionary equation \(\dot{u} + Au = f(u)\) for \(u \in \mathbb{H}\) assuming that \(A\) is a linear, self-adjoint, positive operator on \(\mathbb{H}\) with compact resolvent. The nonlinearity \(f\) is assumed to be \(C^k\) from \(D(A^\alpha)\) to \(D(A^\beta)\) with \(0 \leq \alpha - \beta \leq \frac{1}{2}\). Let \(P_N\) be the orthogonal projection of \(\mathbb{H}\) onto the subspace generated by the eigenvectors corresponding to the first \(N\) eigenvalues \(\lambda_i\) of \(A\), and let \(Q_N = \text{id} - P_N\).

We state an existence theorem for an inertial manifold \(\text{graph}(\phi) \cap \{u \in D(A^\alpha) : |A^\alpha P_N u| < r, |A^\alpha Q_N u| < r\}\) with \(\phi \in C^k(P_N D(A^\alpha), Q_N D(A^\alpha))\) using an almost
sharp spectral gap condition

\[ \lambda_{N+1} - k\lambda_N > \sqrt{2}K_1 \left( \lambda_{N+1}^{\alpha-\beta} + k\lambda_N^{\alpha-\beta} \right) \]

where \( K_1 \) is a Lipschitz constant of \( f \) on \( B(\sqrt{2}r), r > r_0 \), and \( B(r) \) contains an absorbing set. The proof of the theorem bases on comparison theorems for special two-point boundary value problems and for inequalities in ordered Banach spaces.

**ON OSCILLATION OF SOLUTIONS OF FUNCTIONAL DIFFERENTIAL EQUATIONS**

Roman Koplatadze, Tbilisi, Georgia

AMS Class.: 34K15

The equation

\[ u^{(n)}(t) = F(u)(t), \quad (1) \]

is considered where \( F : C(R_+; R) \rightarrow L_{loc}(R_+; R) \) is continuous. Sufficient conditions are established for the equation (1) to have Properties A and B.

**THE PROPERTY (A) FOR A CERTAIN CLASS OF THE THIRD ORDER ODE**

Monika Kováčová, Bratislava, Slovakia

AMS Class.: 34C10 (34C15)

We study oscillatory and non-oscillatory solutions of the third order ODE

\[ [g(t)(u''(t) + p(t)u(t))]' = f(t, u, u', u'') \quad (*) \]

where \( g, p : [T, \infty) \rightarrow [0, \infty) \) are bounded functions, \( g \geq \delta > 0 \). The function \( f \) is assumed to be continuous and \( f(x_1, x_2, x_3) \cdot x_1 \leq 0 \).

Many authors have consider ODE’s of the form (*) where the main part, i.e. the term \( u'' + pu \) is nonoscillatory. By contrast to these results we consider here the case of the oscillatory kernel function \( u'' + pu \).

The main goal is to show that any solution \( u \) of (*) is either oscillatory or it is a solution of the second order ODE \( u''(t) + p(t)u(t) = \beta(t) \) with vanishing right hand side \( \beta \geq 0, \beta(t) \rightarrow 0 \) as \( t \rightarrow \infty \). In the latter case all the derivatives \( u^{(n)}(t) \) up to the second order tend to zero as \( t \rightarrow \infty \), i.e. eq. (*) has the property (A).

The results are generalizations of these obtained by I. T. Kiguradze.
THE BODY WITH AN ELLIPSOIDAL CAVITY ROTATIONS ON A STRING
STABILITY NECESSARY CONDITIONS ANALYSIS

Victor Koval, Konstantin Ruchkin, Makeyevka, Ukraine

AMS Class.: 34D20 (93D15)

The paper considers Lagrange’s gyroscope motion, having an ellipsoidal cavity, completely filled with an ideal incompressible liquid, and suspended on an absolutely flexible inextensible and noninertial string. Motion equations of such a mechanical system are described by ODE system, accepting partial solution, corresponding to carrier – body and liquid round vertical line uniform rotations. These stationary motions stability investigation has been conducted by means of Lyapunov’s direct method. These motions stability necessary conditions coincide with the existence conditions of all the real roots of the fifth degree characteristic equation for linear differential equations system.

In the present paper these conditions were obtained with the help of the innors theory and analyzed in the case when the carrier – body is a weightless momentless envelope, which is above the fixed point. As a result of the analysis conducted the domains of stability necessary conditions realization in plane of dimensionless parameters have been built.

INvariant AND ORIENTED MANIFOLDS OF DYNAMICAL SYSTEMS

Alexander Kovalev, Donetsk, Ukraine

AMS Class.: 34C30 (93B05)

The notion of invariant manifolds (IM) plays the important role in the theory of dynamical systems. Levi - Civita got IM equations in the form: \( \partial V_j(t,x) = \sum_{i=0}^{k} \lambda_{ij}(t,x)V_i(t,x), \) where \( \partial \) is the operator of differentiation by virtue of the system. Some another approach was suggested by Poincare and Kharlamov, who reduced the problem to the investigation of functional dependence of the relations \( \partial^i \varphi(t,x) = 0, \) \( i = 0,1,2,... \) New form of IM equations is being suggested: \( \partial^{k+1}V(t,x) = \sum_{i=0}^{k} \lambda_i(t,x) \partial^i V(t,x), \) which establishes the interconnection between these two approaches. The new form of the equations of the manifolds oriented with respect to the systems is obtained as well and in their terms the controllability criterion of nonlinear dynamical systems is proved. The illustrative examples are considered and the control problem of the rotational motion of the rigid body is being solved.
ABOUT ENERGY KINETIC ACCUMULATOR (EKA) STATIONARY MOTIONS STABILITY

Igor Kovalev, Makeyevka, Ukraine

AMS Class.: 34D20 (90D15)

Recently the interest to energy kinetic accumulators as relatively new means of energy accumulation with high density and ecologically friendly energy source for different transport means has essentially increased. Energy kinetic accumulator (EKA) consists of a flexible shaft with a massive disk. EKA is modelled by the system of connected by universal elastic hinges five rigid bodies. The proposed by A.Ya. Savchenko model takes into consideration the non-linear effects, reasoned by the hinges big deflections. Such a system motion is described by the system of five non-linear ordinary differential equations.

The paper studies these differential equations system stationary solutions existence to which correspond to EKA working regimes. Their stability property has been investigated in the neighborhood rotations resonance frequencies under the small asymmetry presence in the system. Four classes of such solutions have been found. For one class of the solutions it has been swon that any asymmetry appearance in the system may lead to the unstability intervals appearance in the resonance frequencies neighborhood.

FINDING THE MOMENTS OF A SOLUTION OF A SYSTEM OF DIFFERENTIAL EQUATIONS WITH A RANDOM NON-GAUSSIAN PERTURBATION

Irina Kovtun, Kyiv, Ukraine

AMS Class.: 34 (47E05, 60H10)

For \( n \)-order systems perturbed by delta-correlated processes, we obtain the closed moment equations. For the first moment, this is a system whose order is the same as that of the initial system. For the second moment we have \( n \) systems which are alike, but with different initial data. The initial data, that is the dispersions, are determined from a certain system of \( \frac{n(n+1)}{2} \)-order ordinary differential equations, and so on.
A NOTE ON EXISTENCE OF BOUNDED SOLUTIONS OF ODE ON THE WHOLE REAL LINE

Bohumil Krajc, Ostrava, Czech Republic

AMS Class.: 34C11

Consider the $n$-th order scalar differential equation

$$x^{(n)} + \sum_{j=1}^{n} a_j x^{(n-j)} = f(t, x),$$

where $a_1, \ldots, a_n$ are real constants and $f$ is a function defined on $\mathbb{R}^2$. Suppose that all roots of the characteristic polynomial $\lambda^n + \sum_{j=1}^{n} a_j \lambda^{n-j}$ have non-zero real parts. Denote by $C(\mathbb{R})$ the Fréchet space of all continuous functions on the real line. For every $q \in C(\mathbb{R})$ we can define function $b_q : \mathbb{R} \to \mathbb{R}$, of the form $b_q(t) = f(t, q(t))$. Let there exist a subset $Q$ of $C(\mathbb{R})$ such that the function $b_q$ is Lebesgue measurable and bounded on $\mathbb{R}$ for every $q \in Q$. Then the linear equation $x^{(n)} + \sum_{j=1}^{n} a_j x^{(n-j)} = b_q(t)$ has exactly one solution $\varphi_q$ which is bounded on $\mathbb{R}$ and we can define an operator $T : Q \to C(\mathbb{R})$, $T(q) = \varphi_q$. Clearly, every fixed point of $T$ will be a bounded solution of the equation (1). The existence of bounded solution of (1) can be obtained by using the Tychonoff fixed point theorem.

In such a way the results making use Banach contractive principle can be improved. Thus we can weaken assumptions of classical Bohl theorem, especially Lipschitz condition on the right-hand side $f$.

PERIODIC BOUNDARY VALUE PROBLEM FOR RICCATI’S MATRICES EQUATION

Sergiy Krivosheya, Kyiv, Ukraine

AMS Class.: 34B30

Consider the periodic boundary value problem

$$\dot{Z} = AZ + ZB + \Phi(t) + \varepsilon Z \Psi(t) Z,$$

$$Z(0) = Z(\omega),$$

where $Z = Z(t)$ is an unknown $(n \times n)$-dimensional matrix, $A, B$ are constant, $\Phi(t), \Psi(t)$ are $\omega$-periodic, $(n \times n)$-dimensional continuous on $[0, \omega]$ matrices, $\omega = \text{const} > 0$, $\varepsilon \geq 0$ is small parameter.

By virtue of the theory of generalized inverse operators technique [BZS], we establish a criterion for solvability and study the structure of the set of solutions for generating problem (1), (2) ($\varepsilon = 0$). It is shown that in the case, when matrices $A$ and $B$ have common eigen values (critical case), the generating problem has a $n_0$-parametrical set of solutions and $n_0$ is determined by the Jordan’s structure of
matrices $A$ and $B$. The necessary condition for solvability of the problem (1), (2) ($\varepsilon > 0$) is established in the critical case.


APERIODIC REGIMES IN MODELLING OF PERIODIC FLOW REVERSAL OPERATION OF A TUBULAR REACTOR

Milan Kubíček, Praha, Czech Republic  
Jan Řeháček, Los Alamos, U.S.A.

AMS Class.: 58G28, 58G40, 35K55

A one dimensional two-phase model of a tubular reactor consists of a system of four parabolic PDE’s. Periodic flow reversal operation, i.e. the switching of the direction of the flow into the reactor, then means that the system of PDE’s and boundary conditions are periodically changed between two descriptions. Results of extensive numerical simulations show that periodic, quasiperiodic and chaotic symmetric and unsymmetric spatiotemporal patterns arise. Coexistence of various types of attractors was found. Chaotic solutions arising via the torus breaking were also observed. The transitions among individual types of patterns will be shown by means of Poincare maps.

NUMERICAL STUDY OF HOMOCLINIC AND HETEROCLINIC ORBITS REPRESENTING PULSE AND FRONT WAVES

Milan Kubíček, Igor Schreiber, Praha, Czech Republic

AMS Class.: 58F39, 34C37, 35K57, 65L99

We extend our earlier result on continuation of homoclinic orbits of autonomous ODE systems to heteroclinic orbits. The numerical method is based on a multiple shooting approach and a proper formulation of boundary conditions leading to an overdetermined system. Using numerical continuation we present several examples of dependences of homoclinic and heteroclinic orbits on a parameter. We also analyze by this method pulse–wave and front–wave solutions to reaction–diffusion equations. In this way dependences of wave solutions on a parameter can be conveniently calculated. Several examples are presented.
ON A TIME GLOBAL CLASSICAL SOLUTION
OF THE BOUNDARY VALUE PROBLEM
FOR $\Box u - \mu u + au^m = 0$ IN THE INTERIOR DOMAIN

Akisato Kubo, Aichi, Japan

AMS Class.: 35L20, 35L70

This paper studies the boundary value problem the non-linear wave equation

$$u_{tt}(t,x) - \Delta u(t,x) - \mu u(t,x) + au^m(t,x) = 0, (t,x) \in [0, \infty) \times \Omega$$  \hspace{1cm} (1.1)

$$u(t,x)|_{\partial \Omega} = 0,$$ \hspace{1cm} (1.2)

where $a$ is a real number, $m = 2, 3, \ldots$ and $\Omega \subset \mathbb{R}^n$ is a bounded domain with a smooth boundary $\partial \Omega$. We will show that for a constant $\mu > 0$, one can obtain time global solutions for (1.1) and (1.2) with exponential decay property. Denote eigen values of $-\Delta$ by $\{\lambda_j\}$ with $0 < \lambda_1 < \lambda_2 < \cdots \to \infty$. The eigen function corresponding to $\lambda_j$ is written by $\varphi_\lambda(x)$. Assume the following assumption.

(A) For an eigen value $\lambda \in \{\lambda_j\}$ and a number $\alpha > 1/2$ we set $\mu = \lambda + \alpha^2$. Put $L = \lfloor n/2 \rfloor + 2$. Then we have the following results.

**Theorem 1.** Assume that (A) holds. There exists a sufficiently large $m$ such that there exists a solution $u(t,x)$ of (1.1) and (1.2) in $\bigcap_{i=0}^{L+1} C_i([0, \infty) ; H_{L+1-i}(\Omega))$ satisfying $\lim_{t \to \infty} e^{\alpha t}u = \varphi_\lambda(x)$ in $H_{L+1}(\Omega)$.

Let $d =$diameter$(\Omega)$. Then we have the following result.

**Theorem 2.** Assume that (A) holds for $\alpha = 1$ and that $m$ is an integer $\geq 2$. There exists a sufficiently large $d$ such that there exists a solution $u(t,x)$ of (1.1) and (1.2) in $\bigcap_{i=0}^{L+1} C_i([0, \infty) ; H_{L+1-i}(\Omega))$ satisfying $\lim_{t \to \infty} e^{t}u = \varphi_\lambda(x)$ in $H_{L+1}(\Omega)$.

THE TIME PERIODICAL SOLUTION
OF NAVIER-STOKES EQUATIONS
WITH MIXED BOUNDARY CONDITIONS

P. Kučera, Prague, Czech Republic

AMS Class.: 35Q10 (58E35)

We study a flow of viscous incompressible fluid in the bounded channel $\Omega \subset \mathbb{R}^n$, $n = 2, 3$. We prescribe on both input and output

$$-P \cdot n_i + \nu \cdot e_{ij}(u) \cdot n_j = \omega_i, \quad i = 1, \ldots, n,$$

where $u = (u_1, \ldots, u_n)$ - velocity, $p$ - pressure, $\nu$ - viscosity, $(n_1, \ldots, n_n)$ - normal vector, $\omega = (\omega_1, \ldots, \omega_n)$ - a given function and $e_{ij}(u) = \frac{1}{2} \cdot (\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i})$. We suppose the non-slip boundary conditions on the fixed wall.

We study qualitative properties of the system of time-periodical Navier-Stokes equations and continuity equation with the boundary conditions mentioned above.
EQUATIONS OF MIXED TYPE: BOUNDARY-VALUE PROBLEMS AND APPLICATIONS

Alexander Kuz‘min, St.Petersburg, Russia

AMS Class.: 35M10 (76H05)

This paper is concerned with boundary-value problems for the partial differential equation of elliptic-hyperbolic type
\[ k(x,t)u_{tt} + \sum_{i,j=1}^{n} [a_{ij}(x,t)u_{x_i}u_{x_j}] + \alpha(x,t)u_t + \sum_{i=1}^{n} \beta_i(x,t)u_{x_i} + c(x,t)u - \lambda u = f(x,t), \]
where \( x = (x_1, x_2, ..., x_n) \).

The operator \( \sum a_{ij}(x,y)u_{x_i}u_{x_j} \) of differentiation with respect to the spatial variables is a uniformly elliptic one, while the set of points where the coefficient \( k(x,t) \) vanishes, and hence, the type of equation degenerates, can be of essentially general kind.

Weak conditions for the coefficients \( k(x,t) \) and \( \alpha(x,t) \), which provide Fredholm solvability of appropriate boundary-value problems for the above-mentioned equation, are found using a priori estimates in Sobolev spaces. The existence of singularities of the solution at certain points of the surface of type degeneracy is shown.

Also, boundary-value problems for a nonlinear equation of mixed type, which governs perturbation of steady two-dimensional flow of inviscid compressible gas are studied. The obtained results contribute to the development of high resolution numerical algorithms for computation of transonic flows over airfoils/wings.

ON EXISTENCE OF A POSITIVE SOLUTION OF THE FIRST ORDER LINEAR DIFFERENTIAL EQUATION WITH A DEVIATING ARGUMENT

George Kvinikadze, Tbilisi, Georgia

AMS Class.: 34K15

Consider the equation
\[ u'(t) = p(t)u(\tau(t)), \]
where \( p : R_+ \rightarrow R_+ \) is locally integrable, \( \tau : R_+ \rightarrow R \) is measurable and \( \lim_{t \to +\infty} \tau(t) = +\infty \).

**Theorem.** Let for some \( t_0 \in R_+ \)
\[ \int_t^{\tau^*} p(s) \, ds \leq \frac{1}{e} \quad \text{for} \quad t \geq t_0, \]
where \( \tau^*(t) = \max\{t, \tau(t)\} \). Then the equation (1) has a solution \( u : [t_0, +\infty[ \rightarrow ]0, +\infty[ \) satisfying
\[ \exp \left\{ \int_{t_0}^t p(s) \, ds \right\} \leq u(t) \leq \exp \left\{ e \int_{t_0}^t p(s) \, ds \right\} \quad \text{for} \quad t \geq t_0. \]
HYDRODYNAMIC LIMITS OF KINETIC EQUATIONS

Mirosław Lachowicz, Warsaw, Poland

AMS Class.: 82A40, 45K05, 76P05

An exposition of some recent rigorous results (as well as open problems) concerning the connection between two different descriptions of matter:

I.) the microscopic (statistical) one in the framework of kinetic theory,

II.) the macroscopic one in the framework of continuum theory,

is given. An analysis of the asymptotic behavior of kinetic equations towards various hydrodynamic limits is outlined. Different relations between the small parameters of kinetic models yield different versions of hydrodynamic equations.

In particular, some kinetic equations result, in some limit, in a compressible Navier–Stokes–type hydrodynamic system, with elliptic (i.e. viscosity and heat conduction) terms which are independent of all small parameters of the kinetic model.

EXISTENCE THEOREMS FOR A HEAT EQUATION WITH FUNCTIONAL DEPENDENCE

Henryk Leszczyński, Gdańsk, Poland

AMS Class.: 35K05 (35K10)

We reduce the Cauchy problem for a heat equation with the non-linear right-hand side which depends on some functionals acting on both, the unknown function and its gradient, to an equivalent system of integral equations whose solutions are sought in some Banach spaces of continuous, bounded and/or exponentially bounded functions. We prove that the solution to this system of integral equations consists of a solution to the differential-functional equation and its spatial derivatives. We obtain weak and classical solutions.

MINIMAX PROBLEMS OF ESTIMATION OF SOLUTIONS OF STOCHASTIC DIFFERENTIAL EQUATIONS BY INFORMATION CRITERIONS

O. L. Levoshich

We investigated problems of estimation of stochastic differential equations by information criterions. The information is understood in the Shennon’s sense. Under different assumptions the determination of minimax estimations have been reduced to solving some recurrent equations of filtration.
NONLINEAR SIMULATION OF MOVING BOUNDARY PROBLEM COUPLED WITH LIQUID WAVE MOTION

Oleg Limarchenko, Kiev, Ukraine

AMS Class.: 35R35 (76M30)

We consider nonlinear boundary problem about liquid wave motion in closed tank. The distinctive peculiarity of this problem is the existence of free surface of liquid with two boundary conditions on it. These boundary conditions allow to determine position of free surface and satisfy pressure equilibrium condition.

Mathematical description of system dynamics is based on Hamilton variational principle. The construction of solving model was carried out by nonlinear algorithm developed by author for perfect incompressible liquid that partially fills cavity of rigid body. Account of sufficiently large number of liquid free surface oscillation modes is the main achievement of the approach. Some test problems show that for transient processes main nonlinear effects connected greater with nonlinear interaction between oscillation modes than with accounting more high order of nonlinearities.

We investigate behavior of such mechanical system by means of construction effective ordinary differential equation system and apply such approach for analysis of nonlinear applied problems of liquid wave motion in moving tank under different type of impact loading.

ON THE NUMERICAL SOLUTION OF CERTAIN SECOND ORDER LINEAR DIFFERENTIAL EQUATIONS ON SEMI-INFINITE INTERVAL

Vu Hoang Linh, Budapest, Hungary

AMS Class.: 65D20 (34E05)

We consider the differential equation

\[ y''(x) + U(x)y(x) = 0, \quad x_0 \leq x < \infty. \]

Function \( U(x) \) is assumed to be at least piecewise continuous and to satisfy the asymptotic formula

\[ U(x) = x^\sigma \sum_{j=0}^n U_j x^{-jp} + O(x^{\sigma-(n+1)p}), \quad x \to \infty, \]

where \( p, \sigma, U_j, j = 0, 1, 2, \ldots, n \), are given constants, \( U_0 \neq 0, \sigma \geq 0, p > 0 \) and \( (n + 1)p - \sigma/2 > 1 \). The qualitative behavior of solutions for large \( x \) depends on the sign of \( U_0 \). Asymptotic formula of the solutions can be given when \( x \) tends to infinity. In the case of a positive \( U_0 \), a new variant of amplitude-phase methods has been recently proposed for computing the solutions in the papers of N.B. Konyukhova, I.B. Staroverova and this author. Analogous tricks will be suggested for negative \( U_0 \), as well. Smoothness of the modified amplitude and phase functions is provided by the statements and verified in numerical integration, too.
METHOD OF P–ANALITICAL FUNCTIONS
IN BOUNDARY–VALUE PROBLEM
OF MATHEMATICAL PHYSICS

L. Lomonos, Kiev, Ukraine

Method of p-analitical functions is one of the most effective methods for solution of boundary-value problems of axially symmetric elasticity theory. Solving the first and second main axially symmetric tasks of elastic prolate rotation ellipsoid with ball cavity, when there centers are not coincided, the formulas of N.G.Pologia are used. These formulas are analogous to well known formulas Kolosov–Mushelishvili in the plane elastic theory. They differ from them in the following. They evaluate tensor components of tension and of mixture vector of tension position circulay symmetry of the rotation body not through analitical, but through two p-anlitical functions of \( z = x + iy \) with characteristic \( p = x \) in the region of meridian section of rotation body. With a help of basic integral presentation of p-anlitical functions, formulas of reexpansion for solutions of Laplas equation in spherical and prolate spherical coordinates the solutios of each tasks are reduced to solution of quasi-entirely regular infinite system of linear algebraic equations with limited above free terms, which tends to zero, when index increases. Hence these systems may be solved by reduction method.

NUMERICAL SOLUTION OF COMPRESSIBLE FLOW

Mária Lukáčová – Medviďová, Brno, Czech Republic / Magdeburg, Germany

AMS Class.: 65M12, 65M60, 35K60, 76M10, 76M25

The contribution deals with numerical solution of inviscid as well as viscous compressible flow. The results were obtained in the cooperation with Feistauer, Felcman (Charles University, Prague) and Warnecke (University Magdeburg).

The Navier–Stokes equations are numerically solved by the combined finite element – finite volume method via operator inviscid–viscous splitting. The main idea of the method is to discretize nonlinear convective terms by means of the finite volume method, whereas the rest pure diffusion system is discretized by the conforming piecewise linear finite element method. The nonlinear convective terms can be also solved by the method of characteristics. Numerical solution obtained by latter method is trully multidimensional and independent of mesh character. Some result of numerical experiments are presented.

Convergence and error estimate of the combined finite element – finite volume method for the nonlinear scalar conservation law with a diffusion term are proved.
STABILITY INVESTIGATION
FOR TWO POPULATIONS CULTIVATING

Elena Lyashenko, Kiev, Ukraine

The possibility of joint cultivating in running cultivator for two nonantagonistic populations of microorganisms, which use the common resource of nutrition, is investigated. The mathematical problem is to investigate the stability of stationary points of differential equations system with hyperbolic nonlinearity. The correlations between parameters of population growth and parameters of cultivator, under which the unique nontrivial stationary point are obtained. Unfortunately, this point turns out to be nonstable by Lyapunov, that shows nonrealizability of joint cultivating of two populations with common nutrition, and also the importance of attaining the purity of populations.

OPTIMAL PULSED CONTROL OF UNDERGROUND MASS TRANSPORT

S. I. Lyashko D. A. Kljushin

AMS Class.: 35B30

We study optimal pulsed control of the underground mass transport processes described by linear partial differential equations of parabolic and pseudo–parabolic type. The theorems of existence of the solutions of these problems in the class of generalized finite–order functions are proved, approximate solutions are obtained and convergence of the sequences of the approximations to the generalized solutions in the various spaces are proved. Obtained results are applied to solve the problems of optimal control of water flow and contaminant dissemination in the homogeneous and heterogeneous media (stratified aquifers, fissured media and so on).

SINGULAR DIFFERENTIAL EQUATIONS AND LAURENT TYPE SERIES

Grzegorz Lysik, Warszawa, Poland

AMS Class.: 34A20, 34A30

Let \( P \) be a linear ordinary differential operator with analytic coefficients at zero. It is well known that however there exist formal power series solutions of \( Pu = f \) where \( f \) is analytic at zero they, in general, do not converge.

In my work I present a method of finding solutions of \( Pu = f \) in a form of convergent series of Laurent type. Namely we have

**Theorem.** Let \( P \) be an operator of the first order or such that the Newton diagram for \( P \) lies above or below the characteristic level, or a composition of such operators. If \( f \) is a Laurent type series of Gevrey order then every solution of \( Pu = f \) is a Laurent type series of Gevrey order.
REGULARITY OF THE SOLUTION OF THE SECOND BOUNDARY-VALUE PROBLEM FOR THE AIRY EQUATION

E. Majewska, J. Popiołek, Warsaw, Białystok, Poland

AMS Class.: 35J15, (35Q53)

Let \( \mathcal{D} = \{(x,t) \in \mathbb{R}^2 : 0 < x < 1, 0 < t \leq T\} \), \( T = \text{const.} > 0 \). We pose the following boundary–value problem: find a function \( u \) being the solution of equation

\[
\mathcal{L}[u(x,t)] = D_x^3 u(x,t) - D_t u(x,t) = f(x,t)
\]

in the domain \( \mathcal{D} \), belonging to the class \( C^{3,1}_x, C^{2,0}_t(\mathcal{D}) \) and satisfying the following boundary conditions:

\[
\begin{align*}
\phi_j(t) &= \psi_j(x), & 0 \leq x \leq 1, & u(0,t) = \phi_0(t), & 0 \leq t \leq T, & u(1,t) = \phi_1(t), & 0 \leq t \leq T, & D_x^2 u(0,t) = \phi_2(t), & 0 \leq t \leq T,
\end{align*}
\]

where \( \phi_j, j = 0, 1, 2 \), and \( \psi \) are given functions and satisfy the compatibility conditions:

\[
\phi_0(0) = \psi(0), \quad \phi_1(0) = \psi(1), \quad \phi_2(0) = \psi''(0).
\]

In [1] we have proved the existence of solutions of the problem (1) – (5). Now, we shall examine some regularity properties of solutions of the said problem.


FINITE–DIMENSIONAL DYNAMICAL SYSTEMS WITH COMPLEX BEHAVIOR AND OSCILLATIONS AS THE MODELS OF HYDRODYNAMICS WITH MEMORY EFFECTS

Alexander Makarenko, Kiev, Ukraine

AMS Class.: 35B30 (35B37)

One of the approaches to the investigation of hydrodynamics equations (for example Navier–Stokes equations) is Galerkin method. Using this method it is easy to construct a low–dimensional dynamical system. In particular, for the Navier–Stokes equations one of such system is well known Lorentz system. But as was described previously by us more accurate is generalized hydrodynamics with memory effects. In this paper there are exposed some results on the construction of a low–dimensional analogs for such hydrodynamics and their comparison with common equations obtained under the same initial and boundary conditions.

In 1979 Boldrighini and Franceschini investigated five–dimensional systems on torus. For the sake of comparison we derive the analogous systems for the generalized hydrodynamics.
BOUNDDED SOLUTIONS OF A CLASS OF ORDINARY DIFFERENTIAL EQUATIONS AND APPLICATIONS TO A BOUNDARY VALUE PROBLEM

Luisa Malaguti, Modena, Italy

AMS Class.: 34C11

We consider the second order equation:

\[(r(x)u')' + q(x)g(u) = 0\]

under the assumptions \(r, q : [0, +\infty) \to \mathbb{R}\) continuous, \(r(x) > 0, q(x) > 0\) for all \(x\) and \(g \in C(\mathbb{R})\) bounded, nonnegative, nonvanishing and locally Lipschitz. By means of various topological methods, we obtain integral criteria, involving only the functions \(r\) and \(q\), which garantee both the existence of bounded solutions when \(x\) tends to +\(\infty\) and the existence of unbounded ones. Such integral criteria have already been used in Cecchi-Marini-Villari J. Diff. Equat. 118 (1995), in a different context. Then we apply the obtained results to study a boundary value problem, on an infinite interval, appearing both in combustion models and in population genetics.

OSCILATORY SOLUTIONS OF EMDEN–FOWLER SYSTEM OF DIFFERENTIAL EQUATIONS

Jelena Manojlović, Niš, Yugoslavia

The differential equation

\[u''(t) = a(t)|u(t)|^{\lambda} \text{sgn} u(t), \quad \lambda \neq 1\]  (1)

is known in the literature as the equation of the Emden-Fowler type. An investigation of the equation of this type began in connection with the astrophysical investigations around the turn of the century. The oscillatory and nonoscillatory behavior of solutions of the Emden-Fowler equation have been investigated by many authors.

We shall consider the nonlinear system of differential equations

\[u'_1 = a_1(t)|u_2|^{\lambda_1} \text{sgn} u_2, \quad u'_2 = a_2(t)|u_1|^{\lambda_2} \text{sgn} u_1\]  (2)

where the functions \(a_i(t), (i = 1, 2)\) are summable on each finite segment of the interval \([0, +\infty)\) and \(\lambda_i > 0, (i = 1, 2)\), as an extension of the Emden-Fowler equation, since for \(a_1(t) \equiv 1, \lambda_1 = 1\) from the system (2) we obtain the equation (1).

We shall establish some oscillation criterias for the system (2) whose are extensions of some results for the equation of the Emden–Fowler type.
ASYMPTOTIC PROPERTIES OF QUASILINEAR NEUTRAL DIFFERENTIAL EQUATIONS

Pavol Marušiak, Žilina, Slovak Republic

AMS Class.: 34K40 (34K25)

There is investigated neutral nonlinear differential equations in the form
\[(L_0^\alpha x(t))' + f(t, x(g(t))) = 0, \quad t \geq a > 0,\]  
where
\[L_0 x(t) = x(t) - p(t)x(h(t)),\]
\[L_0^\alpha x(t) = r(t)|L_0'x(t)|^{\alpha-1}L_0'x(t),\]
\(\alpha\) is a positive real constant.

It is assumed that \(r : [a, \infty) \to (a, \infty), p, h, g : [a, \infty) \to R, f : [a, \infty) \times R \to R\) are continuous functions; \(0 \leq p(t) \leq \lambda < 1\) on \([a, \infty)\), \(h(t)\) is strictly increasing function, \(h(t) < t\) on \([a, \infty)\) and \(\lim_{t \to \infty} h(t) = \lim_{t \to \infty} g(t) = \infty; \quad uf(t, u) > 0\) for \(u \neq 0\), all \(t \geq a\) and \(f\) is nondecreasing in the second variable for all \(t \geq a\).

\[\int_a^\infty (r(t))^{\frac{1}{\alpha}} dt < \infty.\]

In the lecture will be given the sufficient conditions for the existence of several types of nonoscillatory solutions of (E) which specified asymptotic behavior.

NON-NEWTONIAN FLOW THROUGH A THIN FILTER

Sanja Marušić, Zagreb, Croatia

AMS Class.: 35B27

We study a purely viscous flow of a quasi-newtonian fluid with viscosity obeying the Carreau’s law, passing through a periodic thin filter with period and thickness \(\varepsilon\). Denoting the median plane of the filter by \(\Sigma\) we use the boundary layer method to study the flow in the tiny layer around \(\Sigma\). After matching the inner asymptotic expansions in the boundary layer in vicinity of the filter, depending on the fast variable, with the exterior expansion depending on the slow variable we obtain the global behavior of the flow. We find that the limit as \(\varepsilon \to 0\) of the velocity and of the pressure satisfy the same system of equations as the ones that we had before the limiting procedure but only in the domain above and below the median plane \(\Sigma\), while on \(\Sigma\) the velocity is equal to some constant vector depending only on the filter’s local geometry and on the mean normal injection. Moreover the expansion in the boundary layer shows that in the vicinity of \(\Sigma\) the velocity gradient and the pressure are of order \(1/\varepsilon\). The result corresponds to the results by Sanchez-Palencia and Conca found in the newtonian case. (Joint work with A. Bourgeat and E. Marušić-Paloka.)
EFFECTS OF INERTIA FOR A FLUID FLOW IN A POROUS MEDIUM

Eduard Marušić-Paloka, Zagreb, Croatia

AMS Class.: 35B27

The filtration of the viscous fluid in porous medium is usually described by the Darcy’s system. However it is well known that, for sufficiently high flow rates in porous media, nonlinear filtration effects are observed. Theoretical explanations for such effects were not totally understood, up to recently, when there was a surge of interest for obtaining non-linear laws by means of two scale asymptotic expansions. Starting from the Navier Stokes system in a periodic porous medium $\Omega_{\varepsilon}$ where $\varepsilon$ is the characteristic pore size the homogenization method gives different results depending on the relation between the Reynolds number, Freude’s number and the period.

If both, the Reynolds number and the inverse of the Froude’s number are of order $1/\varepsilon$, then the formal asymptotic expansion gives a homogenized system containing the fast and slow variables named Navier-Stokes system with two pressures. More precisely the filtration law is nonlocal and nonlinear. Supposing that the data are not too large we prove the existence of a unique solution for the homogenized problem. Furthermore, the convergence of the homogenization process is proved and the error estimate is established.

We also prove that the obtained law is not a singular phenomenon and that there exists a continuity between linear Darcy’s law and the proposed nonlinear law. (Joint work with A. Bourgeat and A. Mikelić.)

INTEGRATED SEMIGROUPS AND A CAUCHY PROBLEM

Milorad Mijatović, Stevan Pilipović, Novi Sad, Yugoslavia

AMS Class.: 47D06 (34G10) Let $L(E) = L(E,E)$ be a Banach space of bounded linear operators from a normed space $E$ into $E$. We study an $\alpha$-times integrated semigroup ($\alpha \in \mathbb{R}$) and the related abstract Cauchy problem

$$u'(t) = Au(t) + f(t), \quad u(0) = u_0,$$

where $A : D(A) \subset E \to E$ is the infinitesimal generator of the $\alpha$-times integrated semigroup. More specially, for $\alpha = 0$ we analyze a class of 0-integrated semigroups (0-i.s., in short) and apply the theory to

$$u' = Au + T, \quad T \in \mathcal{K}'_{1+}(\overline{D(A)})$$

in $\mathcal{K}'_{1+}(\overline{D(A)})$, where $\mathcal{K}'_{1+}$ is the space of exponential distributions which are supported by $[0, \infty)$.

The density of $D(A)$ in $E$ and of a set $\{S(\varphi, x), x \in E, \varphi \in D_0\}$ in $D(A)$, where $S$ is a 0-i.s. with the infinitesimal generator $A$ are the properties which essentially characterize 0-i.s. We explore these properties of the domain $D(A)$ and utilize the relations between $n$-times integrated semigroups and distributional semigroups.
ANDRONOV-HOPF BIFURCATION OF CUTTING PROCESS

Branko M. Milisavljević, Novi Sad, Yugoslavia

AMS Class.: 34A34 (34C15)

Self-excited vibrations appeared during metal cutting process (chatter) are examined in this paper. A two-degrees-of-freedom model is introduced which includes the non-linear part of a cutting force. This non-linear model is one generalization of linear mode coupling J. Thustý’s model of chatter with regard to the A. P. Sokolovskii’s non-linear model.

It is shown that self-excited vibration in metal cutting arises softly, through the stable Andronov-Hopf or supercritical bifurcation. Various characteristic phenomena are explored in general terms using center manifold reduction, Poincaré’s normal form and Floquét theory.

INVESTIGATION OF THE RESONANCE SITUATIONS IN THE PROBLEM OF TWO CONNECTED LAGRANGE’S GYROSCOPES

Tateyana Mosiayash, Makeyevka, Ukraine

AMS Class.: 34D20 (93D15)

The present paper consider the problem of stability of two Lagrange’s gyroscopes connected by ideal hinge. Such a mechanical system’s motion equations are the ODE system. In this case characteristic equation (1) have been the all real roots.

\[
\begin{align*}
\left[ l' g_1^* \mu^5 - l' g_2^* \mu^4 + (l' g_4 - g_2) \mu^2 - g_3 \mu + g_4 \right] \\
\left[ l' g_1^* \mu^5 - l' g_2^* \mu^4 + (l' g_4 - g_2) \mu^2 - g_3 \mu + g_4 \right] = 0
\end{align*}
\]

where \( \mu = \omega + \lambda; \ \mu' = \omega - \lambda \). It may happen that this the roots are equal. In the first case \( \lambda_1 = \lambda_6 \), then we have the resonance situation of the first order, in the second case \( \lambda_1 = \lambda_7 \) - of the second order. This paper is devoted to the investigation of resonance curves both of the first and second order. Algorithm of construction of the curves \( L_1 \) and \( L_2 \) is given in the paper.
NONLINEAR HYPERBOLIC EQUATIONS
WITH DISSIPATIVE TEMPORAL AND SPATIAL
NON-LOCAL MEMORY

František Mošna, Prague, Czech Republic

AMS Class.: 45K05, 73F15

The equation governing the evolution of the displacement vector in an elastic
body
\[ \ddot{u}_i - \frac{\partial}{\partial x_j} \sigma_{ij} = f_i \quad i = 1, \ldots, N \]
is considered. The Cauchy stress tensor consists of two parts, the instantaneous
stress \( \sigma^I \) is extended by the memory part \( \sigma^M \), generated by a singular but inte-
grable kernel:
\[ \sigma_{ij}^M = -\lambda \int_0^t \int_\Omega \left( e_{ij} u(\xi, \tau) - e_{ij} u(\xi, t) \right) \frac{h(t-\tau)}{|x-\xi|^\alpha} \, d\xi \, d\tau, \quad h(t) = e^{-t^{-\nu}}, \]
where \( 0 < \nu < \frac{1}{2}, N-1 < \alpha < N, N-\alpha < \nu, \) and \( e_{ij} u \) is the symmetrized gradient.

The existence of a global weak solution to the associated initial-boundary prob-
lem is established by constructing Galerkin approximations and deriving a suitable
energy estimate:
\[ \|\dot{u}\|_{W^{1+\frac{\nu}{2}, 2}(0; T; W^{-1, \frac{2}{2-N}\alpha}(\Omega; \mathbb{R}^N))} \leq C(T). \]
We use properties of various function spaces (the Slobodeckij spaces), the Fourier
Transform and interpolation.

UNIQUENESS AND NONUNIQUENESS OF WEAK
SOLUTIONS FOR PARABOLIC EQUATIONS
IN DOMAINS WHICH VARY IN TIME

Hirozo Okumura, Ueda, Japan

AMS Class.: 35K10 (35K20)

Let \( \psi(x) \) be a convex \( C^\infty \) function in \( \mathbb{R}^n \) with \( \psi(0) = 0 \) and \( \psi(x) > 0 \) for
\( x \neq 0 \). For \( T > 0 \), set \( \Omega_T = \{(t, x) \in \mathbb{R}^{n+1} | \psi(x) < t < T \} \) and \( S_T = \{(t, x) \in \overline{\Omega_T} \mid t = \psi(x) \} \). We consider the following boundary value problem in \( \Omega_T \):
\[ Lu = \sum_{i,j=1}^n a_{ij}(x, t) \frac{\partial^2 u}{\partial x_i \partial x_j} + \sum_{j=1}^n b_j(x, t) \frac{\partial u}{\partial x_j} + c(x, t) u - \frac{\partial u}{\partial t} = f(x, t) \quad \text{in } \Omega_T, \]
\[ u = 0 \quad \text{on } S_T, \]
where \( a_{ij} = a_{ji}, b_j \) and \( c \) are real valued \( C^\infty \) functions and \( L \) is parabolic in \( \overline{\Omega_T} \).

In this talk we are interested in the critical constant \( p \) such that the uniqueness of
weak solutions belonging to \( L^p(\Omega_T) \) holds. Some new results of this problem will
be given.
ON STABILITY OF SOLUTIONS OF ABSTRACT DIFFERENTIAL EQUATIONS

Vladimir Orlov, Voronezh, Russia

AMS Class.: 35Q99

Let $E$ be arbitrary Banach space, $A$, $B$ linear unbounded and $BA^{-1}$ bounded operators acting in $E$, $A$ generate the exponentialy decreasing analytic semigroup $T(t) = \exp(-tA)$. The Cauchy’s problem

$$u''(t) + \mu_1 Au'(t) + \mu_2 Bu(t) = \varphi(t), \quad t \geq 0, \quad u(0) = u_0, \quad u'(0) = u_1$$

in the Banach space $L_q(0, \infty; E)$ is under consideration. The conditions of solvability and estimate

$$\|u''(t)\|_{L_q(0, \infty; E)} + \|Au(t)\|_{L_q(0, \infty; E)} + \sup_{t \geq 0} u'(t) + \sup_{t \geq 0} \|Au(t)\| \leq M(\|\varphi\|_{L_q(0, \infty; E)} + \|Au_0\| + |u_1|).$$

are established. Here $|u_1| = \left(\int_0^\infty \|AT(t)u_1\|^q dt\right)^{1/q}$. In the special case $A = B$ the exact conditions containig coeffitients of equation and spectral parameters of $A$ are given. The obtained estimates are applied to the investigation of the solvability and stability of nonlinear mathematical model of thermoviscoelasticity.

ALMOST PERIODICALLY FORCED LOCALLY CONVEX LAGRANGIAN SYSTEM

Igor Parasyuk, Kyiv, Ukraine  Sergei Zakharin, Kyiv, Ukraine

AMS Class.: 34C27 (49J40)

Consider the equation:

$$\frac{d^2x}{dt^2} + \nabla v(x) = e(t)$$

(1)

where $x(t) = (x_1, ..., x_n)^T$, $e = (e_1, ..., e_n)^T$, $e \in CAP(R, R^n)$, i.e., each of $e_i$, $i = 1, n$, belongs to the space $CAP(R)$ of Bohr-almost periodic functions; $v : R^n \to R$, $v \in C^2(R^n, R)$ - a locally convex function.

Almost periodic (a. p.) solutions of equation (1) are studied. A variational method is used. A convex non-local problem of minimization is considered. Under certain assumptions equation (1) is shown to have a Besicovitch-a.p. weak solution.
ON SOME PROPERTIES OF SOLUTIONS
OF TWO–DIMENSIONAL DIFFERENTIAL SYSTEMS
WITH DEVIATED ARGUMENTS

Nino Partsvania, Tbilisi, Georgia

AMS Class.: 34K15

The system of differential equations

\[ u'_1(t) = f_1(t, u_1(\tau(t)), u_2(\sigma(t))) \]
\[ u'_2(t) = f_2(t, u_1(\tau(t)), u_2(\sigma(t))) \]

is considered where \( f_i : \mathbb{R}_+ \times \mathbb{R}^2 \to \mathbb{R} \) (\( i = 1, 2 \)) satisfy the local Carathéodory conditions and \( \tau, \sigma : \mathbb{R}_+ \to \mathbb{R} \) are nondecreasing continuous functions such that \( \sigma(t) \leq t, \sigma(\tau(t)) \leq t \) for \( t \in \mathbb{R}_+ \), \( \lim_{t \to +\infty} \tau(t) = +\infty, \lim_{t \to +\infty} \sigma(t) = +\infty \). Sufficient conditions are established for the oscillation of proper solutions of system (1).

MINIMAX ESTIMATION AND PREDICTION
FOR SOLUTIONS
OF PARTIAL DIFFERENTIAL EQUATIONS

Olga Pavluchenko, Kiev, Ukraine

We consider systems described by parabolic and hyperbolic partial differential equations. The problem is to find optimal prediction estimates for functionals to solution of boundary-value problems for these equations, and also minimax estimates for functionals to solutions of boundary-value problems diffraction in plane-parallel wave conductor. It is established that determination of optimal prediction estimates are reduced to solving some systems of integro-differential equations. From noisy observations of the state of systems on a finite interval of time, we find optimal in certain sense prediction estimates for functionals from solution of these equations at an arbitrary moment of time. It is assume here that the right-hand sides of equations, boundary and initial conditions and also errors of measurements are determined not exactly but only the sets to which they belong are known. This method can be applied for example to modeling of diffusion of an admixture in the atmosphere.
\textbf{G-QUASIASYMPOTOTICS AT INFINITY TO SEMILINEAR HYPERBOLIC SYSTEM}

S. Pilipović, M. Stojanović, \textit{Novi Sad, Yugoslavia}

AMS Class.: 46F05 (46F10, 46F99, 35L40)

We give the definition of $G$-quasiasymptotics at infinity in a frame of Colombeau’s space of generalized functions $G$ (cf. [1]) and give the main properties and characterizations in this notion. We give an application to a Cauchy problem for a strictly semilinear hyperbolic system. We prove that the quasiasymptotic behaviour at infinity of the solution inherits the quasiasymptotic behaviour at infinity of initial data under suitable assumptions on the nonlinear term.


\textbf{SHADOWING IN STRUCTURALLY STABLE FLOWS}

S. Yu. Pilyugin, \textit{St. Petersburg, Russia}

AMS Class.: 58F10 (58F15, 58F30)

It was shown by Anosov and Bowen that approximate trajectories (pseudotrajectories) of a diffeomorphism in a neighborhood of a hyperbolic set are shadowed by real trajectories. Later different authors proved that a structurally stable diffeomorphism on a closed manifold $M$ has the shadowing property on $M$. It follows from results of Bowen and of Franke-Selgrade that pseudotrajectories of a flow near a hyperbolic set (separated from rest points) are shadowed by real trajectories.

We prove that pseudotrajectories of a structurally stable flow on a closed manifold $M$ are shadowed by real trajectories, and the shadowing is Lipschitz with respect to the “error”.

The main difficulty of the considered problem is created by the possible existence of rest points. A special abstract shadowing result for a sequence of noninvertible mappings of Banach spaces is established to overcome this difficulty.
BOUNDEDNESS OF SOLUTIONS OF SOME SYSTEMS OF NONLINEAR DIFFERENTIAL EQUATIONS

Viktor Pirč, Košice, Slovakia

AMS Class.: 34C11, 34K20

This paper presents some mathematical methods for studying of specific class of analog processor networks. We present some theoretical results to the dynamic properties of dynamic feedback neural networks. The obtained results are in the form suitable for applications.

Hopfield has presented a type of neural network which is represented by the following systems of ordinary differential equations

\[ x'_i = a_{ii}x_i + \sum_{j=1}^{n} b_{ij}g(x_j) + k_i, \]  

(1)

where \( i = 1, \ldots, n \) \( a_{ii} \) are negative, real constants, \( b_{ij} \) are positive, real constants and \( k_i \) are real constants. Nonlinear function \( g \) is bounded, monotonous and continuous.

THE HARTMAN-WINTNER THEOREM FOR FUNCTIONAL DIFFERENTIAL EQUATIONS

Mihály Pituk, Veszprém, Hungary

AMS Class.: 34K15

We present a generalization of the classical theorem of Hartman and Wintner on asymptotic integration of the solutions of linear ordinary differential equations to functional differential equations. Our result improves and unifies several recent theorems concerning asymptotically autonomous linear functional differential equations.

ROTHE’S METHOD FOR DEGENERATE QUASILINEAR PARABOLIC EQUATIONS

Volker Pluschke, Halle (Saale), Germany

AMS Class.: 35K65 (65M20, 35K20)

We investigate local existence of a weak solution to the degenerate equation

\[ g(x,t,u) u_t + A(t,u)u = f(x,t,u) \quad \text{in} \ G \times (0,T) \]

with Dirichlet boundary conditions. Degeneration occurs in the coefficient \( g \geq 0 \) which is not assumed to be bounded below and above, resp., by some positive
constants. Instead, \( g(\cdot, t, u) \) may vanish on subsets \( D_{t,u} \subset G \) with \( \text{meas} (D_{t,u}) = 0 \) for any \( t \in [0, T], |u| \leq M \). The data of the problem belong to Lebesque spaces.

We approximate the problem by semidiscretization in time (Rothe’s method) and prove uniform convergence of the approximations to a Hölder continuous weak solution. The assumptions with respect to \( u \) for the nonlinear coefficients and the right-hand side are supposed only in a neighbourhood of the initial function, hence we have to derive a \( L_{\infty} \)-estimate for the approximates. This estimate is obtained by a Moser technique. One difficulty that appears because of nonlinearity of \( g \) is that we need for this also an estimate of discrete time derivatives in \( L_r(G) \) with certain \( r > 2 \) uniformly for all subdivisions.

**BOUNDARY VALUE PROBLEMS OF DIFFRACTION IN THE WEDGE**

**Yuri Podlipenko, Kiev, Ukraine**

AMS Class.: 35L05

We construct the theory of boundary value problems for the Helmholtz equation in the three-dimensional wedge from which a finite number of domains bounded by closed surfaces is removed. It is assumed that the solutions of these problems take the prescribed values on surfaces mentioned above and vanish on the wedge’s sides and that they satisfy the Sommerfeld’s radiation condition and to condition at the edge of the wedge.

Such boundary value problems appear in the study of diffraction of acoustical waves on obstacles contained within the wedge.

In particular we obtain the following results:

(i) the structure of the spectra of operators corresponding to the boundary value problems in the considered domain is investigated;

(ii) the uniqueness and existence theorems for solutions of these boundary value problems are proved and the principle of limiting absorption is justified for this situation;

(iii) the potential theory has been developed making it possible to reduce the above boundary value problems to Fredholm integral equations on the obstacle’s boundary;

(iv) the existence and uniqueness of solutions for these integral equations have been also proved.
TRANSITION FROM GLOBAL EXISTENCE TO BLOW-UP IN PARABOLIC PROBLEMS

Pavol Quittner, Bratislava, Slovakia

AMS Class.: 35K60 (35J65, 35B40)

In many parabolic problems possessing blowing-up solutions, there also exist global bounded solutions. For these problems, we study long-time behavior of solutions lying on the borderline between global existence and blow-up.

In particular, we consider initial boundary value problems for the equation $u_t = \Delta u + |u|^{p-1}u + f(x, t, u, \nabla u)$, where $p > 1$ and $f$ is a perturbation term, and problems for systems of equations with similar dynamics. We find sufficient conditions for global existence and boundedness of the solutions mentioned above and we apply these results to the proof of existence of unstable sign-changing stationary and periodic solutions.

A REDUCTION OF IMPULSIVE DIFFERENTIAL EQUATIONS

Andrejs Reinfelds, Rīga, Latvia

AMS Class.: 34A37, 34C20

Impulsive differential equations provide an adequate mathematical model of evolutionary processes that suddenly change their state at certain moments. We find sufficient conditions under which the system of impulsive differential equations in the Banach space $X \times Y$ is dynamically equivalent in the large to the impulsive system that is simpler than the given one in terms of decoupling and linearization. The second system splits into two parts. The first of them does not contain the variable $y \in Y$, while the second one does not contain the variable $x \in X$ and is linear. This result generalizes Hartman–Grobman theorem as well as the principle of reduction for systems of semilinear ordinary differential equations to corresponding one for systems of impulsive differential equations in Banach spaces. Relevant results concerning partial decoupling and simplifying of the systems of impulsive differential equations are given also.

$L_p - L_q$ ESTIMATES FOR SOLUTIONS OF HYPERBOLIC EQUATIONS WITH COEFFICIENTS DEPENDING ON TIME

Michael Reissig, Freiberg, Germany

AMS Class.: 35L10 (35L15, 34E20)

One of the most important questions in the theory of nonlinear wave equations is that for global existence of solutions. An essential tool is the Strichartz inequality for special solutions of the wave equation. In the last time different results were proved generalizing the classical one of Strichartz.

In the present lecture $L_p - L_q$ estimates are proved for the solutions of strictly hyperbolic equations of second order with time dependent coefficients, where these are unbounded or vanishing at infinity. In the first step the WKB method is applied to the construction of a fundamental system of solutions for ordinary differential equations depending on a parameter. In a second step the method of stationary phase yields the asymptotical behaviour of Fourier multipliers with nonstandard phase functions depending on a parameter.

OSCILLATION CRITERIA FOR PERTURBED NONLINEAR DIFFERENTIAL EQUATIONS

Yuri Rogovchenko, Kyiv, Ukraine

AMS Class.: 34C10

We present new oscillation criteria for the second order nonlinear perturbed differential equation

$$[r(t)x'(t)']' + p(t)x'(t) + Q(t, x(t)) = P(t, x(t), x'(t))$$

(1)

obtained by the integral averaging technique which goes back as far as to classical results of Wintner, Hartman, and Kamenev giving sufficient conditions for oscillation of the linear equation

$$x''(t) + q(t)x(t) = 0.$$  

(2)

The obtained criteria are of a high degree of generality and they extend and unify a number of existing results.

We also study asymptotic behavior of solutions for the forced nonlinear differential equation

$$[r(t)x'(t)']' + p(t)x'(t) + Q(t, x(t)) = P(t, x(t), x'(t)) + e(t).$$

(3)
GREEN’S FORMULA AND THEOREMS ON COMPLETE COLLECTIONS OF ISOMORPHISMS FOR GENERAL ELLIPTIC BOUNDARY VALUE PROBLEMS FOR DOUGLIS-NIRENBERG SYSTEMS

Inna Roitberg, Yakov Roitberg, Chernigov, Ukraine

AMS Class.: 35J55 (35J40)

The Green’s formula, similar to the formula of M. Shechter for one equation with normal boundary conditions, is obtained for the general boundary value problems for Douglis-Nirenberg elliptic systems.

The formally adjoint problem is studied, the theorem on its ellipticity is proved; the cokernel of given problem is described.

All these results are obtained also for parameter elliptic and parabolic problems for general system of equations.

The different theorems on complete collections of isomorphisms are found for the problems under consideration. Relations between these theorems are studied.

Different applications are considered. In particular, the questions on the smoothness of weak solutions of such problems in the whole domain are studied. These investigations were stimulated by works of M. S. Agranovich and A. N. Kozhevnikov in the spectral theory of such problems.

A-PRIORI ERROR ESTIMATES FOR CONVECTION DOMINATED DIFFUSION PROBLEMS

Mirko Rokyta, Praha, Czech Republic  Dietmar Kröner, Freiburg, Germany

AMS Class.: 65N15 (35J25, 76M25)

We study a higher order MUSCL type finite volume schemes applied to a linear convection dominated diffusion problem in a convex bounded domain Ω in \( \mathbb{R}^2 \). We work on a regular triangular mesh using the Engquist-Osher numerical flux. Although the original problem is linear, the numerical problem becomes non-linear, due to MUSCL type reconstruction/limiter technique.

For first order schemes, an a-priori estimate of order \( h \) for the discrete \( L^2 \) and \( \varepsilon \)-weighted \( W^{1,2} \) norms are obtained, while for second order schemes this estimate is improved to \( h^2 \). In such a way, the convergence rate for higher order MUSCL type schemes is theoretically justified.
CYCLICITY OF CENTER AND FOCUS FOR SOME CUBIC SYSTEMS
Valery Romanovski, Minsk, Belarus

AMS Class.: 34C05

Two problems, closely related to the second part of the 16-th Hilbert problem, namely, the centre-focus problem and the problem of bifurcation of small-amplitude limit cycles from a singular point (the cyclicity problem) for polynomial vector fields \( i\dot{w} = w - \sum_{i+j=1}^{n} a_{ij} w^{i} \bar{w}^{j} \) \( (w = x + iy) \) are considered. With every such system we associate a monoid ring (polynomial subalgebra) and show that these problems are equivalent to algebraic ones of finding the variety and a basis and of ideals in the monoid rings. The developed method can be applied to resolving of the center-focus and cyclicity problems for 6-parametric polynomial vector field and for wide families of other polynomial vector fields with "simple" structure of corresponding monoids. As an example of application of the method the following proposition is proved.

**Theorem.** The cyclicity of the origin of system

\[ i\dot{w} = w(1 - a_{10} w - a_{20} w^{2} - a_{11} w \bar{w}) \]

with respect to its space of coefficients is less or equal 2.

ON PERIODIC SOLUTIONS OF DIFFERENTIAL EQUATIONS WITH LIPSCHITZIAN RIGHT–HAND SIDES
Andrei Ronto, Kiev, Ukraine

AMS Class.: 34B10 (34B15)

Differential equations of the form

\[ x^{(N)} = F(x), \]

where \( N \geq 1, x : R \to X \) and \( F : X \to X \), are considered. Here \( (X, \preceq, |\cdot|) \) is a partially ordered Banach space, endowed with an ‘abstract norm’ \(|\cdot| : X \to X^{+} := \{x \in X \mid x \geq 0\}\), such that, for all \( \{x, x_{1}, x_{2}\} \subseteq X, \mu \in R \), it holds that \(|x| = 0 \iff x = 0, |\mu x| = |\mu| |x|\), and \(|x_{1} + x_{2}| \preceq |x_{1}| + |x_{2}|\), \( X \times X \subseteq \leq \) being the order in \( X \).

Under the assumption that the Lipschitz type condition,

\[ |F(x_{1}) - F(x_{2})| \preceq L |x_{1} - x_{2}| \quad (\forall \{x_{1}, x_{2}\} \subseteq X) \]

holds true with a linear and continuous operator \( L : X^{+} \to X \), some results on periods of periodic solutions of such equations are obtained. A review of related facts is given.
ON IRREGULARITY OF SOLUTIONS OF MONGE – AMPÈRE EQUATIONS

Olga Rozanova, Belgorod, Russia

AMS Class.: 35L70

On the cilinder $B = R^1 \times S^1_a$ (or on the torus $T = S^1_b \times S^1_a$) we consider the Monge – Ampère equation of mixed type in the form

$$z''_{xx} z''_{yy} - (z''_{xy})^2 = Q(x, y, z, z'_x, z'_y).$$

Equations of such type are used in many geometric and geophysic applications.

**Theorem.** Let the following conditions hold:

1) $\int_{S^1} Q \, dy \geq \mu = \text{const}$;  
2) there is a value of $x = x_0$, that $\beta^2 \geq -\mu \alpha / 2$ (if $\mu \neq 0$) or $\beta \neq 0$ (if $\mu = 0$),

where $\alpha = \int_{S^1} (z'_y(x_0, y))^2 \, dy$, $\beta = \int_{S^1} z'_y(x_0, y) z''_{xy}(x_0, y) \, dy$.

Then the solution of ($*$) can not be of class $C^2(B)$ (or $C^2(T)$).

Also it is possible to estimate the region of the regularity from above.

ASYMPTOTIC PROPERTIES OF SOLUTIONS OF INFINITY ORDER DIFFERENCE EQUATIONS

Ewa Schmeidel, Poznań, Poland

AMS Class.: 39A10

We study difference equation

$$\Delta x_n = \sum_{i=1}^{\infty} a^i_n f(x_{n+i}) + b_n.$$  

We find sufficient conditions to insure that given $c \in R$ there exists a solution $x$ of Eq(1) such that $\lim x_n = c$. We find conditions under which every solution of Eq(1) is a sequence convergent in $R$, and we also find sufficient conditions to insure that every nontrivial solution of Eq(1) (if any) diverges to infinity.
A POSTERIORI ERROR ESTIMATES
FOR A NONLINEAR PARABOLIC EQUATION

Karel Segeth, Prague, Czech Republic

AMS Class.: 65M15 (65M20)

A posteriori error estimates form a reliable basis for adaptive approximation techniques for modeling various physical phenomena. The estimates developed recently in the finite element method of lines for solving a parabolic differential equation are simple, accurate, and cheap enough to be easily computed along with the approximate solution and applied to provide the optimum number and optimum distribution of space grid nodes.

The contribution is concerned with a posteriori error estimates needed for the adaptive construction of a space grid for solving an initial-boundary value problem for a nonlinear parabolic partial differential equation by the method of lines.

FAMILIES OF THREE-DIMENSIONAL
PHASE PORTRAITS
IN DYNAMICS OF A RIGID BODY

Maxim V. Shamolin, Moscow, Russia

AMS Class.: 34C05 (34C25)

The well-known Lyapunov function method for the determining of a character of the regular point stability in the systems on the plane (for example in the "centre-focus" problem) or investigating of the closed curves which consist the trajectories the system considered is the particular (or ajoint) case of the topographical Poincare systems method on the plane for the study of the existence and stability of the closed orbits and singular points.

In this work the generalization of topographical systems method in many dimensional spaces is considered. Furthermore, the odd-dimensional and even-dimansional cases of the application of new method considered have the different nature.

A lots of the applications are arising from the interacting with a medium rigid body dynamics. Furthermore, the important families of the phase patterns in three-dimensional spaces are obtained. Thus families have a lots of non-trivial properties.
GREEN’S FORMULA FOR GENERAL ELLIPTIC PROBLEMS WITH A SHIFT WITHOUT NORMALITY ASSUMPTION

Zinovi Sheftel, Chernigov, Ukraine

AMS Class.: 35R10 (35J40, 41A65)

Let $G \subset \mathbb{R}^n$ be a bounded domain with boundary $\Gamma \in C^\infty$, $\alpha : \Gamma \rightarrow \Gamma$ be a diffeomorphism, where $\alpha(\alpha x) = x$ for any $x \in \Gamma$. The transformation $\alpha$ defines in a natural way the shift operator $J$ on $\Gamma$ which transforms any function $u$ defined on $\Gamma$ into the function $Ju$ according to the formula $Ju(x) = u(\alpha x)$. In $G$ a linear differential expression $L(x, D)$ is assigned, ord$L = 2m$; on $\Gamma$ linear differential expressions $B_{jr}(x, D)$ ($j = 1, \ldots, 2m; r = 1, 2$) satisfying certain matching condition are assigned. We study the boundary elliptic problem with a shift

$$Lu(x) = f(x) \quad (x \in G), \quad (1)$$

$$B_{j1}u(x) + JB_{j2}u(x) = \phi_j(x) \quad (x \in \Gamma; j = 1, \ldots, 2m). \quad (2)$$

In previous author’s papers the Green’s formula was proved under additional assumption of $\alpha$-normality of the boundary conditions (2). In this contribution we prove a similar formula in the case of arbitrary boundary conditions with a shift. Some applications of this formula are considered.

LOCAL DYNAMICS OF SOLUTIONS TO NON-LINEAR HYPERBOLIC EQUATIONS

Armen Shirikyan, Moscow, Russia   Leonid Volevich, Moscow, Russia

AMS Class.: 35L25 (35B40)

We consider a high-order non-linear hyperbolic equation that is a small perturbation of an equation with constant coefficients. It is assumed that the unperturbed equation has a characteristic polynomial whose roots, with respect to the variable dual to time, are outside an open strip containing the imaginary axis. We prove that the homogeneous equation possesses the property of exponential dichotomy, i.e., there are two manifolds $\mathcal{M}^+$ and $\mathcal{M}^-$ in the phase space $\mathbb{E}$ of the equation in question such that the following assertions take place: 1) any solution with initial data on $\mathcal{M}^\pm$ decays exponentially as $t \to \pm\infty$; 2) the manifolds $\mathcal{M}^+$ and $\mathcal{M}^-$ intersect only at the origin and are weakly differentiable at this point; 3) the phase space $\mathbb{E}$ can be represented as the direct sum of the corresponding tangent spaces. Under the additional condition that the roots belong to the left half-plane we prove that the zero solution is asymptotically stable as $t \to +\infty$.

We consider also the case of a non-homogeneous equation whose right-hand side is a uniformly bounded function of time and establish the existence of a uniformly bounded solution. On condition that the right-hand side is almost periodic in time, we prove the almost periodicity of the constructed solution.
SOLVABILITY OF NONLINEAR DATA
ASSIMILATION PROBLEMS IN A SCALE OF
HILBERT SPACES AND NUMERICAL ALGORITHMS

V. Shutyaev, Moscow, Russia

AMS Class.: 35B37 (65M30)

A class of evolution data assimilation problems for the state equation involving a non–linear operator is considered. It is required to determine an ‘initial value function’ so that the solution of the problem minimize a functional related to observational data. The necessary optimality condition is obtained in the form of a set of equations which link the main and the adjoint states, and some additional equalities. The theorems on solvability of this system are proved in a scale of Hilbert spaces. The regularity of solutions is studied. A class of iterative algorithms for solving the system derived is considered, the convergence conditions of these algorithms are studied, and the convergence rate estimates are derived. Numerical examples are presented.

NONLOCAL REDUCTION PROBLEM IN GENERAL
MATRIX KADOMTSEV–PETVIASHVILI HIERARCHY

Yu. M. Sidorenko, Lviv, Ukraine

AMS Class.: 35B58

The problem of reduction seems to be one of the most important in the theory of integrable dynamical systems. Different aspects of this problem were discussed in lots of well-known papers by famous scholars researching the theory of solitons.

In recent papers by Konopelchenko B., Sidorenko Yu., Strampp W. and ¨Ovel, W. the idea of reductions to non-local submanifold has been generalized to reduce soliton equations in multi-dimensions to simpler integrable partial differential equations. In particular, for the Kadomtsev-Petviashvili hierarchy the space-derivative \( (\sum_i \Phi_i \Psi_i)_x \) of squared eigenfunctions (i.e. the products of eigenfunctions \( \Phi_i \) and adjoint eigenfunctions \( \Psi_i \)) associated with linear scattering problems represents a symmetry generator. Hence, using any symmetry \( U_{t_N} \) of the KP-hierarchy the constraint \( U_{t_n} = (\sum_i \Phi_i \Psi_i)_x \) is compatible with all (2+1)-dimensional flows of the KP-hierarchy, thus leading to a hierarchy of commuting integrable equations in (1+1)-dimensions. In particular, the resulting system for \( t_N = x \) was shown to represent the multi-component AKNS hierarchy, the use of the next higher KP symmetries lead to multi-component versions of the hierarchies described by Yajima-Oikawa, Melnikoov, Drinfeld-Sokolov and Kuperschmidt.
ON DIFFERENT TYPES OF NONLINEAR PARABOLIC FUNCTIONAL DIFFERENTIAL EQUATIONS

László Simon, Budapest, Hungary

AMS Class.: 35K55 (35K60)

It will be considered weak solutions of initial-boundary value problems for the equation

$$\begin{align*}
D_t u(t, x) + \sum_{|\alpha| \leq m} (-1)^{|\alpha|} D^\alpha_x [f_\alpha(t, x, ..., D^\beta_x u(t, x), ...)] + \\
\sum_{|\alpha| \leq m-1} D^\alpha_x [H_\alpha(u)](t, x) = F(t, x), \quad (t, x) \in Q_{T_0} = (0, T_0) \times \Omega
\end{align*}$$

where $|\beta| \leq m$, $\Omega \subset \mathbb{R}^n$ is a bounded domain and $f_\alpha$, $H_\alpha$ may be quickly increasing in $u$. The operators $H_\alpha$ have general form such that they cover different types of delay operators (including delay in the boundary condition); it will be given several examples.

SOME EXAMPLES FOR THE EXTENDED USE OF THE PARAMETRIC REPRESENTATION METHOD

Peter L. Simon, Henrik Farkas, Budapest, Hungary

AMS Class.: 58F14 (34C23, 35B32)

The Parametric Representation Method had been applied successfully to construct bifurcation diagrams relating to equilibria of dynamical systems whenever the equilibria are determined from a single equation containing two control parameters linearly. The Discriminant–curve (that is the saddle–node bifurcation curve parametrized by the state variable remained after the elimination) is the base of this method, as it had been shown. The number and even the value of the stationary state variables can be derived from that.

Here we show some possible extensions of the method via three examples.

a.: Nonlinear parameter dependence
b.: Systems of equations for equilibria
c.: Reaction–diffusion equations, condition for multistationarity.

Similarly to the above simple case, the PRM provides us with information about the stationary solutions. Although some features do not remain valid for these extensions.
LINEAR TIME–OPTIMAL PROBLEM
AS MARKOV MOMENT MIN–PROBLEM
WITH PERIODIC GAPS

G. M. Sklyar, Kharkov, Ukraine  I. L. Velkovsky, Kharkov, Ukraine

AMS Class.: 42A70

Let us consider the time-optimal problem for the oscillatory system
\[ \dot{x} = Ax + bu, \quad x \in \mathbb{R}^{2n+2}, \quad |u| \leq 1, \quad x(0) = y, \quad x(T) = 0, \quad T \to \min, \]
where \( \text{rank}(b, Ab, \ldots, A^{n-1}b) = n, \) \( \text{Spec}(A) = \{im_k, -im_k, k = 0, \ldots, n\}, \)
and \( \{m_k\}_{k=0}^{n} \) is non-decreasing \( p \)-periodic sequence of natural numbers.

The precise analytical method of solving the problem in the case when the functions \( \{\sin(m_0t), \cos(m_0t), \ldots, \sin(m_nt), \cos(m_nt)\} \)
form a Tchebycheff system (T-system) on some segment \([0, W]\) is obtained based on the investigation of the equivalent non-classical moment problem called the Markov trigonometric moment min-problem with periodic gaps. The introduced approach is based on investigation of the special subclass of the Caratheodory function class corresponding to given \( p \)-periodic law.

ABSTRACT EVOLUTION EQUATIONS
RELATED TO FLUID MECHANICS

Lech Sławik, Kraków, Poland

AMS Class.: 35Q

Following the well–known T.Kato’s papers we consider a quasilinear evolution equation of the form
\[ u' + A(t, u)u = f(t, u), \quad u(0) = u_0, \quad 0 \leq t \leq T, \]
where the unknown \( u \) takes values \( u(t) \) in a Banach space \( X, \) \( A(t, y) \) is a linear operator in \( X \) on \( t \) and certain \( y \in X, \) and \( f(t, y) \in X. \)

The aim of this communication is to present some new results related to local existence and qualitative properties of the solution.
DIFFERENTIAL EQUATIONS WITH COUPLED MULTIPLE INTEGRALS IN THERMOVISCOELASTICITY

Zdeněk Sobotka, Prague, Czech Republic

AMS Class.: 34A05

The thermoviscoelastic behaviour of isotropic bodies is described by the ordinary differential tensor equations with variable coefficients or coupled multiple integrals, respectively. It is shown that the deviatoric and volumetric relations \( \tilde{A}s_{ij} = \tilde{B}e_{ij}, \tilde{A}_v\sigma_M = \tilde{B}_V(\varepsilon_M - \beta T) \) with linear differential operators \( \tilde{A}, \tilde{B}, \tilde{A}_v \) and \( \tilde{B}_V \) can be expressed by multiple integrals. For instance:

\[
s_{ij}(t) = \frac{1}{R_1(t)} \left[ \int_{t_0}^t R_1(\tau_1) d\tau_1 \cdot \left\{ \int_{t_0}^{\tau_1} \frac{R_2(\tau_2)}{R_3(\tau_2)} d\tau_2 \left[ \int_{t_0}^{\tau_2} \tilde{B}e_{ij}(\tau_3) R_3(\tau_3) d\tau_3 + C_{3ij} \right] + C_{2ij} \right\} + C_{1ij} \right].
\]

(1)

Eliminating the tensor-valued constants of integration \( C_{Kij} \) and integrals again yields the differential equation for the stress deviator \( s_{ij} \):

\[
\frac{1}{R_3(t)} \frac{d}{dt} \left[ \left( \frac{R_3(t)}{R_2(t)} \right) \frac{d}{dt} \left( \frac{R_2(t)}{R_1(t)} \frac{d}{dt} \left( \frac{R_1(t)}{R_3(t)} s_{ij} \right) \right) \right] = \tilde{B}e_{ij}.
\]

(2)

ON A CLASS OF FUNCTIONAL BOUNDARY VALUE PROBLEMS

Svatoslav Staněk, Olomouc, Czech Republic

AMS Class.: 34K10

A natural generalization of the differential equation \( (g(x'))' = f(t, x, x') \) where \( g : \mathbb{R} \to \mathbb{R} \) is an increasing homeomorphism and \( f \) satisfies the Caratheodory conditions on \([a, b] \times \mathbb{R}^2\) is the functional differential equation

\[
(g(x'(t)))' = (Fx)(t)
\]

(1)

with a continuous operator \( F : C^1([a, b]) \to L_1([a, b]) \). Many existence results for (1) with \( g(t) = 1 \) and nonlinear functional boundary conditions were given for example by N. V. Azbelev, S. A. Brykalov, I. T. Kiguradze and L. F. Rakhmatullina.

Our boundary conditions for (1) are formulated by some continuous increasing functionals. The proofs of existence results are based on the topological degree theory and the Borsuk theorem.
REDUCTION AND EXACT SOLUTION OF THE KRAMERS EQUATION

Valerii Stognii, Kyiv, Ukraine

AMS Class.: 35K

We consider the two–dimensional Fokker–Planck equation which describes the motion of particle in fluctuating medium

\[
\frac{\partial u}{\partial t} = -\frac{\partial}{\partial x}(y u) + \frac{\partial}{\partial y}(V'(x)u) + \gamma \frac{\partial}{\partial y}(yu + \frac{\partial u}{\partial y}) \tag{1}
\]

where \( u = u(t,x,y) \) is the probability density, \( \gamma \) is a constant and \( V(x) \) is a potential. Equation (1) is known as Kramers equation [1].

By means of operators of invariance algebras and using the method of continuous one parameter point symmetry invariants we have made reduction and we have derived exact solutions of the Kramers equation.


OSCILLATIONS OF LINEAR DIFFERENTIAL EQUATIONS CAUSED BY DELAY ARGUMENTS

A. Szawiola, Poznań, Poland, J. Werbowski, Poznań, Poland

AMS Class.: 34K15

The purpose of this paper is to study the oscillatory properties of solutions of differential equations as follows:

\[ (-1)^{\sigma}L_n x(t) = \sum_{i=1}^{m} q_i(t)x(g_i(t)), \]

where \( n \geq 2, \sigma = 1, 2, L_n \) is defined by \( L_0 x = x, L_i x = \frac{1}{p_i} \frac{d}{dt}L_{i-1}x, p_n = 1 \). The functions \( p_i, q_k, g_k : \mathcal{R}_+ \rightarrow \mathcal{R}_+ = (0,\infty) \) are continuous with \( g_k(t) \leq t \) on \( \mathcal{R}_+ \), \( g_k(t) \rightarrow \infty \) as \( t \rightarrow \infty \), and \( \int_{-\infty}^{\infty} p_i(t)dt = \infty \) (i = 1, \ldots, n), (k = 1, \ldots, m).

A characteristic feature of this paper is the fact that the results obtained here are not valid for corresponding ordinary differential equations.
PERIODIC SOLUTIONS OF A DIFFERENTIAL INCLUSION OF ORDER 4n

Marko Švec, Bratislava, Slovakia

AMS Class.: 34B15 (34K10, 34A60)

The following boundary value problem will be discussed:
(1) \[ L_{4n} y + a(t)y \in F(t, y(\varphi(t))), \ n > 0, \ t \in J = [a, b], \]
(2) \[ L_i y(a) = L_i y(b), \ i = 0, 1, \ldots, 4n - 1, \]
where \[ L_0 y(t) = a_0(t)y(t), \]
\[ L_i y(t) = a_i(t)(L_{i-1} y(t))', \ i = 1, 2, \ldots, 4n - 1. \]
We assume that:
1. \[ F(t, y) : J \times R \to \text{nonempty convex compact subsets of } R = (-\infty, \infty); \]
2. \[ F(t, y) \] is upper semicontinuous on \( J \times R; \)
3. to each measurable function \( z(t) : J \to R \) there exists a measurable selector \( v(t) \in F(t, z(t)) \ a.e. \ on J; \)
4. \( a_k(t) = a_{4n-k}(t), k = 0, 1, \ldots, 2n - 1 \) on \( J, a(t) \neq 0 \) on \( J. \)

BARRIER METHOD FOR ELLIPTIC EQUATIONS WITH DECREASING NONLINEARITY

Tadie, Copenhagen, Denmark

AMS Class.: 35J70 (35J65, 34C10)

Let \( a > 1, p \in (1, 2) \) and \( D_a^p u := (r^a |u'|^{p-2} u')'. \) Let \( f \in C^1([0, \infty) \times (0, \infty)) \) be such that \( \forall r > 0 \ f(r, T)_+ := \max\{0, f(r, T)\} \) is non increasing in \( T. \) Consider the problem (Ea) which reads
\[ E_a u := D_a^p u + r^a f(r, u)_+ = 0, \quad r > 0; \quad u(0) > 0; \quad u'(0) = 0. \]
A non increasing \( v \in C^1 := C^1(\mathbb{R}_+), \) piecewise \( C^2 \) is said to be a supersolution (subsolution) of (Ea) if \( E_a v \geq 0 \ (\leq 0 \ \text{respect.}). \) A supersolution \( v \) and a subsolution \( w \) of (Ea) will be said \( \text{Ea-compatible} \) if \( 0 \leq w \leq v \) and \( w' \leq v' \leq 0 \) in \( \mathbb{R}_+. \)
In this note we find conditions under which the existence of compatible \( v \) and \( w \) of (Ea) implies the existence of a classical solution \( (C^2) \) \( u \) of (Ea) such that \( w \leq u \leq v. \) We also find conditions under which the existence of such supersolution (respectively such subsolution) leads to the existence of a similar solution.
**SUSPENSION BRIDGES – EXISTENCE AND UNIQUENESS OF SOLUTION**

_Gabriela Tajčová, Plzeň, Czech Republic_

AMS Class.: 35B10, 70K30, 73K05

One of the very problematic and not fully explained areas of mathematical modelling involves nonlinear dynamical systems, especially systems with a jumping nonlinearity. It can be seen that its presence brings into the whole problem unexpected difficulties and very often it is a cause of multiple solutions.

An example of such a dynamical system can be a suspension bridge. The nonlinear aspect is caused by the presence of supporting cable stays which restrain the movement of the center span of the bridge in a downward direction, but have no influence on its behaviour in an opposite direction.

In our research, we consider models describing the behaviour of the suspension bridge by a single beam equation, or by a system of two coupled equations of “string-beam” type, respectively, and we try to determine under what conditions the existence of a unique stable solution is guaranteed. We use two different attitudes. The first one is based on the Banach contraction theorem which needs some restrictions on the bridge parameters and the second one works in greater generality but with an additional assumption of sufficiently small external forces.

**ASYMPTOTIC EXPANSIONS FOR EIGENVALUES AND EIGENFUNCTIONS OF BOUNDARY VALUE ELLIPTIC PROBLEMS WITH RAPIDLY OSCILLATING COEFFICIENTS**

_Olexiy Teplinskyy, Kyiv, Ukraine_

AMS Class.: 35B27 (35C20, 35P05)

The homogenization theory deals with problems in the mechanics of strongly nonhomogeneous media, so the coefficients of equations have, in general case, the form $a(x, x/\varepsilon)$, here $\varepsilon > 0$ is a small parameter. The methods for the construction of asymptotic expansions, with respect to the powers of $\varepsilon$, for eigenvalues and eigenfunctions of such elliptic self-adjoint problems were worked out in the ‘80s by O. A. Oleinik, A. S. Shamaev, and G. A. Yosifian. But hitherto these methods have been in force only for the case of a simple spectrum of the corresponding homogenized problem. T. A. Melnyk has shown that certain symmetry relations imposed on the coefficients allow to overcome this obstacle.

The new result is a discovery of certain symmetry present in the algorithm itself irrespective of the form of coefficients. This symmetry lies in a self-adjointness of differential operators appearing. Using this fact the periodical problem (it is the simplest problem of such a type because it does not draw necessity to study the boundary layers) is solved completely.
SOME NEW SOLUTIONS IN A PROBLEM ABOUT MOTION OF GYROSTATE IN A MAGNETIC FIELD

Natalia V. Tkachenko, Donetsk, Ukraine

AMS Class.: 34C30 (70E15)

In a problem about gyrostate’s motion in a magnetic field with account of Barnett — London effect a new exact solutions describing some classes of precession motions are investigated. The motion’s equations of gyrostate are the system of ordinary differential equations.

Being based on a general method of research of precession, in which the determining role is played by types of motions, for this problem the exact solutions and conditions of their existence are received. In particular, these solutions describe precession isoconical motions of general kind and Bressan-Hess precession motions. Some new cases of integration of motion’s equations are obtained in the case, when the gyrostate’s motion is the precession motion of general kind about vertical vector.

ON ALMOST PERIODIC SYSTEMS WITH BOUNDED SOLUTIONS

Viktor I. Tkachenko, Kyiv, Ukraine

AMS Class.: 34C27 (34A30)

Let us consider the system of linear differential equations

$$\frac{dx}{dt} = A(t)x,$$

where $x \in \mathbb{C}^n$, $A(t)$ is the $n$-dimensional skew-adjoint matrix, $A(t) + A^*(t) = 0$, $t \in \mathbb{R}$, and we also suppose that function $A(t)$ is Bohr almost periodic with frequency module $\mathcal{F}$. Let $X_A(t)$ be a fundamental matrix for system (1).

Theorem 1. In any neighbourhood of the matrix-function $A(t)$ (in uniform on the real axis topology) there exists a skew-adjoint matrix-function $C(t)$ such that $C(t)$ and $X_C(t)$ are almost periodic with frequencies belonging to a rational hull of frequency module $\mathcal{F}$.

Theorem 2. Systems (1) with the $k$-dimensional frequency basis of almost periodic function $A(t)$ whose solutions are not almost periodic form a subset of the second category (an intersection of a countable set of everywhere dense subsets) in the space of all systems with $k$-dimensional frequency basis of $A(t)$.

NUMERICAL ANALYSIS OF HIGH-TEMPERATURE
STRAINS AND STRESSES IN SUPERALLOYS

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AMS Class.: 73F05 (73F15, 65M99)

The most effective way how the creep resistance of metals at high temperature can be improved is their reinforcement by hard particles. Unlike the case of metal matrix composites, in the case of superalloys the particles can precipitate in the matrix during the heat treatment.

The geometric configuration changes, their rates \(v\) and corresponding development of strains and stresses can be calculated from the following system of equations:

(i) equations of the physical principle of virtual displacement rates for materials consisting of two phases,
(ii) constitutive equations of the nonlinear Maxwell viscoelastic model (for elastic deformation in hard particles \(\Omega_1, \ldots, \Omega_n\) and creep flow in slip directions in a matrix \(\Omega_0\)),
(iii) equations for the generation and dynamic recovery of dislocations at matrix/particle interfaces.

Let \(V\) be the space of configuration change rates (certain subspace of the Sobolev space \(\prod_{i=0}^n W^1_2(\Omega_i, \mathbb{R}^3)\) =, special interface discontinuities must be allowed) and \(H = \prod_{i=0}^n L^2_2(\Omega_i, \mathbb{R}^3)\). For any time interval \(I\) and admissible initial status the Rothe method with special linearized iterative scheme gives \(v \in L^\infty(I, V) \cap C_L(I, H)\) and \(\dot{v} \in L^\infty(I, H)\) (time derivatives). Numerical results obtained with help of the author’s FEM software package C= DS will be demonstrated.

ON SOME EFFECTIVE METHOD OF
SOLVING OF MIXED BOUNDARY VALUE
PROBLEMS FOR THE PARTIAL
DIFFERENTIAL EQUATIONS

Nina Virchenko, Kyiv, Ukraine

AMS Class.: 35B30

Among different methods of solving of mixed boundary value problems of mathematical physics the method of dual (triple, \(N\)-ary) integral (or series) equations is proved to be the most effective one contemporary analytical method. This method is widely used last years [1].

System of \(m\) of \(N\)-integral equations:

\[
\int_{\Delta_i} \sum_{k=1}^m a_{ik}^j(\tau) \varphi_k(\tau) K^i_j(\tau, x) d\tau = f_i^j(x),
\]
Some new types of dual (triple) equations with generalized Legendre’s functions \( P_{-\frac{1}{2}+ir}(\sin \alpha) \), with generalized Watson functions are solved and investigated. These results have made it possible to solve serial new mixed boundary value problems in the cases of special curvilinear coordinates (confluent, ellipsoidal, toroidal, bipolar, spherical etc.).


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ON THE QUALITATIVE PROPERTIES OF SOLUTIONS OF QUASILINEAR PERIODIC MIXED PROBLEMS FOR PARABOLIC EQUATION

Mikhail P. Vishnevskii, Novosibirsk, Russia

AMS Class.: 35K45 (35K50)

In this communication we study the behavior at the large time of solutions to a boundary value problem for quasilinear, periodic, parabolic equations. The class of parabolic equations include, in particular, reaction-diffusion equations and more others parabolic equations which are of great importance of applications.

After some additional assumptions, we prove the existence of maximal and minimal solutions to the problem. We describe the attractor of asymptotically stable periodic solutions and prove the existence of unstable periodic solutions on the boundary of attractor set. On the set of all periodic solutions we introduce the partial order.

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ON THE QUALITATIVE AND NUMERICAL INVESTIGATIONS OF A HYDRODYNAMIC SYSTEM DESCRIBING MULTICOMPONENT MEDIA WITH ONE RELAXING COMPONENT

Vsevolod A. Vladimirov, Kyiv, Ukraine

AMS Class.: 35B30

A system of PDE describing non-linear waves propagation in relaxing media is considered. Our purpose is to study the wave patterns formation. This is achieved by the analytical means for special class of self-similar solutions, that satisfy a system of ODE obtained from the initial one by group-theory reduction. Solutions of the ODE system were taken as initial values in solving Cauchy problem for the initial PDE system. Of special interest is the behavior of solutions, corresponding
to the homoclinic loops. These solutions were shown to manifest the features of solitary waves. Besides, they play role of intermediate asymptotics for a large class of Cauchy data simulating pulse loads afteraction in relaxing media.

**ON PRODUCTS OF NON-COMMUTING OPERATORS IN BANACH SPACES**

Frank Weber, Halle (Saale), Germany

AMS Class.: 35K65 (47A50)

Products of non-commuting sectorial operators $A, B$ in a Banach space $X$ are investigated to provide functional-analytical tools for the treatment of multiplicative perturbations and degenerations.

Our considerations are based on the “Operator Sum Method” which has been applied to products i.e. by A. Favini. By means of this method, inverse operators $(I + AB)^{-1}$ are constructed using an extended Dunford functional calculus. In order to show the convergence of the involved cotour integrals, A. Favini employed real interpolation spaces between $X$ and $\mathcal{D}(A)$.

Combining the approach of A. Favini with the theory of operators with bounded imaginary powers (in UMD - Banach spaces), we obtain analogous results in the underlying Banach space $X$ itself. Namely, it is shown:

If sectorial operators $A, B$, acting in an UMD-Banach space $X$, admit bounded imaginary powers, satisfy a parabolicity condition, and fulfill certain commutator estimates, then $\nu + AB$ is also sectorial for a sufficiently large $\nu \geq 0$.

Examples show that the result can be applied to degenerate parabolic problems.

**OSCILLATIONS OF SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS WITH ADVANCED ARGUMENTS**

J. Werbowski, Poznań, Poland, A. Szawiola, Poznań, Poland

AMS Class.: 34K15

In this paper we consider the influence of advanced arguments on the oscillatory behaviour of solutions of differential equations as below

$$(-1)^\sigma L_n x(t) \text{sgn} x(t) = q(t) \prod_{i=1}^{m} |x(g_i(t))|^{\alpha_i},$$

where $n \geq 2, \sigma = 1, 2$, $L_n$ is defined by $L_0 x = x$, $L_i x = \frac{1}{p_i} \frac{d}{dt} L_{i-1} x$, $p_n \equiv 1$. The functions $q, p_i, g_k : \mathcal{R}_+ \rightarrow \mathcal{R}_+ = (0, \infty)$ are continuous with $g_k(t) \geq t$ on $\mathcal{R}_+$, and $\int_{\infty} p_i(t) dt = \infty$, $\alpha_k \in \mathcal{R}_+$, $\alpha_1 + \cdots + \alpha_m = 1$, $(i = 1, \ldots, n)$, $(k = 1, \ldots, m)$.

The main results are not valid for differential equations without deviated arguments.
ASYMPTOTIC BEHAVIOUR OF SOLUTIONS OF PARTIAL DIFFERENCE INEQUALITIES

Patricia J. Y. Wong, Nanyang Technological University, Singapore

AMS Class.: 39A10

We shall offer criteria for the nonexistence of eventually positive (negative) solutions of the following partial difference inequalities

\[ y(m-1,n) + \beta(m,n)y(m,n-1) - \delta(m,n)y(m,n) + P(m,n,y(m+k,n+\ell)) \]

\[ \leq (\geq) Q(m,n,y(m+k,n+\ell)), \quad m \geq m_0, \quad n \geq n_0 \quad (1) \]

\[ y(m-1,n) + \beta(m,n)y(m,n-1) - \delta(m,n)y(m,n) + \sum_{i=1}^{\tau} P_i(m,n,y(m+k_i,n+\ell_i)) \]

\[ \leq (\geq) \sum_{i=1}^{\tau} Q_i(m,n,y(m+k_i,n+\ell_i)), \quad m \geq m_0, \quad n \geq n_0 \quad (2) \]

where \(\beta(m,n), \delta(m,n)\) satisfy \(\beta(m,n) \geq \beta > 0\) and \(\delta(m,n) \leq \delta (> 0)\) for all large \(m\) and \(n\). It is noted that \(\delta(m,n)\) is not required to be positive eventually.

Our results readily give rise to oscillation theorems for the difference equations (1) and (2). Several examples which dwell upon the importance of our results are also included.

INFINITE SYSTEM OF REACTION-DIFFUSION EQUATIONS.

COAGULATION-FragmentATION MODEL WITH DIFFUSION

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AMS Class.: 35K57 (92E20)

The coagulation-fragmentation model with diffusion reads

\[ \frac{\partial u_i}{\partial t} - d_i \Delta u_i = \frac{1}{2} \sum_{j=1}^{i-1} (a_{i-j,j} u_{i-j} u_j - b_{i-j,j} u_i) \quad \text{in} \quad \Omega \times (0,T) \]

\[ - \sum_{j=1}^{\infty} a_{i,j} u_j + \sum_{j=1}^{\infty} b_{i,j} u_{i+j}, \quad i = 1, 2, 3, \ldots \]

with initial and 'no-flux' boundary conditions under the convention that for \(i = 1\) the first and the second term on the r.h.s. do not appear. The system describes the dynamics of clusters growth such that \(u_i\) represents the concentration of \(i\)-clusters.
which consist of $i$ identical elementary particles, $i = 1, 2, 3 \ldots$. In most of physically relevant situations $a_{i,j} \to \infty$ as $i, j \to \infty$ which leads to many interesting analytical problems e.g. break-down of mass conservation (so-l-gel transition).

The main questions concern existence and uniqueness of solutions as well as long time behaviour and mass preservation. A survey of recent results obtained by the author with Ph. Bénilan and with Ph. Laurençot is provided.

**ASYMPTOTIC AND OSCILLATION BEHAVIOUR OF SOLUTIONS OF NONLINEAR DIFFERENTIAL EQUATIONS**

**Aleksandra Wyrwińska, Poznań, Poland**

AMS Class.: 34K15

The purpose of this paper is to present the asymptotic and oscillation behaviour of functionally integrable solutions of the following differential equations

$$x^{(n)}(t) + f(t, x(t)) = h(t)$$

for $n \geq 2$, where the functions $h : R_+ \to R, f : R_+ \times R \to R$ are continuous.

**ON OPERATORS AND OPERATOR EQUATIONS IN BOCHNER SPACE**

**Nina A. Yerzakova, Khabarovsk, Russia**

AMS Class.: 47H09 (46E35, 47B07, 47B38)

In this paper estimates for measure of noncompactness of any sets in space of integrable functions by Bochner are obtained, new class of condensing operators is given and solvability of ordinary operator equation is proved. An extension of any results by J. L. Lions and Yu. A. Dubinskii to noncompact operators is secured. The main result may be regarded as a generalization of Fredholm alternative to noncompact and nonlinear operators, i.e. in the particular case, the problem formulated by S. I. Pohozaev and J. Nečas is solved.

**ON A TRAJECTORY’S AVOIDING PROBLEM IN CONFLICT CONTROLLED PROCESSES**

**Lev Yugai, Tashkent, Uzbekistan**

The avoiding problem for trajectories of nonlinear conflict controlled systems is considered [1]. Every trajectory is formed by two sides (players) having antagonistic goals and it is the absolutely continuous differential equations with control parameters.
For considered systems sufficient conditions are obtained that provide avoiding from given terminal set all trajectories with initial positions not belonging to terminal set.

A way of construction of avoid control is offered. All results are illustrated on examples.


STABILITY AND PERSISTENCE OF TIME DELAYS MODEL IN POPULATION DYNAMICS

A. A. S. Zaghrout, Cairo, Egypt

AMS Class.: 34K15 (34K20, 34D20, 45J05)

Sufficient conditions are obtained for the global asymptotic stability in a competition system of four competing species modelled by autonomous delay differential equations. Also sufficient conditions for the persistence and stability of an autonomous integrodifferential system are given.

STABILITY OF NEUTRAL DELAY DIFFERENCE SYSTEMS

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AMS Class.: 39A11

Consider the neutral delay difference systems of the general form:

\begin{equation}
\Delta(D(n,x_n)) = f(n,x_n), \quad n \in Z^+,
\end{equation}

where \(Z^+\) denotes the set of non-negative integers, \(x \in R^k\) with some positive integer \(k\), \(\Delta\) is the forward difference operator, let

\[C = \{\varphi : \{-r,-r+1,\ldots,-1,0\} \to R^k\} \text{ with some } r \in Z^+.
\]

For \(\varphi \in C\), define \(\|\varphi\| = \max_{s=-r,-r+1,\ldots,0} |\varphi(s)|\) with \(\cdot\) a norm in \(R^k\); while \(x_n \in C\) is defined as \(x_n(s) = x(n+s)\) for \(s = -r,-r+1,\ldots,0\).

Then two kinds of stability criteria for (1) are established in this paper. One is by means of the discrete Liapunov functionals; while the other is with the discrete Liapunov functions. Both make use of Razumikhin techniques.
ON BOUNDARY VALUE PROBLEMS WITH REFLECTION OF ARGUMENT

Mirosława Zima, Rzeszów, Poland

AMS Class.: 34K10

We will consider two-point boundary value problems for the following differential equation:

\[ x''(t) = f(t, x(t), x(-t)), \quad t \in [-1, 1]. \]

Using the fixed point theorem in partially ordered Banach spaces and the properties of a spectral radius of linear bounded operator we will show existence and uniqueness results when the function \( f \) satisfies a Lipschitz condition.

GLOBAL GENERALIZED SOLUTIONS FOR THE EQUATIONS OF A VISCOUS HEAT-CONDUCTING GAS ONE-DIMENSIONAL MOTION

Alexander A. Zlotnik, Andrew A. Amosov, Moscow, Russia

AMS Class.: 35D (35Q, 76N)

Nonhomogeneous initial-boundary value problems are considered for the composite type system of quasilinear equations

\[ \eta_t = u_x, \]
\[ u_t = (\nu u_x - p)_x + g, \quad p = k \rho \theta, \quad \rho = 1/\eta, \]
\[ c_v \theta_t = (\lambda \rho \theta_x)_x + (\nu u_x - p) u_x + f. \]

The initial data are only such that the total initial energy and entropy are finite whereas the initial specific volume is bounded from above and from below by positive constants. Boundary data, free terms, and coefficients are also nonsmooth (particularly, discontinuous) functions.

Not only the global existence of the generalized solutions for the problems but also their uniqueness and strong continuous dependence on data are proved.

The work is partially supported by the RFBR (grant 97–01–00214).
ENLARGED ABSTRACTS
STABILITY THEOREMS FOR NONLINEAR FUNCTIONAL DIFFERENTIAL EQUATIONS

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AMS Class.: 34K20

Consider nonlinear FDE with finite delay

(4) \[ \dot{x} = f(t, x_t), \]

where \( f : G^h_{H} \rightarrow \mathbb{R}^n, \ G^h_{H} = \mathbb{R}^n \times \Omega^h_{H}, \ \mathbb{R}^n = [0, \infty), \ \Omega^h_{H} \) is the open \( H \) - ball in the Banach space \( C_h \) of continuous functions \( \varphi : [-h, 0] \rightarrow \mathbb{R}^n \) with supremum norm= \( \| \cdot \| \) will denote a norm in \( \mathbb{R}^n \). Let \( \mathcal{K} \) be a set of all continuous strictly increasing functions \( a : \mathbb{R}^n \rightarrow \mathbb{R}^n, \ a(0) = 0, \) and \( \mathcal{A}^h_{H} = \{ \varphi \in C_h : \| \varphi \| \leq R|\varphi(0)| \} \). Main theorems are formulated as follows.

Suppose that there exist constants \( d_0 > 1, m_0 > 0 \) and an integrable function \( M_0 : \mathbb{R}^n \rightarrow \mathbb{R}^n \) such that \( \int_{t_1}^{t_2} M_0(t) \ dt \leq m_0(t_2 - t_1) \) for any finite segment \([t_1, t_2] \) and \( |f(t, \varphi)| \leq M_0(t)\|\varphi\|^{d_0} \) for each \( (t, \varphi) \in G^h_{H}. \) For some \( h_0 \geq h \) and \( R > 1 \) there exist functionals \( v, \Phi : G^h_{H} \rightarrow \mathbb{R} \) such that: 1) \( \dot{v}_{(1)}(t, \varphi) = \lim_{\Delta t \downarrow 0} (v(t + \Delta t, x(t + \Delta t, t, \varphi)) - v(t, \varphi))/\Delta t \leq \Phi(t, \varphi) \) for each \( (t, \varphi) \in G^h_{H}; \) 2) for some \( a, b \in \mathcal{K} v(t, \varphi) \leq b(\|\varphi\|) \) for each \( (t, \varphi) \in G^h_{H} \) and \( v(t, \varphi) \geq a(\|\varphi\|) \) for any \( t \geq 0 \) and \( \varphi \in \mathcal{A}^h_{R} \cap \Omega^h_{H} \); 3) there exist constants \( d > 1, m > 0 \) and an integrable function \( M : \mathbb{R}^n \rightarrow \mathbb{R}^n \) such that \( \int_{t_1}^{t_2} M(t) \ dt \leq m(t_2 - t_1) \) for any finite segment \([t_1, t_2], |\Phi(t, \varphi)| \leq M(t)\|\varphi\|^d \) for any \( (t, \varphi) \in G^h_{H}, |\Phi(t, \varphi) - \Phi(t, \psi)| \leq M(t)\|\varphi - \psi\|^d \) for all \( t \geq 0 \) and \( \varphi, \psi \in \Omega^h_{r}, 0 < r < H; \) 4) there exist constants \( T > 0, \beta > 0 \) and \( \delta > 0 \) such that for any \( t_0 \geq h_0, x_0 \in B_\beta \) and \( \Delta t \geq T \)

\[ \mathcal{I}(\Delta t, t_0, x_0) = \int_{t_0}^{t_0 + \Delta t} \Phi(t, x_0) \ dt \leq -2\delta|x_0|^d \Delta t. \]

Then the zero solution of (1) is uniformly asymptotically stable.

Suppose that for some \( h_0 \geq h, \ \beta > 0, \ \sigma > 0 \) and \( R > 1 \) there exist functionals \( v, \Phi : [\sigma, \infty) \times \Omega^h_{\beta} \rightarrow \mathbb{R} \) such that: 1) \( \dot{v}_{(1)}(t, \varphi) \geq \Phi(t, \varphi) \) for each \( (t, \varphi) \in [\sigma, \infty) \times \Omega^h_{\beta}; \) 2) for each \( t \geq \sigma \) and \( \eta, 0 < \eta < \beta, \) there exists \( \varphi \in \mathcal{A}^h_{R} \cap \Omega^h_{\eta} \) such that \( v(t, \varphi) > 0; \) 3) there exists \( b \in \mathcal{K} \) such that \( v(t, \varphi) \leq b(\|\varphi\|) \) for each \( (t, \varphi) \in [\sigma, \infty) \times \mathcal{A}^h_{R} \cap \Omega^h_{\beta}; \) 4) the functional \( \Phi \) satisfies condition 3) of Theorem 1 for \( (t, \varphi) \in [\sigma, \infty) \times \Omega^h_{\beta} \) and \( 0 < r < \beta; \) 5) there exist constants \( T > 0 \) and \( \delta > 0 \) such that for any \( t_0 \geq \sigma, x_0 \in B_\beta \) and \( \Delta t \geq T \)

\[ \mathcal{I}(\Delta t, t_0, x_0) \geq 2\delta|x_0|^d \Delta t. \]

Then the zero solution of the system (1) is unstable.
THREE–POINT SINGULAR BOUNDARY–VALUE PROBLEM
FOR A SYSTEM OF THREE DIFFERENTIAL EQUATIONS

Jaromír Baštinec

AMS Class.: 34B10 (34B15)

Let us consider the following Cauchy-Nicoletti problem

\[ y_i'(x) = \omega_i(x)y_i + f_i(x, y_1, y_2, y_3), \quad i = 1, 2, 3, \]  
\[ y_1(x_1^+) = A_1, \quad y_2(x_2^+) = A_2, \quad y_3(x_3^-) = A_3, \]  

where \( x \in I = [a, b], \ a = x_1 < x_2 < x_3 = b \) and \( A_i, i = 1, 2, 3 \) are real = constants.

Let us define

\[ I_i = I \setminus \{ x_i \}, \ \Omega = \cap_{i=1}^3 I_i. \]

Denote \( \alpha_i(x), \beta_i(x) \in C(I, \mathbb{R}), \ i = 1, 2, 3, \) \( \alpha_i(x) \leq y_i \leq \beta_i(x), \) and

\[ \Omega_i = \{ (x, y_1, y_2, y_3) : x \in I_i, (x, y_1, y_2, y_3) \in \Omega \}, \]

\[ \Omega_i = \{ (x, y_1, y_2, y_3) : x \in I_i, (x, y_1, y_2, y_3) \in \Omega \}, \ i = 1, 2, 3, \]  
\[ \alpha_i(x), \beta_i(x) \in C(I, \mathbb{R}), \ i = 1, 2, 3, \]

Denote, moreover, as \( \Omega_{ij}, i, j = 1, 2, 3, i < j \) the set

\[ \Omega_{ij} = \{ (x, y_i, y_j) : x \in I_k, k \in \{1, 2, 3\}, k \neq i, j, \alpha_s(x) \leq y_s \leq \beta_s(x), \ s = i, j \}. \]

Definition. A vector-function \( y(x) = (y_1(x), y_2(x), y_3(x)) \in C(I, \mathbb{R}) \) where

\( y_1(x) \in C^1(I, \mathbb{R}) \) is said to be a solution of the problem (1), (2) if it satisfies the system (1) on \( J \) and, moreover, \( y_1(x_1^+) = A_1, \ y_2(x_2^+) = A_2, \ y_3(x_3^-) = A_3. \)

Let us define

\[ F_i(y_1, y_2, y_3) = \omega_i(x)y_i - y_i' + f_i(x, y_1, y_2, y_3), \ i = 1, 2, 3. \]

Assume that

\[ F_1(\alpha_1(x), y_2, y_3) \cdot F_1(\beta_1(x), y_2, y_3) < 0 \]

if \( (x, y_2, y_3) \in \Omega_{2,3} \),

\[ F_2(y_1, \alpha_2(x), y_3) \cdot F_2(y_1, \beta_2(x), y_3) < 0 \]

if \( (x, y_1, y_3) \in \Omega_{1,3} \)

and

\[ F_3(y_1, y_2, \alpha_3(x)) \cdot F_3(y_1, y_2, \beta_3(x)) < 0 \]

if \( (x, y_1, y_2) \in \Omega_{1,2} \).

Let, moreover,

\[ |f_i(x, y_1, y_2, y_3) - f_i(x, z_1, z_2, z_3)| \leq M_i(x)|y_1 - z_1| + N_i(x)|y_2 - z_2| + P_i(x)|y_3 - z_3| \]
for any $(x, y_1, y_2, y_3), (z_1, z_2, z_3) \in \Omega_i$ where $M_i(x), N_i(x), P_i(x) \in C(I, \mathbb{R}), i = 1, 2, 3$,

$$|\omega_i(x)| > M_i(x) + N_i(x) + P_i(x), \ i = 1, 2, 3, \ x \in I_i$$

and for any $(x, y_1, y_2, y_3) \in \Omega_i$

$$\text{sign}\omega_1(x) = \text{sign}F_1(\beta_1(x), y_2, y_3),$$
$$\text{sign}\omega_2(x) = \text{sign}F_2(y_1, \beta_2(x), y_3),$$
$$\text{sign}\omega_3(x) = \text{sign}F_3(y_1, y_2, \beta_3(x)).$$

**Theorem.** Under above assumptions there is at least one solution $y(x) = (y_1(x), y_2(x), y_3(x))$ of the problem (1), (2) such that $\alpha_i(x) < y_i(x) < \beta_i(x)$ where $x \in I_i, \ i = 1, 2, 3.$


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ON THE DEFINITION OF THE DIRAC SYSTEM
BY TWO INCOMPLETE GIVEN COLLECTION
OF EIGENVALUES

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AMS Class.: 34B25 (45A05)

The problem of degenerating of classical Sturm-Liouville operator by two partially given spectra was considered in H. Hochstadt’s work [1].

Later on B.M. Levitan [2] and E.S. Panakhov proposed another solution to Hochstadt’s problem.

In this work authors consider the inverse problem for the canonical Dirac system
\begin{align}
y_2' + p(x)y_1 &= \lambda y_1 \\
-y_1' + q(x)y_2 &= \lambda y_2
\end{align}

by two partially given spectrum. It is shown that the nucleus of the operator
\begin{equation}
\phi(x, \lambda) - R(x)\phi(x, \lambda) + \int_0^x K(x, s)\phi(s, \lambda)ds
\end{equation}
of the transformation is generalized degenerate, where \( R(x) \) and \( K(x, s) \) are real matrices.

In the work, the analogue of the Hochstadt Theorem is also studied and an explicit formula for the difference of the potentials is obtained.


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QUASILINEAR ELLIPTIC PROBLEM

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AMS Class.: 35J65 (34B05, 34C10)

We are concerned with problems of the following type

\[ (P) \begin{cases} 
- \left( |u'|^{p-2} u' \right)' = g(x, u, u') \text{ in } (0,1) \\
    u(0) = u(1) = 0
\end{cases} \]

The operator \( - \left( |u'|^{p-2} u' \right)' \) is elliptic but nonlinear for \( p \neq 2 \). In the linear case \( (p = 2) \) several results are obtained concerning existence, multiplicity and oscillation properties of solutions (see for instance [3])

Some new results to the problem \((P)\) are submitted using quadrature method. This approach enable us to find a lower bounds on the number of solutions of \((P)\) and some supplementary informations; their periodicity, their oscillations, the number of their zeros. Some applications based on phase plane analysis, close this note.


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THE ABSTRACT CAUCHY PROBLEM IN PLASTICITY

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The questions of correct formulation and effective numerical solution of the plasticity boundary value problem are examined.

In the mathematical theory of plasticity the material is described by the constitutive relation for speeds

\[ \dot{\sigma}_{ij} = S_{ij}(\varepsilon, \dot{\varepsilon}) = C_{ijkm} \left( \dot{\varepsilon}_{km} - \dot{P}_{km}(\varepsilon, \dot{\varepsilon}) \right), \]

where \( \sigma_{ij} \) and \( \varepsilon_{ij} \) are the components of the Cauchy stress and strain tensors, respectively; \( P_{ij} \) are the components of the nonelastic part of strain tensor, \( C_{ijkm} \) are the components of the elasticity acoustic tensor, the above point is \( d/dt \) and \( t \in [0, 1] \) is the parameter of external loading. Here and in what follows we use the rule of summing from 1 to 3 over repeated indeces and the designation \( |A| = (A_{ij}A_{ij})^{1/2} \) for the modulus of \( 3 \times 3 \) matrix \( A \).

Let a solid occupy a bounded domain \( \Omega \subset \mathbb{R}^3 \) with the Lipschitz boundary \( \partial \Omega = \Gamma_1 \cup \Gamma_2, \Gamma_1 \cap \Gamma_2 = \emptyset, \) area \( (\Gamma_1) > 0. \) The plasticity boundary value problem is formulated as the evolution variational equation (EVE): the sought displacement corresponds to the abstract function \( u^*(t) = u^0(t) + u(t), \) where the piecewise smooth abstract function \( u^0 : [0, 1] \rightarrow W^{1,2}(\Omega, \mathbb{R}^3) \) with \( u^0(0) = 0 \) corresponds to the given external displacement on \( \Gamma_1, \) and unknown abstract function \( u : [0, 1] \rightarrow V^0 \) must satisfy the initial condition \( u(0) = 0 \) and the differential equation for every \( \phi \in V^0 \) and almost every \( t \in (0, 1) \)

\[
\int_{\Omega} S_{ij} \left( \varepsilon(u^0(t) + u(t)), \varepsilon(\dot{u}^0(t) + \dot{u}(t)) \right) \partial^j \phi_i \, dx = \int_{\Gamma_2} f_i(t) \phi_i \, d\gamma + \int_{\Gamma_1} \dot{F}_i(t) \phi_i \, d\gamma. \tag{1}
\]

Here \( V^0 = \{ \phi \in W^{1,2}(\Omega, \mathbb{R}^3) : \phi(x) = 0, x \in \Gamma_1 \} \) — is the set of admissible variations of displacement, it being supposed that \( \Gamma_1 = \text{const} (t), \) \( \varepsilon_{ij}(u) = \frac{1}{2} (\partial^j u_i + \partial^i u_j), \) \( \partial^j = \partial/\partial x_j, \) the piecewise smooth abstract functions \( f : [0, 1] \rightarrow L^2(\Omega, \mathbb{R}^3) \) and \( F : [0, 1] \rightarrow L^2(\Gamma_2, \mathbb{R}^3) \) correspond to the given external mass and surface forces, respectively \( f(0) = 0, F(0) = 0 \). For real plasticity models the equation (1) is principally unsolved regarding \( \dot{u} \).

The general existence and uniqueness theorem for the plasticity EVE (1) is formulated. The proof is based on the monotonous operators theory and the theory of the abstract Cauchy problem in the Hilbert space \([1].\) The main necessary and sufficient condition is independent and does not coincide with the classic Drucker’s...
and thermodynamical postulates. It has the following simple algebraic form: for every symmetric $3 \times 3$ matrices $A, B^1$ and $B^2$

$$C_{ijkm} \left( \dot{P}_{km}(A, B^1) - \dot{P}_{km}(A, B^2) \right) (B^1_{ij} - B^2_{ij}) < 2\mu_* \left| B^1 - B^2 \right|^2, \quad (2)$$

where $\mu_* > 0$ is the effective (i.e. smallest) shear modulus. This condition does not coincide with the Lipschitz condition of the matrix function $\dot{P}(A, B)$ over second matrix argument. It is easily to get convinced that the Lipschitz condition is stronger than the condition (2).

The independence of the condition (2) is illustrated for the plasticity model of linear isotropic-kinematic hardening with the ideal Bauschinger’s effect, dilatation and internal friction [1].

After the finite element approximation the plasticity EVE (1) is transformed into the finite dimensional Cauchy problem for the nonlinear system of ordinary differential equations unsolved regarding the derivative. Moreover, this system can be stiff for some models [1–3]. Therefore, for the numerical solution the implicit Euler scheme with the decomposition method of adaptive block relaxation (ABR) is used [1–3]. The main idea of the ABR method consists of iterative improvement of zones with "proportional" deformation by special \textit{adaptive decomposition} of variables on every iteration, and separate calculation on these variables. The numerical results show that, for finding the deformed configuration and the time of calculation, this technique has advantages over the standard method.


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VIABILITY RESULTS FOR SEMILINEAR DIFFERENTIAL INCLUSIONS

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AMS Class.: 34A60 (47D06)

Let $X$ be a Banach space, $A : D(A) \subset X \to X$ the infinitesimal generator of a $C_0$-semigroup, $D$ a nonempty subset in $X$ and $F : D \to 2^X$ a nonempty valued mapping. We consider the semilinear perturbed differential inclusion

$$u'(t) \in Au(t) + F(u(t)) \quad t \geq 0$$

and we are interested in finding necessary and sufficient conditions in order that $D$ be a viable domain for ($DI$), i.e. that for each $\xi \in D$ there exists at least one mild solution $u : [0, T] \to D$ of ($DI$) satisfying the initial condition

$$u(0) = \xi.$$  

($IC$)

A problem of this kind has been solved for the first time by Nagumo [4] in 1942. Since then many efforts have been made to extend Nagumo’s result to more general settings by using various techniques and we refer the reader to [2] for historical comments. See also [1], [3] and the references therein.

For the basic concepts on multivalued mappings and $C_0$-semigroups used here see [1], [5] and [6]. Our main results are stated below.

**Theorem 1.** Let $X$ be a reflexive and separable Banach space, $D$ a nonempty, locally closed subset in $X$ and $F : D \to 2^X$ a nonempty, closed, convex and bounded valued mapping which is weakly-weakly u.s.c. Let $A : D(A) \subset X \to X$ be the infinitesimal generator of a $C_0$-semigroup $S(t) : X \to X$, $t \geq 0$. Then a necessary and sufficient condition in order that $D$ be a viable domain for ($DI$) is the following weak tangency condition:

($BwTC$) There exists a locally bounded function $M : D \to \mathbb{R}_+^*$ enjoying the property that for each $\xi \in D$ there exists $y \in F(\xi)$ such that for each $\delta > 0$ and each weak neighborhood $V$ of 0 there exist $t \in (0, \delta]$ and $p \in V$ with $\|p\| \leq M(\xi)$ and satisfying

$$S(t)\xi + t(y + p) \in D.$$

**Theorem 2.** Let $X$ be a reflexive Banach space, $D$ a nonempty, strongly locally closed subset in $X$ and $F : D \to 2^X$ a nonempty, closed and convex valued mapping which is strongly-weakly u.s.c. and locally bounded. Let $A : D(A) \subset X \to X$ be the infinitesimal generator of a compact $C_0$-semigroup $S(t) : X \to X$, $t \geq 0$. Then a sufficient condition in order that $D$ be a viable domain for ($DI$) is the following strong tangency condition:

($sTC$) For each $\xi \in D$ there exists $y \in F(\xi)$ such that for each $\delta > 0$ and each neighborhood $V$ of 0 there exist $t \in (0, \delta]$ and $p \in V$ satisfying

$$S(t)\xi + t(y + p) \in D.$$  

The main step in the proof of both Theorems 1 and 2 consists in using Zorn’s Lemma to show that, for a suitably chosen $T > 0$, there exists a sequence of
“approximate solutions” defined on $[0, T]$ which converges either in $C([0, T]; X_w)$, or in $C([0, T]; X)$ to a mild solution of $(DI)$ and $(IC)$.

In order to formulate an existence result for global solutions we recall:

**Definition 1.** A mapping $F : D \rightarrow 2^X$ is called *positively sublinear* if there exist $a > 0$, $b \in \mathbb{R}$ and $k > 0$ such that $$\sup \{ \|y\|; y \in F(\xi) \} \leq a\|\xi\| + b$$ for each $\xi \in X^k_+(F)$, where $$X^k_+(F) := \{ \xi \in D; \sup \{ [\xi, y]_+; y \in F(\xi) \} > 0, \|\xi\| > k \}.$$

**Theorem 3.** Let $X$ be a reflexive and separable Banach space, $D$ a nonempty, locally closed subset in $X$ and $F : D \rightarrow 2^X$ a nonempty, closed, convex and bounded valued mapping which is weakly-weakly u.s.c. and positively sublinear. Let $A : D(A) \subset X \rightarrow X$ be the infinitesimal generator of a $C_0$-semigroup. Then a necessary and sufficient condition in order that for each $\xi \in D$ there exists at least one global mild solution of $(DI)$ satisfying $(IC)$ is the weak tangency condition $(BwTC)$.

A similar global existence result holds also under the general hypotheses of Theorem 2.

For the complete proofs of all these theorems see [2].


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VIABILITY RESULTS FOR NONLINEAR DIFFERENTIAL INCLUSIONS

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AMS Class.: 34A60 (47H20)

Let \( X \) be a Banach space, \( A : D(A) \subset X \to 2^X \) the generator of nonlinear semigroup of nonexpansive operators \( S(t) : D(A) \to D(A) \), \( t \geq 0 \), \( D \) a nonempty subset in \( X \) and \( F : D \to 2^X \) a nonempty valued mapping. We consider the nonlinear perturbed differential inclusion

\[
\dot{u}(t) \in Au(t) + F(u(t)) \quad t \geq 0
\]

and we are interested in finding sufficient conditions in order that \( D \) be a viable domain for (\( DI \)), i.e. that for each \( \xi \in D \) there exists at least one mild solution \( u : [0,T] \to D \) of (\( DI \)) satisfying the initial condition

\[
\dot{u}(0) = \xi. \quad (IC)
\]

Since the pioneering work of Nagumo [4], this problem has been studied by many authors by using various frames and techniques and we refer the reader to [3] for historical comments. See also [1], [2] and the references therein.

We assume familiarity with the basic concepts and results on multivalued mappings and dissipative operators and we refer to [1] and [5] for details. Throughout we denote by \( u(\cdot,0,\xi,f) \) the unique mild solution of \( \dot{u}(t) \in Au(t) + f(t) \) satisfying \( u(0,0,\xi,f) = \xi \). Our main results are stated below.

**Theorem 1.** Let \( X \) be a separable Banach space whose dual is uniformly convex, \( D \) a nonempty, locally closed subset in \( X \) and \( F : D \to 2^X \) a nonempty, closed, convex and bounded valued mapping which is strongly-weakly u.s.c. and locally bounded. Let \( A : D(A) \subset X \to 2^X \) be the infinitesimal generator of a nonlinear compact semigroup \( S(t) : D(A) \to D(A) \), \( t \geq 0 \). Then a sufficient condition in order that \( D \) be a viable domain for (\( DI \)) is the following weak tangency condition:

\[ (MwTC) \text{ There exists a locally bounded function } M : D \to \mathbb{R}_+^* \text{ satisfying: for each } \xi \in D \text{ there exists } y \in F(\xi) \text{ such that for each } \delta > 0 \text{ and each weak neighborhood } V \text{ of } 0 \text{ there exist } t \in (0,\delta] \text{ and } p \in V \text{ with } \|p\| \leq M(\xi) \text{ and } u(t,0,\xi,y+p) \in D. \]

**Theorem 2.** Let \( X \) be a separable Banach space whose dual is uniformly convex, \( D \) a nonempty, locally compact subset in \( X \), \( F : D \to 2^X \) a nonempty, closed, convex and bounded valued mapping which is strongly-weakly u.s.c. on \( D \) and let \( A : D(A) \subset X \to 2^X \) be an \( m \)-dissipative operator. Then a sufficient condition in order that \( D \) be a viable domain for (\( DI \)) is the weak tangency condition (\( MwTC \)) above.

The idea of proof for both Theorems 1 and 2 consists in showing that, for a suitably chosen \( T > 0 \), a sequence of “approximate solutions”, whose existence on \([0,T]\) is ensured by Zorn’s Lemma, converges in \( C([0,T];X) \) to a mild solution of (\( DI \)) and (\( IC \)).
In order to get the existence of global solutions we recall first:

**Definition 1.** A mapping $F : D \rightarrow 2^X$ is called positively sublinear if there exist $a > 0$, $b \in \mathbb{R}$ and $k > 0$ such that

$$\sup \{\|y\| ; y \in F(\xi)\} \leq a\|\xi\| + b$$

for each $\xi \in X^k_+(F)$, where

$$X^k_+(F) := \{\xi \in D ; \sup \{[\xi, y]_+ ; y \in F(\xi)\} > 0, \|\xi\| > k\}.$$

**Theorem 3.** Let $X$ be a separable Banach space whose dual is uniformly convex, $D$ a nonempty, locally closed subset in $X$ and $F : D \rightarrow 2^X$ a nonempty, closed, convex and bounded valued mapping which is strongly-weakly u.s.c., locally bounded and positively sublinear. Let $A : D(A) \subset X \rightarrow 2^X$ be the infinitesimal generator of a nonlinear compact semigroup $S(t) : D(A) \rightarrow \overline{D(A)}$, $t \geq 0$. Then a sufficient condition in order that for each $\xi \in D$ there exists at least one global mild solution of $(DI)$ satisfying $(IC)$ is the weak tangency condition $(MwTC)$.

The statement of a similar global result concerning Theorem 2 is now obvious. For the complete proofs of all these theorems the reader is referred to [3].


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THE METHOD OF DERIVED CONES IN OPTIMAL CONTROL OF DIFFERENTIAL INCLUSIONS

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AMS Class.: 34A60 (49J52)

The concept of derived cone to an arbitrary subset of a normed space has been introduced by M. Hestenes in [5] and successfully used to obtain necessary optimality conditions in Control Theory. However, in the last 25-30 years, this concept has been largely ignored in favor of other concepts of tangents cones, that may intrinsically be associated to a point of a given set : the cone of interior directions, the contingent, the quasitangent and, above all, Clarke’s tangent cone.

Recently Mirică [6] obtained ”an intersection property” of derived cones that allowed a conceptually simple proof and significant extensions of the minimum principle in optimal control; moreover, other properties of derived cones may be used to obtain controllability and other results in the qualitative theory of control systems.

We consider a differential inclusion and a corresponding variational inclusion that generalize the variational equations in the classical theory of Ordinary Differential Equations and we prove that the reachable set of the variational (linearized) inclusion is a derived cone to the reachable set of the differential inclusion considered.

In our previous paper [2] we indentified another derived cones to the reachable sets of differential inclusions in terms of the set of tangent directions to the trajectory of the system, which are enlarged by a process of ”transport” using certain variational inclusion.

Even if the derived cones find in [2] are one of the main tool in obtaining necessary optimality conditions for optimal control problems given by differential inclusions, their construction is rather complicated. The approach proposed here is much simpler, concerning only the reachable set of the linearized inclusion.

We note that the variational inclusion we deal is related to the linearized inclusions in Frankowska [3] and is defined by closed convex processes contained in the quasitangent directional derivative of the convexified multifunction which is larger than the linearized inclusion in [2] given by the quasitangent directional derivative of the multifunction. This special choice of the variational inclusion allows applications of our main result in the theory of necessary optimality conditions and also in controllability theory for differential inclusions.

In order to obtain the continuity property in the definition of a derived cone we shall essentially use the continuous version of Filippov’s theorem on Lipschitzian differential inclusions obtained by Cernea and Mirică [1] and the continuous version of relaxation theorem of Goncharov [4].

As applications of our main result we obtain a simple proof of the Maximum Principle in Optimal Control and sufficient conditions for local controllability along a reference trajectory.


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ABOUT SOME MONOTONICITY CONDITIONS FOR THE PERIOD FUNCTION

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We are interested on a Newtonian Hamiltonian system $H(x, y) = (1/2)y^2 + G(x)$, with a center at the origin. This equation can be written

$$\frac{dx}{dt} = y, \quad \frac{dy}{dt} = -g(x),$$

where $G$ is a primitive of $g$.

Each orbit lies on an energy level, $H(x, y) = c$, and it is uniquely determined by $c$. The period function $T(c)$ is the minimal period of this orbit.

$$T(c) = \sqrt{2} \int_a^b \frac{dx}{\sqrt{c - G(x)}}.$$

Many authors gave conditions on $G$ and its derivatives for the period of the orbit to be monotone. Some of them give sufficient conditions for the period function to be monotonic. Accordingly, we consider the following technical hypothesis $(J)$:

$$\begin{cases}
(i) & \text{There exists two numbers } a < 0 < b, \text{ such that } 0 < G(a) = G(b) = c. \\
(ii) & G(0) = g(0) = 0. \\
(iii) & xg(x) > 0 \text{ if } x \neq 0, x \in (a, b). \\
(iv) & g'(0) > 0.
\end{cases}$$

We give below some known sufficient conditions, each of them imply that $T(c)$ is nondecreasing under the hypothesis $(J)$.

$$(C_0) \quad \left\{ \begin{array}{l}
H(x) = g(x)^2 + \left( \frac{g''(0)}{3g'(0)^2} \right) g(x)^3 - 2G(x)g'(x) \geq 0 \quad \text{for } x \in (a, b).
\end{array} \right.$$$$

$$(C_1) \quad \left\{ \begin{array}{l}
(i) \quad g''(x) > 0 \quad \text{for } x \in (a, b), \\
(ii) \quad \Delta(x) = x(g''(0)g'(x) - g'(0)g''(x)) \geq 0 \quad \text{for } x \in (a, b).
\end{array} \right.$$$$

$$(C_2) \quad \Psi(x) = \frac{G(x)}{g(x)^2} \text{ is a convex function for } x \in (a, b).$$$$

$$(C_3) \quad \left\{ \begin{array}{l}
(i) \quad g''(x) > 0 \text{ for } x \in (a, b), \\
(ii) \quad \Delta_1(x) = 5g''(x)^2 - 3g'(x)g^{(3)}(x) \geq 0 \quad \text{for } x \in (g^{-1}(0), b).
\end{array} \right.$$
\((C_4)\) \( \begin{cases} \quad x[3g'(x)^2 - g(x)g''(x) - (3g'(0)^2)g''(x)] \geq 0 \quad \text{for} \quad x \in (a, b). \end{cases} \)

Conditions \((C_0)\) and \((C_1)\) are given by Chow and Wang [3], \((C_2)\) appears in Chicone [2]. \((C_3)\) is due to R. Schaaf [5]. Finally, \((C_4)\) was exhibited by F. Rothe, it was denoted \(f_4\) (see [4]).

Here is our new criteria (see Chouikha-Kelfa [1]) which is less inclusive than \((C_0)\) but relatively easy to check. Our new condition is more inclusive than condition \((C_1)\) and \((C_3)\), and, with an additional assumption, more general than \((C_2)\) and \((C_4)\)

\((C_5)\) \( \begin{cases} \quad (i) \quad g''(x) > 0 \quad \text{for} \quad x \in (a, b). \\ \quad (ii) \quad 3g'(x)^2 - g(x)g''(x) - (3g'(0)^2)g''(x) \leq 0 \quad \text{for} \quad x \in (g^{-1}(0), 0). \\ \quad (iii) \quad \frac{g'(x)g''(0)}{g''(x)g'(0)^2} \geq \frac{2G(x)}{g(x)^2} \quad \text{for} \quad x \in (0, b). \end{cases} \)


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REGULARITY OF MINIMA 
OF VARIATIONAL INTEGRALS

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In this paper we will consider the problem of the regularity of the derivatives 
of functions minimizing a variational integral

\[ F(u;\Omega) = \int_\Omega A_{ij}^{\alpha\beta}(x)D_\alpha u^i D_\beta u^j \, dx + \int_\Omega g(x, u, Du) \, dx \]  

where \( \Omega \subset \mathbb{R}^n, n > 1 \) is an open set, \( u : \Omega \to \mathbb{R}^N, N > 1 \), \( Du = \{D_\alpha u^i\}, \alpha = 1, \ldots, n, i = 1, \ldots, N \) and \( g : \Omega \times \mathbb{R}^N \times \mathbb{R}^{nN} \to \mathbb{R}, A_{ij}^{\alpha\beta}(x) \) will be stated below. Here and in the following summation over repeated indices is understood.

A local minimum for functional \( F \) is a function \( u \in W^{1,2}_{loc}(\Omega, \mathbb{R}^N) \) such that for every \( \varphi \in W^{1,2}(\Omega, \mathbb{R}^N) \) with \( \operatorname{supp} \varphi \subset \Omega \) have

\[ F(u; \operatorname{supp} \varphi) \leq F(u + \varphi; \operatorname{supp} \varphi) \]  

In his article [2] first from authors has proved the \( L^{2,n}_{loc} \)-regularity for gradient of solutions of some nonlinear elliptic system. This article and that one [3] gave to us some motive to investigate the \( L^{2,n}_{loc} \)-regularity for gradient of functional (1).

We shall consider a local minimum of the functional (1) where the coefficients \( A_{ij}^{\alpha\beta} \) are continuous in \( \overline{\Omega} \) and satisfy the Legendre-Hadamard condition:

\[ A_{ij}^{\alpha\beta}(x)\xi_\alpha \xi_\beta \eta^i \eta^j \geq \nu |\xi|^2 |\eta|^2, x \in \Omega, \xi \in \mathbb{R}^n, \eta \in \mathbb{R}^N; \nu > 0 \]  

About function \( g \) we suppose that for almost \( x \in \Omega \) and all \( (u, z) \in \mathbb{R}^N \times \mathbb{R}^{nN} \) the following condition hold:

\[ -f(x) - l(|u|^{\delta} + |z|^{\gamma}) \leq g(x, u, z) \leq f(x) + l(|u|^2 + |z|^\gamma) \]  

where \( l \geq 0, 0 \leq \delta < 2n/(n-2) \) for \( n > 2, \delta \geq 0 \) for \( n \leq 2, 0 \leq \gamma < 2 \) and \( f \in L^p(\Omega), p > 1 \).

Let \( u \in W^{1,2}(\Omega, \mathbb{R}^N) \) be a local minimum for the functional (1). We shall get estimates of the derivatives of \( u \) in the spaces \( L^{2,\lambda} \) and \( L^{2,n} \). For a detailed information for these function spaces see [1].

The following theorem may be seen as some small generalization of theorem 4.1 in [3].

**Theorem 1.** Let \( u \in W^{1,2}(\Omega, \mathbb{R}^N) \) be a local minimum of the functional (1) and let (3), (4) be satisfied. Then \( Du \in L^{2,n(1-1/p)}_{loc}(\Omega, \mathbb{R}^{nN}) \).

For obtaining of \( L^{2,n} \)-regularity of \( Du \) we strengthen the conditions on function \( g \). We will suppose that for a.e. \( x, y \in \Omega \) and all \( u, v \in \mathbb{R}^N, z, q \in \mathbb{R}^{nN} \)

\[ |g(x, u, z) - g(y, v, q)| \leq |f(x) - f(y)| + l((|u| + |v|)^\delta + |z - q|^{\gamma}) \]
Now we can formulate the main result of this paper:

**Theorem 2.** Let $u \in W^{1,2}(\Omega, \mathbb{R}^N)$ be a local minimum of the functional (1). Suppose that the conditions (3) with coefficients $A^{\alpha \beta}_{ij} \in C^{0,\mu}(\Omega)$ and (5) are satisfied for $f \in L^{1,n}(\Omega)$. Then $Du \in L^{2,n}_{loc}(\Omega, \mathbb{R}^{nN})$.

**Corollary.** If the assumptions of Theorem 2 are satisfied then $u \in \Lambda^{1}_{loc}(\Omega, \mathbb{R}^N)$.

Here Zygmund class $\Lambda^1(\overline{\Omega}, \mathbb{R}^N)$ is the subspace of such functions $u \in C^0(\overline{\Omega}, \mathbb{R}^N)$ for which

$$\sup \left\{ \frac{|u(x) + u(y) - 2u(x + y/2)|}{|x - y|} : x, y, (x + y)/2 \in \overline{\Omega} \right\} < \infty$$

and the following imbeddings

$$C^{0,1}(\overline{\Omega}, \mathbb{R}^N) \subseteq \Lambda^1(\overline{\Omega}, \mathbb{R}^N) \subseteq \cap_{0<\alpha<1} C^{0,\alpha}(\overline{\Omega}, \mathbb{R}^N)$$

hold.


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ON THE VARIATIONAL PRINCIPLE FOR THE LOADED DIFFERENTIAL-OPERATOR EQUATION

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The variational principle is widely used for the investigation of equations of the mathematical physics. With help of the variational principle founded on physical law of energy preservation it is possible, as a rule, to state the correct problems for a given class of equations; to prove they solvability, to investigate differentiability properties of the solution and to form a requisite solution with any degree of the exactness.

The special Hilbert space \( \{ W; [\cdot, \cdot]\} \) and operator \( B \) are constructed, which permits to reduce Cauchy problem for the first-order loaded differential-operator equation:

\[
L(t)u \equiv u'(t) + A(t)u(t) + \sum_{k=1}^{m} A_k(t)u(t_k) = f(t) \quad \text{at} \quad t \in (0, 1), u(0) = u_0,
\]

to a variational problem:

\[
J(v) \equiv [v, v] - 2l_e(v) \to \min_{v \in W}.
\]

Here \( l_e(v) \) is a linear continuous form in \( W \) defined by the original Cauchy problem and operator \( B; A(t), A_k(t) \) are the families of the given linear operators acting in corresponding spaces; points \( \{ t_k \} \) of the interval \( [0, 1] \) are fixed and \( 0 < t_1 < ... < t_m \leq 1 \).

Let the Hilbert space \( \{ H, | \cdot |, (\cdot, \cdot) \} \) be dense in the reflex Banach space \( \{ V, \| \cdot \| \} \) and \( | u | \leq \gamma \| u \| \quad \forall u \in V, \gamma = \text{const} > 0 \).

**Assumption I.** (i). There exists a inverse operator \( A^{-1}(t) : H \to H \) for almost every \( t \) which is self-conjugate and nonnegative. (ii). There exist linear manifolds \( \Phi \subset V \), positive constant \( \alpha \) and essentially bounded functions \( a_k(t), k = 1, ..., m \), such that \( A(t)v \in H, (A(t)v, v) \geq \alpha \| v \|^2, |A_k(t)v| \leq \sqrt{a_k(t)} \| v \| \quad \forall v \in \Phi \) and for a.e. \( t \in [0, 1] \). (iii). Family of operators \( A^{-1/2}(t), 0 \leq t \leq 1 \), is uniformly bounded for a.e. \( t \). (iv). There exists a mapping of functions \( v : [0, 1] \to \Phi \) which is a subspace in \( C^1([0, 1] : H) \) such that \( A(t)v(t), A^{-1}(t)v'(t), A^{-1}(t)A_k(t)v(t_k), K(t)v(t) \in L_2(0, 1; H) \) at \( v \in \Psi \), where \( K(t) = A^{-1/2}(t)L(t) \).

We have proved following lemma and theorems.

**Lemma.** Let an absolutely continuous in interval \( [0, 1] \) nonnegative function \( y(t) \) satisfies inequalities:

\[
y'(t) \leq ay(t) + \sum_{k=1}^{m} a_k(t)y(t_k) + f(t) \quad \text{at} \quad (0, 1), \quad y(0) \leq y_0,
\]

where \( a = \text{const}, 0 \leq f \in L_1(0, 1), y_0 \geq 0, a_{0k} = \| a_k \|_{L_{\infty}}. \) Let there hold conditions

\[
a_{0k} \leq \delta_1 \cdots \delta_{k-1}(1 - \delta_k)\chi_k, \quad 0 < \delta_k < 1, \quad k = 1, ..., m,
\]
where $\chi_k = \exp(-at_k)/(1 - \exp(-at_k))$. We have

$$y(t) \leq k \left( y_0 + \|f\|_{L_1(0,1)} \right), \ t \in [0, 1],$$

where a constant $k$ is defined only by values $\delta_k$, $t_k$, $k = 1, ..., m$.

Let us assume that $L \in \Delta(\varepsilon,a)$ iff $a_0k/\varepsilon \leq \delta_1 \cdot \delta_{k-1}(1 - \delta_k)\chi_k$, $0 < \delta_k < 1$, $k = 1, ..., m$.

**Theorem 1.** The minimality problem for a functional $J(v)$ in $W$ has the unique solution $u \in W$, that is

$$J(u) = \inf_{v \in W} J(v).$$

**Theorem 2.** Let us assume that an Assumption I hold, $L \in \Delta(\varepsilon,a)$ iff $a_0k/\varepsilon \leq \delta_1 \cdot \delta_{k-1}(1 - \delta_k)\chi_k$, $0 < \delta_k < 1$, $k = 1, ..., m$.

**Assumption II.** The set $\{Bu, \ v \in \Psi\}$ is dense in $L_2(0,1;H) \times H$.

**Theorem 3.** Let us assume that Assumptions I, II, hold $L \in \Delta(\varepsilon,m\varepsilon - 2\alpha\delta), \varepsilon > 0, 0 < \delta < 1, q \equiv \{f,u_0\} \in L_2(0,1;H) \times H$. Then we have the existence for operator $C_k$ K. O. Friedrichs extension $C_k$; and $C_ku = q$ is a L. Euler equation for minimality problem of functional $J(v)$ in $W$. Proofs of these theorems have been given in work [1].


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ON TWO-POINT BOUNDARY-VALUE PROBLEMS IN A LAYER FOR PARTIAL DIFFERENTIAL EQUATIONS WITH VARIABLE COEFFICIENTS

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AMS Class.: 35G15 (35B30, 35B65, 35A22)

The following boundary-value problem in the layer $\Pi(T) = \mathbb{R}^n \times [0, T]$ is investigated in the present work:

$$\frac{\partial u(x,t)}{\partial t} = P(D_x)u(x,t) + f(x,t), (x,t) \in \Pi(T);$$

$$A(D_x)u(x,0) + B(D_x)u(x,T) = w(x), x \in \mathbb{R}^n;$$

where $D_x = (-i\frac{\partial}{\partial x_1}, \ldots, -i\frac{\partial}{\partial x_n})$; $P(s,t)$ is an arbitrary polynomial with respect to $s$, that has continuous on $[0, T]$ complex-valued coefficients; $A(s), B(s)$ are arbitrary polynomials with constant complex coefficients, $T > 0$.

This problem is investigated proceeding from the general theory of partial differential equations. This approach is characterised by the absence of a priori restrictions for the equations and the boundary conditions. Problem (1), (2) is non-local generally speaking. A lot of mathematicians have studied non-local boundary-value problems.

Let $\|g\|_{p^r} = \sup\{|D_x^\alpha g(x,t) | (1+ | x |)^{-r} : |\alpha| \leq p \land (x,t) \in \Pi(T)\}$;

$\|h\|_{p^r} = \sup\{|D_x^\alpha h(x) | (1+ | x |)^{-r} : |\alpha| \leq p \land x \in \mathbb{R}^n\}$ (when we write such norms, we assume that if $|\alpha| \leq p$ then $D_x^\alpha g \in C(\Pi(T))$ and $D_x^\alpha h \in C(\mathbb{R}^n)$), here $\alpha$ is multi-index with non-negative components.

**Definition 1.** Problem (1), (2) is said to be well posed in the class of functions of polynomial growth $r \geq 0$ if $\forall \varphi \in \mathbb{N} \exists q \in \mathbb{N} \exists C > 0$ such that for each pair of functions $f(\| f \|_{q^r} < +\infty)$ and $w(\| w \|_{q^r} < +\infty)$ there exists a unique solution $u(\| u \|_{p^r} < +\infty)$ and this solution satisfies the condition:

$$\| u \|_{p^r} \leq C(\| f \|_{q^r} + \| w \|_{q^r}).$$

Let $Q(s, \tau, t) \equiv \int_T^t P(s, \xi) d\xi$, $\varphi(s, \tau, t) \equiv \Re Q(s, \tau, t)/\ln(e+ | s |)$.

**Theorem 1.** Problem (1), (2) is well posed in the class of functions of polynomial growth $r \geq 0$ if and only if the following four conditions are fulfilled:

$$\Delta(s) \equiv A(s) + B(s)\exp\{Q(s,0,T)\} \neq 0(\forall s \in \mathbb{R}^n);$$

$$\inf\{\varphi(s,t,\tau) : s \in \mathbb{R}^n \land A(s) = 0 \land 0 \leq t \leq \tau \leq T\} > -\infty;$$

$$\sup\{\varphi(s,\tau,t) : s \in \mathbb{R}^n \land B(s) = 0 \land 0 \leq \tau \leq t \leq T\} < +\infty;$$

$$\exists C \geq 0, \forall (s,t) \in \Pi(T) \exists \xi \in \{0,T\} \exists \zeta \in \{0,T\}$$

$$[(\forall \tau \in [0,t] \varphi(s,\tau,t) - \varphi(s,\xi,T) \leq C) \land$$
\[(\forall \tau \in [t, T] \varphi(s, t, \tau) - \varphi(s, \zeta, T) \geq -C)]. \tag{6}\]

To prove theorem 1 we use Fourier transform method [1]. The proof is based on the analysis of the explicit formula for the solution of the "dual" problem obtained as a result of Fourier transform (with respect to $x$) of problem (1), (2). The main point of the proof is obtaining polynomial estimates from below for $|\Delta(s)|$ in $\mathbb{R}^n$ and then obtaining polynomial estimates from above for $|\exp\{Q(s, \tau, t)\}/\Delta(s)|$ in $\mathbb{R}^n$ from conditions (3)–(6). The focal point of the proof of these estimates is the use of Tarski-Seidenberg theorem [2] and its corollaries [3], Łojasiewicz inequality [4], Whitney’s results concerning algebraic sets [5], Khovanskiy’s results concerning systems of transcendental equations [6], results and methods of works [7, 8].


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OSCILLATIONS IN LINEAR DIFFERENCE EQUATIONS

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Consider the linear delay difference system

\[ x(t) = \sum_{j=1}^{p} A_j(t)x(t - r_j), \]

where \( x(t) \in \mathbb{R}^n \), the \( A_j(t) \) are continuous functions in \([0, +\infty[\) with values in \( \mathbb{R}^{n \times n} \) and the \( r_j \) are strictly positive real numbers such that \( r_j \neq r_k \) for \( j \neq k, j, k \in \{1, \ldots, p\} \). Taking \( R = \max\{r_1, \ldots, r_p\} \), a continuous solution \( x : [-R, +\infty[ \to \mathbb{R}^n \) of (1), \( x(t) = (x_1(t), \ldots, x_n(t))^T \), is said to be oscillatory if either

\[ \sum_{i=1}^{n} |x_i(t)| = 0 \quad \text{for} \quad t \text{ sufficiently large}, \]

or at least one of the coordinates \( x_i(t) \), of \( x(t) \), satisfies \( \limsup_{t \to +\infty} \text{sgn} x_i(t) \neq \liminf_{t \to +\infty} \text{sgn} x_i(t) \).

When all solutions of (1) are oscillatory, it will be said a totally oscillatory system.

**Theorem.** Let, for each \( j \in \{1, \ldots, p\} \), \( A_j(t) \) be a bounded matrix function on \([0, +\infty[\) and denote by \( D_j \) and \( C_j \) \( n \)-by-\( n \) real matrices such that \( D_j \leq A_j(t) \leq C_j \), for every \( t \geq 0 \). If, for \( D_j \leq B_j \leq C_j \), \( j = 1, \ldots, p \), each system

\[ x(t) = \sum_{j=1}^{p} B_jx(t - r_j), \]

is totally oscillatory, then (1) is also totally oscillatory.

**Theorem.** In the conditions of the preceding theorem let, for each \( j \in \{1, \ldots, p\} \), \( D_j \) and \( C_j \) be \( n \)-by-\( n \) real essentially nonnegative matrices. If the system

\[ x(t) = \sum_{j=1}^{p} C_jx(t - r_j) \]

is totally oscillatory, then (1) is also totally oscillatory.

**Corollary.** In the conditions above, if for \( j \in \{1, \ldots, p\} \), \( \text{Re} \sigma(A_j) \subset [-\infty, 0] \) and the matrices \( C_j \) are either symmetric or commute one each other, then (1) is totally oscillatory globally in the delays.

A particular case of this situation occurs for a scalar equation

\[ x(t) = \sum_{j=1}^{p} a_j(t)x(t - r_j), \]

where, for \( j = 1, \ldots, p \), \( a_j(t) \) are real continuous functions on \([0, +\infty[\).

**Theorem.** Let, for each \( j \in \{1, \ldots, p\} \), \( a_j(t) \) be a bounded function on \([0, +\infty[\) and denote by \( d_j \) and \( c_j \) real numbers such that,

\[ d_j \leq a_j(t) \leq c_j, \quad \text{for every} \quad t \geq 0. \]
(i) If the equation
\[ x(t) = \sum_{j=1}^{p} c_j x(t - r_j), \]
is totally oscillatory, then (2) is also totally oscillatory.

(ii) If \( c_j \leq 0 \) for each \( j \in \{1, ..., p\} \), then (2) is totally oscillatory globally in the delays.

When all the delays in (2) are commensurable, that is, when there exists a \( \beta > 0 \) and positive integers, \( n_j \), such that \( r_j = n_j \beta \), for each \( j = 1, ..., p \), the totally oscillatory behaviour of (2) for the discretized case is studied in [1, Chapter 7].
A result of different kind is obtained.

**Theorem.** If, for \( j = 1, ..., p \), there exist the limits \( \lim_{t \to +\infty} a_j(t) = a_j \) and with \( N = \max\{n_1, ..., n_p\} \), all roots of the polynomial
\[ P(z) = z^N - \sum_{j=1}^{p} a_j z^{N - n_j}, \]
are real, negative and simple then the corresponding equation (2) is totally oscillatory.


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THE COMBINATION OF THE GRAPHICAL AND NUMERICAL METHODS FOR THE SOLUTION OF LINEAR AND NON–LINEAR BOUNDARY VALUE PROBLEMS

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AMS Class.: 34B15 (65L10)

This contribution considers combination of the numerical and graphical methods for solution boundary value problems by methods of superposition and transformation.

1. Consider the reduction of third–order ordinary differential equation:
\[
\frac{d^3 y}{dx^3} + f_1(x) \frac{d^2 y}{dx^2} + f_2(x) \frac{dy}{dx} + f_3(x) y = r(x)
\]
subject to the boundary conditions:
\[
y(0) = 0, \quad \frac{dy(0)}{dx} = 0, \quad y(1) = 0
\]
to the initial value problems by method of superposition [1].

The reduction of third–order boundary value problems to initial value problems by methods of superposition is well known and very simple. Its using is illustrated by the examples from which we select sandwich beams for which the distribution of shear deformation \(y\) is governed by the linear ordinary differential equation
\[
\frac{d^3 y}{dx^3} - k^2 \frac{dy}{dx} + a = 0,
\]
where \(k^2\) and \(a\) are physical constants which depend on the elastic properties of the lamina.

For the free ends the condition of zero shear bimoment at both ends leads to the boundary conditions
\[
\frac{dy(0)}{dx} = \frac{dy(1)}{dx} = 0,
\]
from symmetry considerations,
\[
y\left(\frac{1}{2}\right) = 0.
\]

Equation (3) subject to the boundary condition (4) and (5) constitutes a three–point boundary value problem for which we shall apply the method of superposition. In detail refer to [1].

The contribution presents the principles of this method only and the reasons of its being effective for solving problems of the type mentioned above. Detailed elaboration of numerical algorithm cannot be presented here because of the limited size of the contribution but it will be offered in the further paper.

The familiar methods for solving individual problems of the type introduced in [1] seem to be the most effective ones, however, the class of equations that can be solved by means of them is, in general, rather limited.

Our object is to extend the possibilities of these methods to solutions of problems comprising some physical parameter either in a differential equation or in boundary conditions and, what is most important, to establish a complex of solutions at
a required interval of parameter values that may be represented graphically, or
presented in the form of tables, according to the investigation purposes.

2. Let us note that the combination of the graphical and numerical method
can be used for method of transformation nonlinear boundary value problems. In
paper [1] this method is considered for differential equation

\[
\frac{d^3 y}{dx^3} + \frac{1}{2} \frac{d^2 y}{dx^2} = 0
\]  

subjected to the boundary conditions

\[
y(0) = C, \quad \frac{dy(0)}{dx} = B, \quad \frac{dy(\infty)}{dx} = 1.
\]

By this method can be found solutions which are useful for boundary value
problems involving only one parameter. The difficulty for boundary value problems
lies in the fact that when solving the transformed initial value problem, we assign
the values of \(B\) and \(C\) and not of \(B\) and \(C\) and therefore, up to the completion of
calculations, we do not know what pair of values \(B\) and \(C\) will correspond to the
result. Moreover, if \(B\) is constant and \(C\) is altered, none of the found parameters
\(B\) or \(C\) will be constant. To overcome this difficulty a graphical method can be
used. For given dependence we construct nomogram by which we can get any
future solution of (6) for existing parameters \(B\) and \(C\). The construction of this
nomogram will be discussed in further paper.

The combination of the graphical and numerical methods for solution of linear
and nonlinear differential equations in engineering boundary value problems as a
highly efficient procedure for searching for multiple solutions and analyzing prop-
certies thereof. By means of graphical methods, it is possible to show the effect of
one parameter upon another, to investigate experimental dependencies. Thereby,
it can be useful to combine numerical methods with graphical methods. If graphi-
cal methods do not afford the calculation accuracy required, they can be employed
for attaining initial approximations.


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equations.* Russian Academy of Sciences Steklov Mathematical Institute. In-
ternational Conference on Functional Spaces, Approximation Theory, Non-

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ABOUT MIXED BOUNDARY PROBLEM
FOR HELMHOLTZ EQUATION

Olga Gavrilyako

In this report we are going to discuss the problem of diffraction of a plane electromagnetic wave by a set of gaps in the imperfectly conductive shield. We assume that edges of the gaps are perfectly and surfaces of the shield are imperfectly conductive. The geometry of the problem is shown on the figure.

The case of perfectly conductive shield was analyzed in [1].

The authors solved this problem by the technique of dual integral equations. We would like to present the method of Boundary Singular Integral Equations (SIE) [2], developed by prof. Yu. Gandel.

We start with a dual integro-series equations, and introducing a new function, we come up with two singular integral equations on the set of segments with two additional conditions. To accomplish this we apply the parametric representation of Hilbert transform to the integral part of the equation and Hilbert transform with cotangent kernel to the series one.

The digitization of the integral equations and additional conditions is carried out by means of a Gauss type quadrature formula, presented and proved in [3].

To estimate the effectiveness of the built up mathematical model the set of numerical experiments was carried out. The analytical formulae deduced from the method of Boundary SIE allow us to investigate the scattered field in far zone, scattered pattern and diffracted field in the gaps with high range of accuracy.

In summary it may be said that the given method can be applied to a wide class of electromagnetic and acoustic wave diffraction problems. Nowadays the method of Boundary SIE is being actively developed in Kharkov State University.


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BRANCHING OF SOLUTIONS
OF NONLOCAL BOUNDARY VALUE PROBLEMS
WITH FREE PARAMETERS

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On the segment $[0,1]$ the system of $n$ integrodifferential equations with $n$ unknowns $x(t)$ and parameter $\xi \in \mathbb{R}^k$

$$Bz \equiv \dot{x} + A(t)x + \int_0^1 K(t,s)x(s)ds + C(t)\xi = y(t)$$  \hspace{1cm} (1)

with nonlocal boundary conditions ($\alpha \in \mathbb{R}^l$)

$$\beta z \equiv B_0x(0) + B_1x(1) + \int_0^1 \Phi(s)x(s)ds + B_2\xi = \alpha$$  \hspace{1cm} (2)

is considered. Here the matrices $A,K,C$ and $\Phi$ are continuous on their variables, $z=(x,\xi)$ and unknown functions $x(t)$ are continuous on $[0,1]$.

The conjugate by Lagrange to (1),(2) problem is constructed. The conditions for its unique solvability, Fredholm and Noetherian properties are obtained.

The nonlinear boundary value problem $Bz=R(z,\lambda),\beta z = r(z,\lambda)$ with small parameter $\lambda$ and sufficiently smooth operators $R$ and $r$ is considered. Here is supposed that $R(0,0)=0, r(0,0)=0, R_z(0,0) = 0, r_z(0,0) = 0$. The general facts of branching theory for the solution $z=0$ of this problem are discussed.

Our investigations are supported RFFI, grant 96-01-00512.


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ASYMPTOTIC BEHAVIOUR OF NONOSCILLATORY SOLUTIONS OF GENERALIZED LIÉNARD DIFFERENTIAL EQUATION

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We investigate the asymptotic behaviour of nonoscillatory solutions of the equation
\[ x'' + f(x)x' + g(x) = 0, \quad \text{where } f, g \in C(R, R), R = (-\infty, \infty), \]
\[ f(x)x > 0, \quad g(x)x > 0 \quad \text{for } x \neq 0. \]

Denote \( F(x) = \int_0^x f(u)du, G(x) = \int_0^x g(u)du, x \in R \). Evidently, \( F(x) \) and \( G(x) \) are continuous on \( R \), \( F(0) = G(0) = 0 \) and \( F(x) > 0, G(x) > 0 \) for \( x \neq 0, x \in R \) and \( F(x) \) and \( G(x) \) are monotone on \( (-\infty, 0) \) and \( (0, \infty) \), too. Many properties of the solutions of (1) were investigated in the paper [1]. Now we focus our attention to the ultimately positive (negative) solutions of (1).

1. Ultimately positive solutions.

**Lemma 1.** Let \( x(t) > 0 \) on \([T, \infty)\). Then only the following two cases are possible:
   a) \( x(t) > 0, x'(t) > 0 \) for \( t > T_1 \geq T \);
   b) \( x(t) > 0, x'(t) < 0 \) for \( t > T_2 = T \).

**Proof.** It follows from the fact that \( x'(t) \) cannot have two zeros on \([T, \infty)\).

**Theorem 1.** In the case a) \( x(t) \) increases and \( x'(t) \) decreases on \([T_1, \infty)\) and \( 0 = \lim x'(t) < \infty, \lim x(t) = \infty \) as \( t \to \infty \). The case a) cannot occur if \( F(\infty) = \infty \).

**Proof.** It follows from the equation (1) that \( x''(t) < 0 \), thus \( x'(t) \) decreases, being positive. Then from the fact that \( x(t) > 0, x'(t) > 0 \) it follows that \( \lim x(t) = \infty \) as \( t \to \infty \). Further, \( [x'(t) + F(x(t))]'' = x'' + f(x(t))x'(t) = g(x(t)) < 0 \). Thus, in the case a) \( x'(t) + F(x(t)) \) decreases, being positive. But, \( 0 \leq \lim x'(t) < \infty \) as \( t \to \infty \) and \( 0 \leq \lim x'(t) + \lim F(x(t)) \leq x'(T_1) + F(x(T_1)) \) as \( t \to \infty \), which implies that \( \lim F(x(t)) < \infty \) as \( t \to \infty \).

**Theorem 2.** Let \( x(t) \) be a solution of (1) such that the case b) occurs, i.e. \( x(t) > 0, x'(t) < 0 \) for \( t \in [T, \infty) \). Then \( \lim x(t) = \lim x'(t) = 0 \) as \( t \to \infty \).

**Proof.** The assertion follows from Theorem 3.1 and Theorem 3.3 in [1].

2. Ultimately negative solutions.

**Theorem 3.** Let \( x(t) \) be a solution of (1) such that \( x(t) < 0 \) on \([T, \infty)\). Then \( \lim x(t) = -\infty \) as \( t \to \infty \). (See Theorem 4.3 in [1].)
Theorem 4. Let $x(t)$ be a solution of (1) such that $x(t) < 0$ on $[T, \infty)$ and let $F(-\infty) < \infty$. Then $\lim x(t) = -\infty$ and $\lim x'(t)$ as $t \to \infty$ exists and is finite.

Proof. From Theorem 3 we have that $\lim x(t) = -\infty$ as $t \to \infty$ and from the equation (1) we get that $x(t)$ cannot have a local maximum on $[T, \infty)$. But then $x(t)$ cannot have a local minimum on $(T, \infty)$. In fact, if $x(t)$ has a local minimum in $t_1 \in (T, \infty)$, then $x'(t_1) = 0$ and $x'(t) \geq 0$ on $[t_1, \infty)$, which is a contradiction with the fact that $\lim x(t) = -\infty$ as $t \to \infty$. Thus $x(t) < 0$, $x'(t) < 0$ on $(T, \infty)$, $\lim x(t) = -\infty$ as $t \to \infty$. Further, $(x'(t) + F(x(t)))' = g(x(t)) > 0$, $F'(x(t)) = f(x(t))x'(t) > 0$, therefore $x'(t) + F(x(t))$ as well as $F(x(t))$ increase on $(T, \infty)$ and consequently $x'(t) + F(x(t)) < F(x(t)) < F(-\infty) < \infty$, $\lim x'(t) = \lim [x'(t) + F(x(t))] - \lim F(x(t))$ as $t \to \infty$ exists and is finite.

ON SOME PROPERTIES
OF HOLOMORPHIC DYNAMICAL SYSTEMS

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Let $D$ be a nonempty domain in $\mathbb{C}^n$ and let $\{T^t\}, t \in \mathbb{R}$, be a continuous flow in $D$ such that for fixed $t \in \mathbb{R}$ the transformation $T^t$ is a holomorphic automorphism of $D$. Then $\{T^t\}, t \in \mathbb{R}$, is called the holomorphic flow and the pair $(D,\{T^t\}, t \in \mathbb{R})$ is called a dynamical system. Let $p$ be a Lebesgue measurable function on $D$ such that $p(z) > 0 \text{ for m.a.e. } z \in D$, where $m$ denote the Lebesgue measure.

Let $L^2H(D,p) = \{f \in L^2(D,p) : f \text{ is holomorphic in } D\}$. Assume that $p$ is an admissible weight, i.e. $L^2H(D,p)$ is a closed subspace of $L^2(D,p)$. The Hilbert space $L^2H(D,p)$ is called the Bergman space with weight $p$. Note that if the function $1/p$ is locally integrable on $D$ then $p$ is an admissible weight. Let $J_t$ denote the complex Jacobian of $T^t$. Assume that there exists a $p$-Jacobian of $\{T^t\}$ which we denote by $\varphi$. It means that $\varphi$ is a nonzero continuous function such that for every $t \in \mathbb{R}$ the function $\varphi(t, \cdot)$ is holomorphic in $D$ and

$$|\varphi(t,z)|^2 p(z) = p(T^t(z))|J_t(z)|^2 \text{ for m.a.e. } z \in D$$

$$\varphi(t+s,z) = \varphi(s,T^t(z))\varphi(t,z) \text{ for } z \in D, s \in \mathbb{R}.$$ 

We define the group of unitary operators $\{U^t\}, t \in \mathbb{R}$, in the form

$$U^t(f)(z) = f(T^t z)\varphi(t,z), \quad z \in D,$$

where $f \in L^2H(D,p)$ (cf. [1], [2]).

The following theorems are proved

**Theorem 1.** The following statements are equivalent

a) the group $\{U^t\}, t \in \mathbb{R}$, has discrete spectrum.

b) the group $\{U^t\}, t \in \mathbb{R}$, has a nonzero eigenvector.

c) there exists a $\{T^t\}$-invariant probability measure $\lambda$ on $D$ such that $\lambda$ is absolutely continuous with respect to the Lebesgue measure.

d) there exists a $T^1$-invariant probability measure $\lambda$ on $D$ such that $\lambda$ is absolutely continuous with respect to the Lebesgue measure.

**Corollary.** If the group $\{U^t\}, t \in \mathbb{R}$, has discrete spectrum then for fixed $t \in \mathbb{R}$ the transformation $\{T^t\}$ has the Poincaré recurrence property with respect to the Lebesgue measure, i.e. for every Borel set $B \subset D$ the trajectory of $m$ almost all points of $B$ return to $B$. 


Definition. A point \( w \in D \) is called to be wandering with respect to \( \{ T^t \} \), \( t \in \mathbb{R} \), if there exists an open neighbourhood \( G \) of \( w \) such that for some \( s > 0 \)
\[
\bigcup_{|t| \geq s} T^t(G) \cap G = \emptyset.
\]

Theorem 2. Assume that there exists a wandering point with respect to \( \{ T^t \} \), \( t \in \mathbb{R} \). Then the group \( \{ U^t \} \), \( t \in \mathbb{R} \) has absolutely continuous spectrum.

Theorem 3. Let \( D \) be a domain in the complex plane. Suppose that there exists \( w \in D \) such that \( T^s(w) \neq w \) for some \( s > 0 \). Then the induced group of unitary operators \( \{ U^t \} \), \( t \in \mathbb{R} \), has simple spectrum.

Example 1. Let us consider the Fock space. It means that \( D = \mathbb{C}^n \) and \( p(z) = \pi^{-n} e^{-|z|^2} \) for \( z \in \mathbb{C}^n \). It is easy to see that \( p \) is an admissible weight and \( L^2 H(\mathbb{C}^n, p) \neq \{ 0 \} \). For fixed \( a \in \mathbb{C}^n \setminus \{ 0 \} \) we define a holomorphic flow \( \{ T^t \} \) in \( \mathbb{C}^n \) as follows
\[
T^t(z) = z + at, \quad z \in \mathbb{C}^n, t \in \mathbb{R}.
\]

It is easy to show that the function \( \varphi(t, z) = e^{-\frac{1}{2}t^2|a|^2} e^{-t(z.a)} \), \( z \in \mathbb{C}^n, t \in \mathbb{R} \) is the \( p \)-Jacobian of \( \{ T^t \} \) where \( (u, w) = \sum_{j=1}^n u_j \overline{w}_j \) for \( u, w \in \mathbb{C}^n \) and \( |a| \) denotes the Euclidean norm of \( a \). It is obvious that every point in \( \mathbb{C}^n \) is wandering with respect to \( \{ T^t \} \). Then by Theorem 2 the induced group of unitary operators \( \{ U^t \} \), \( t \in \mathbb{R} \), has absolutely continuous spectrum in \( L^2 H(\mathbb{C}^n, p) \).

Example 2. Let \( B \) be a nonempty, open, connected and bounded subset of \( \mathbb{R}^n \) and let \( D = \mathbb{R}^n + iB \) be the tube over \( B \). For \( b \in \mathbb{R}^n \setminus \{ 0 \} \) we define the flow \( \{ T^t \} \) as follows \( T^t(z) = z + bt, t \in \mathbb{R}, z \in D \). Let \( p(z) = |e^{-z^2}|^2 \) for \( z \in D \), where \( z^2 = z_1^2 + z_2^2 + \ldots + z_n^2 \). It is easy to see that the function \( \varphi(t, z) = e^{-2t(b - z^2)|b|^2} \) is the \( p \)-Jacobian of \( \{ T^t \} \). Since every point in \( D \) is wandering then the induced group of unitary operators \( \{ U^t \} \) has absolutely continuous spectrum. Moreover we obtain

Theorem 4. Suppose that \( n > 1 \). Then the group \( \{ U^t \} \), \( t \in \mathbb{R} \), has infinite multiplicity Lebesgue spectrum. If \( n = 1 \) then the group \( \{ U^t \} \), has simple Lebesgue spectrum.


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ON SUBSETS OF BOUNDED SOLUTIONS OF FUNCTIONAL–DIFFERENTIAL EQUATIONS ON A SEMI-INFINITE INTERVAL

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AMS Class.: 34K05 (34K10)

In this paper we represent an approach [1]-[3] to the statement and investigation of singular FDEs. We consider a system of n nonlinear FDEs defined almost everywhere (a.e.) on a semi-infinite interval $T \leq t < \infty$. As $t \to \infty$ either the limit value of the desired function or the boundedness condition of a solution is given. We study existence theorems for a k-parameter family of solutions satisfying given conditions at infinity, $0 \leq k \leq n$. The case $k = 0$ gives theorems on existence and uniqueness of solutions of these singular Cauchy problems. In the case $k = n$ a point at infinity is an attracting point while in the case $1 \leq k \leq n - 1$ in an $n$-dimensional variable phase space the values of solutions form a $k$-dimensional stable initial manifold depending on a parameter $t$. These questions are important for correct statement and approximation of singular initial and boundary value problems including ODEs and integro–differential equations.

Introduce the notation: $I_T = [T, \infty)$, $T \geq T_0$, $T_0$ is fixed; $K \in \{R, C\}$, $| \cdot |$ is a norm in $K^n$ or in $L(K^n)$; $\Omega_n(a) = \{x : x \in K^n, |x| \leq a\}$, where either $a = a_0 > 0$ is fixed or $a > 0$ is arbitrary; $C_n(I_T)$ is the Banach space of bounded continuous functions $\xi(t) : I_T \to K^n$ with the norm $|\xi|_C = \sup_{t \in I_T} |\xi(t)|$; $L^\infty_n(I_T)$ is the Banach space of essentially bounded Lebesgue measurable functions $\xi(t) : I_T \to K^n$ with the norm $|\xi|_\infty = \sup_{t \in I_T} |\xi(t)|$; $AC^loc_n(I_T)$ is the class of locally absolutely continuous on $I_T$ functions $\xi(t) : I_T \to K^n$.

Let $Lip_n = Lip_n(I_{T_0} \times G_n)$ be the class of functions $f(t, x) : I_{T_0} \times G_n \to K^n$, $G_n = \Omega_n(a_0)$ or $G_n = K^n$, such that $f(\cdot, x)$ is continuous $\forall x \in G_n$ and $f(t, \cdot)$ satisfies in $\Omega_{a_0}(a_0)$ ($\Omega_n(a)$ $\forall a > 0$) the Lipschitz condition uniformly with respect to $t \in I_{T_0}$ with a constant $L_f > 0$ ($L_f(a) > 0$). We define $Lip_n = Lip_{n,a_0} \cup Lip_{n,\delta_0}(\varepsilon) \cup Lip_n(a) \cup \tilde{Lip}_n$. Here $Lip_{n,a_0} = Lip_n(I_{T_0} \times \Omega_n(a_0))$ with a constant $L_f > 0$ while $Lip_{n,\delta_0}(\varepsilon) = \{f : f \in Lip_{n,a_0} \text{ and } \forall \varepsilon > 0 \exists \delta_{\varepsilon}, T_0, 0 < \delta \leq a_0, T_0 \geq T_0, \text{ such that for } (t, x) \in I_{T_0} \times \Omega_n(\delta) \text{ we can choose } L_f = \varepsilon\}$. Further we define $Lip_n(a) = \{f : f \in Lip_n(I_{T_0} \times K^n) \text{ and } \sup_{a > 0} L_f(a) = \infty \text{ for any choice of } L_f(a)\}$ and $\tilde{Lip}_n = \{f : f \in Lip_n(I_{T_0} \times K^n) \text{ and there are } L_f(a) \text{ such that } \tilde{L_f} = \sup_{a > 0} L_f(a) < \infty\}$.

We consider a system of n nonlinear FDEs

$x' = A(t)x + \int_{\infty}^{+\infty} M_f(t)(FNx)(t) + g(t)\ a.e.\ on\ I_T,$

with the condition $\sup_{t \in I_T} |x(t)| \leq \omega$ where either $\omega > 0$ (problem 1) or $\omega = \infty$ (problem 2), or with the condition $\lim_{t \to \infty} x(t) = 0$ (problem 3).

Here $x : I_T \to K^n$, $g : I_{T_0} \to K^n$; $A, M_f : I_{T_0} \to L(K^n)$, the entries of $A(t), M_f(t), g(t)$ are locally summable functions on $I_{T_0}$; $(N x)(t) = f(t, x(t)), f \in$$

\text{1Supported by the Russian Foundation for Basic Research, Project N96-01-00951.}
\(Lip_n, f(t,0) \equiv 0\), i.e., \(N\) is the local Nemytskii operator, \(N : C_n(I_T) \to C_n(I_T)\); \(F : C_n(I_T) \to L^\infty_n(I_T)\), \((FNx)(t) = (F \circ f(\cdot, x(\cdot)))(t)\), where the mapping \(F\) (in general, it is nonlinear, nonlocal and depends on the choice of \(T\)) satisfies conditions (Russell D., 1970): \(F(0) = 0, |F(\xi) - F(\xi_\varepsilon)| \leq |\xi - \xi_\varepsilon|_{\text{loc}} \forall \xi, \xi_\varepsilon \in C_n(I_T)\).

The problems 1, 2, 3 are considered for the class \(AC^\infty_n(I_T)\).

The more detailed results are obtained for the cases: a) the mapping \(F : C_n(I_T) \to L^\infty_n(I_T)\), where \(T \geq T_0\) is fixed, is a singular Volterra operator (i.e., for each \(T \geq \tilde{T}\) and \(\xi_1, \xi_2 \in C_n(I_{\tilde{T}})\), if \(\xi_1(t) = \xi_2(t)\) on \(I_T\), then \((F\xi_1)(t) = (F\xi_2)(t)\) a.e. on \(I_T\); b) the mapping \(F : C_n(I_{\tilde{T}}) \to L^\infty_n(I_{\tilde{T}})\) is the local Nemytskii operator, i.e., \((F\xi)(t) = \varphi(t, \xi(t))\), and, as a special case, \(F\) is an embedding of \(C_n(I_{T_0})\) into \(L^\infty_n(I_{T_0})\), i.e., \(F(\xi) \equiv \xi \forall \xi \in C_n(I_{T_0})\).

In the general case we assume \(A(t) = A_-(t) \oplus A_+(t), M_f(t) = M_{f_-(t)} \oplus M_{f_+(t)}\), \(A_-, M_- : I_{T_0} \to L(K^k), A_+, M_+ : I_{T_0} \to L(K^{n-k})\), 0 \(\leq k \leq n\), and we use the notation: \(\Phi_{A_-(t)}(t) \oplus \Phi_{A_+(t)}(t)\) is a fundamental matrix of the system \((*) x'_- = A_-(t)x_- a.e. on I_{T_0}((**)) x'_+ = A_+(t)x_+ a.e. on I_{T_0}^\circ, U_\pm(t, s) = \Phi_{A_\pm(t)}\Phi_{A_\pm}^{-1}(s)\). Let all the solutions to the system \((*)\) be bounded (uniformly bounded) on \(I_{T_0}\) while the system \((**)\) have no solutions vanishing as \(t \to \infty\) except for \(x_+ \equiv 0\) (as a special case, all nontrivial solutions of \((**)\) are unbounded as \(t \to \infty\)). We define \(J_{f_+}(t) = \int_t^\infty |U_+(t, s)M_{f_+}(s)|ds, J_{g_+}(t) = \int_t^\infty |U_+(t, s)g_+(s)|ds, t \geq T_0; J_{f_-}(T) = \sup_{t \in I_T} \int_T^t |U_-(t, s)M_{f_-}(s)|ds, J_{g_-}(T) = \sup_{t \in I_T} \int_T^t |U_-(t, s)g_-(s)|ds,\) where the integrals are considered in the Lebesgue sense. With several restrictions on the behavior of these values we get the solution of the problem 1, 2 or 3 as a fixed point of a mapping \(V : C_n(I_T) \to C_n(I_T)\) defined by \(\langle V(x_-; x_+)\rangle(t) = \left\{U_-(t, T)x_{T_-} + \int_T^t U_-(t, s)M_{f_-}(s)(FNx)_-(s) + g_-(s)|ds; - \int_t^\infty U_+(t, t)[M_{f_+}(s)(FNx)_+(s) + g_+(s)]ds, t \geq T, where x_{T_-} \in K^k is the parameter vector.\)


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APPLICATION OF POTENTIAL THEORY TO INITIAL–BOUNDARY VALUE PROBLEMS FOR AN EQUATION OF COMPOSITE TYPE

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In the plane $x = (x_1, x_2) \in R^2$ we consider the internal or external multiply connected domain $D$ bounded by closed curves $\Gamma \in C^{2,0}$. The following PDE of composite type

$$\frac{\partial^4 u}{\partial t^2 \partial x_1^2} + \frac{\partial^4 u}{\partial t^2 \partial x_2^2} + \omega_1^2 \frac{\partial^2 u}{\partial x_1^2} + \omega_2^2 \frac{\partial^2 u}{\partial x_2^2} = 0; \quad \omega_1, \omega_2 \geq 0,$$

(1)

describes internal waves in the ocean. The potential theory has been constructed for eq.(1) recently. Some applications of potentials to solving problems are presented in [1–8]. In particular, explicit solutions of some problems in canonical domains were obtained in [1–4]. In the present note we study the solvability of the initial-boundary value problems with either Dirichlet or Neumann boundary condition in arbitrary domains with the help of the potential technique and the boundary integral equation method [5–8]. Boundary value problems for equations of composite type in multiply connected domains were not treated before.

**Definition 1.** A function $u(t, x)$ defined on $[0, \infty) \times D$ belongs to the smoothness class $G$ if $u, u_t \in C^0([0, \infty) \times D)$, if at each $t \geq 0$ there exists a limit of $u(t, x)$ along the normal to the boundary $\Gamma$ and if

$$\frac{\partial^k}{\partial t^k} \frac{\partial^p}{\partial x^p_j} u \in C^0((0, \infty) \times D), \quad k, p = 0, 1, 2; \quad j = 1, 2.$$

(2)

Let $u(t, x)$ be a sufficiently smooth function for $t \geq 0$, $x \in D$. Assuming that $n$ is the normal at the point $x(s) \in \Gamma$, we define the differential operator $N_{t,x}$ at points $\bar{x} \in D$ by the relationship

$$N_{t,x} u(t, \bar{x}) = \frac{\partial^2}{\partial t^2} \frac{\partial}{\partial n} u(t, \bar{x}) + \omega_1^2 \cos(n, x_1) u_{x_1}(t, \bar{x}) + \omega_2^2 \cos(n, x_2) u_{x_2}(t, \bar{x}),$$

where $\cos(n, x_j)$ is the cosine of the angle between the normal $n$ and the axis $Ox_j$, $j = 1, 2$.

By $N_{t,x} u(t, x)$ we denote the limiting value of $N_{t,x} u(t, \bar{x})$ as $\bar{x}$ approaches $x(s) \in \Gamma$ along the normal $n$, if the limit exists uniformly for all $x(s) \in \Gamma$.

**Definition 2.** A function $u(t, x)$ defined on $[0, \infty) \times D$ belongs to the smoothness class $G_1$ if condition (2) holds, if $u, u_t \in C^0([0, \infty) \times D)$, $\nabla u, \nabla u_t \in C^0((0, \infty) \times D)$, and if the expression $N_{t,x} u(t, x)$ exists at each point of $\Gamma$ in the sense of the uniform limit for all $x(s) \in \Gamma$.

**Problem N** (Neumann). To find $u(t, x)$ of class $G_1$ satisfying eq.(1) in $(0, \infty) \times D$ and the following conditions

$$u(0, x) = u_t(0, x) = 0,$$

(3)

$$N_{t,x} u|_{\Gamma} = f(t, x)|_{\Gamma} \in C^0([0, \infty); C^0(\Gamma)).$$

By $C^k([0, \infty); H)$ we denote the class of abstract functions $\varphi(t)$, which are $k$ times continuously differentiable in $t$ for $t \geq 0$ and which take values in the Banach space $H$. Besides, we put $C^k_0([0, \infty); H) = \{ \varphi(t) : \varphi(t) \in C^k([0, \infty); H), \varphi(0) = \varphi'(0) = \ldots = \varphi^{(k-1)}(0) = 0 \}$.

**Problem D** (Dirichlet). To find $u(t, x)$ of class $G$ satisfying eq. (1) in $(0, \infty) \times D$, initial conditions (3) and the boundary condition
\[ u|_{\Gamma} = f(t, x)|_{\Gamma} \in C^2_0([0, \infty); C^0(\Gamma)). \]

All conditions of the problems must be satisfied in the classical sense. In the case of the external domain \( D \) the following conditions at infinity must be included in the formulation of the problems

\[ |\partial_t^k u| \leq q_1(t), \quad |\partial_t^k u_{x_j}| \leq q_2(t)|x|^{-2}, \quad k = 0, 1, 2; \quad j = 1, 2; \]

where \( |x| = \sqrt{x_1^2 + x_2^2} \to \infty \) and \( q_1(t), q_2(t) \in C^0([0, \infty)). \)

On the basis of energy equalities we prove the following theorem.

**Theorem 1. (a).** A necessary condition for the solvability of the problem \( N \) is

\[ \int_{\Gamma} f(t, x)dl_x = 0, \quad t \geq 0. \quad (4) \]

If a solution of the problem \( N \) exists, then it is determined up to an arbitrary additive function of time \( c(t) \in C^2_0[0, \infty). \)

(b). There is no more than one solution of the problem \( D \) in the class \( G_1 \).

By means of potentials the problems are reduced to the Fredholm integral equations at the boundary. Studying the solvability of these equations in the appropriate Banach spaces, we obtain the solvability theorem for the problems.

**Theorem 2. (a).** If the condition (4) holds, then the classical solution of the problem \( N \) exists and it is defined up to an arbitrary function of class \( C^2_0[0, \infty) \).

(b). A classical solution of the problem \( D \) exists. Moreover, if \( \Gamma \in C^{2,\lambda}, \quad f(t, x) \in C^2_0([0, \infty); C^{1,\lambda}(\Gamma)), \quad \lambda \in (0, 1], \) then this solution belongs to the class \( G_1 \) and consequently it is unique.

The theorems hold for both internal and external domain \( D \). The problems \( N \) and \( D \) can be easily computed by means of finding numerical solutions of Fredholm boundary integral equations derived for these problems.


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A METHOD FOR DETERMINING
A LIMIT CYCLE OF A HAMILTONIAN
SYSTEM WITH A SPECIAL
SMALL PERTURBATION

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AMS Class.: 58F21 (34C05)

The method is a modification of the criterion given by Li and Zhang in [2]. Consider the system
\[
\begin{align*}
\dot{x} &= \Psi'(y) \\
\dot{y} &= -\Phi'(x) + \varepsilon g(y)(\alpha f_1(x) + f_2(x)),
\end{align*}
\]
(1)
where \(\Phi \in C^2[a_1, b_1], f_1, f_2 \in C^1[a_1, b_1], \Psi, g \in C^2[a_2, b_2].\)

If \(\varepsilon = 0\), then (1) is a Hamiltonian system with energy \(H(x, y) = \Phi(x) + \Psi(y)\). Let the energy levels \(h\) on \(a_1, b_1\) are changing along an interval \((h_1, h_2)\). Each compact component \(\Gamma_h\) of the level curve \(H^{-1}(h), h \in (h_1, h_2)\), corresponds to a closed orbit of the system (1).

For \(\varepsilon \neq 0\) the existence and number of periodic orbits of system (1) can be found out by computing zeros of Mešníkov function
\[
M(h) = \int_{\Gamma_h} g(y)(\alpha f_1(x) + f_2(x)) \, dx
\]
along each orbit \(\Gamma_h\). Instead of solving the equation \(M(h) = 0\) we can examine the function
(1)
\[
A(h) = -\frac{I_2(h)}{I_1(h)}, \quad h \in (h_1, h_2),
\]
where
\[
I_k(h) = \int_{\Gamma_h} f_k(x)g(y)\, dx, \quad k = 1, 2.
\]
If \(A' \neq 0\), then Theorem 4.6.2. in [1] guarantees the existence of exactly one limit cycle in (1), for given values of parameters and for sufficiently small \(\varepsilon\).

Now we describe the criterion of Li and Zhang with original assumptions. Let us denote by \(l(h)\) and \(u(h)\) the border points of the variable \(x\) on the orbit \(\Gamma_h\), while border points of variable \(y\) will be denoted by \(L(h)\) and \(U(h)\). Let there exists a point \((x_0, y_0) \in (a_1, b_1) \times (a_2, b_2)\) such that:

(H1) \(\Phi'(x)(x - x_0) > 0\) (or \(< 0\)) and \(\Psi'(y)(y - y_0) > 0\) (or \(< 0\)), where \((x, y) \in (a_1, b_1) \times (a_2, b_2) \backslash \{(x_0, y_0)\}.

This hypothesis requires the orbits are "symmetric" (in the sense we will describe below) and "monotonic" on intervals \((l(h), x_0), (x_0, u(h)), (L(h), y_0), (y_0, U(h))\) (i.e. \(\Phi'(x)\) and \(\Psi'(x)\) do not change sign on these intervals).

Instead of hypothesis (H1) it is sufficient to take the following assumption on the symmetry:
(H1’) There exist one-to-one mappings

\[ [l(h), x_0] \longrightarrow [x_0, u(h)] : x \mapsto \tilde{x}, \quad [L(h), y_0] \longrightarrow [y_0, U(h)]: y \mapsto \tilde{y}, \]

such that \( \Phi(x) = \Phi(\tilde{x}), \Psi(y) = \Psi(\tilde{y}), \)

\[ \frac{d\tilde{x}}{dx} = \frac{\Phi'(x)}{\Phi'(\tilde{x})} < 0, \quad \frac{d\tilde{y}}{dy} = \frac{\Psi'(y)}{\Psi'(\tilde{y})} < 0. \]

This assumption implies that \( \Gamma_h \) consists of two symmetric branches \( y(x) \) and \( \tilde{y}(x) \), for which \( y(x) = y(\tilde{x}), \tilde{y}(x) = \tilde{y}(\tilde{x}) \) when \( x \in (l(h), x_0). \)

We put one more assumption on the system (1) (the same as in [2]):

(H2) \( f_1(x)f_1(\tilde{x}) > 0, \ g'(\tilde{y})g'(\tilde{y}) > 0 \) for \( (x, y) \in (l(h), x_0) \times (L(h), y_0). \)

Theorem 1 Let (H1’) and (H2) are satisfied and let

\[ \xi(x) = \frac{f_2(x)\Phi'(\tilde{x}) - f_2(\tilde{x})\Phi'(x)}{f_1(x)\Phi'(\tilde{x}) - f_1(\tilde{x})\Phi'(x)}, \quad \eta(y) = \frac{(g(\tilde{y}) - g(y))\Psi'(\tilde{y})\Psi'(y)}{g'(\tilde{y})\Psi'(y) - g'(y)\Psi'(\tilde{y})}. \]

If \( \xi'(x)\eta'(y) > 0 \) (\( < 0 \)) for \( (x, y) \in (l(h), x_0) \times (L(h), y_0), \) then \( A'(h) < 0 \) (\( > 0 \)).

Example: Let \( f_1(x) = 1, \ f_2(x) = \cos x, \ g(y) = y, \ \Phi(x) = 0.5 \cos^2 x - \gamma \cos x, \)

\( \Psi(y) = 0.5y^2 + \gamma - 0.5. \) Then the system (1) has “symmetric” periodic orbits for \( h \in (0, 2\gamma). \) If \( \gamma \geq 1, \) then both (H1) and (H1’) are satisfied; but for \( \gamma < 1 \) only (H1’) is satisfied.


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BIFURCATION PROBLEM ON THE DETERMINATION OF FERROFLUID FREE SURFACE IN MAGNETIC FIELD

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AMS Class.: 58E09

The spatial layer of ferrofluid bounded from below by flat bottom and from above by vacuum is considered. When on the layer acts the vertically directed magnetic field of sufficient strength \( H \), on the upper boundary of ferrofluid arises two-periodical relief, subjected to determination together with magnetic potentials of media. Ferrofluid is supposed to be incompressible, having the finite depth \( h \) and free from exterior currents. In dimensionless variables this problem is described by the following system of differential equations in spatial layer with free boundary

\[
-\Delta \Phi = 0, \quad f(x, y) < z < 1; \quad \frac{\partial \Phi}{\partial z} |_{z=1} = 0;
\]

\[
-\Delta \varphi = 0, \quad -1 < z < f(x, y); \quad \frac{\partial \varphi}{\partial z} |_{z=-1} = 0;
\]

\[
-\gamma f - \frac{1}{2} [\|\nabla (\Phi + \mu Hz)\|^2 - \mu |\nabla (\varphi + Hz)|^2] + (\frac{\partial \Phi}{\partial z} + \mu H) \nabla (\Phi + \mu Hx) \cdot \vec{n} -
\]

\[
\mu (\frac{\partial \varphi}{\partial z} + H) \nabla (\varphi + Hz) \cdot \vec{n} - \frac{1}{2} \mu (\mu - 1) H^2 + (\nabla, \frac{\nabla f}{\sqrt{1 + |\nabla f|^2}}) = 0, \quad z = f(x, y);
\]

\[
\Phi - \varphi + (\mu - 1) Hz = 0, \quad z = f(x, y); \quad \nabla (\Phi - \mu \varphi) \cdot \vec{n} = 0, \quad z = f(x, y).
\]

Here \( \Phi \) and \( \varphi \) are magnetic potentials of upper and lower media, \( f(x, y) \) is a free boundary close to horizontal plane \( z = 0 \), \( \vec{n} \) is the normal to it, \( \mu_c \) is the magnetic constant, \( \mu \) is the magnetic permeability of the ferrofluid, \( \gamma = (\delta \rho)gh^2/\sigma \), \( \delta \rho \) is the difference of the densities of media, \( g \) is the acceleration of gravity, \( \sigma \) is the surface tension coefficient. It is posed the problem of the construction of periodical solutions with the periods \( \frac{2\pi}{a} = a_1 \) and \( \frac{2\pi}{b} = b_1 \) along the coordinate axes, and \( \Pi_0 \) is the rectangle of periodicity.

After straightening of the free boundary \( \zeta = \frac{z-f}{1-z/f} \) and setting \( H = H_0 + \varepsilon \) we obtain nonlinear system linear part of which represents Fredholm [34,35] operator \( B : C^{2+\alpha}([1, 0] \times \Pi_0) + C^{2+\alpha}([-1, 0] \times \Pi_0) + C^{2+\alpha}(\Pi_0) \rightarrow C^{\alpha}([1, 0] \times \Pi_0) + C^{\alpha}([-1, 0] \times \Pi_0) + C^{\alpha}(\Pi_0) \). The bifurcation point \( H_0 \) is defined by the dispersive relation

\[
\frac{\mu(\mu - 1)^2}{\mu + 1} H_0^2 \frac{\sin s_{mn}}{\sqrt{\sin s_{mn}}} = \gamma + s_{mn}^2, \quad s_{mn}^2 = m^2 n^2 + n^2 b^2
\]

We are showing that they are possible: two-multiple degeneration (rolls), four-multiple one (the interaction of two degenerate lattices, or one rectangular lattice), 6-multiple one (hexagonal lattice), 8-multiple (the interaction of rectangular and hexagonal lattices) and 12-multiple (double hexagon–four lattices of periodicity,
two of which are degenerate). Note, that the solutions of the last type were not observed until now in synergetics. The case of infinite depth was considered in [1,2]. Here it is studied the layer of finite depth [3,4]. This problem has applications in cosmonautics [5]. At the usage of group analysis methods [6] and their applications in bifurcation theory [7,8] we construct and investigate the branching equations for each of indicated cases of the operator $B$ degeneracy. Then the asymptotics of periodical solutions are calculated and on the base of [9] their stability is investigated.


AN ALGEBRAIC DISCRETE APPROXIMATION
SCHEME FOR EVOLUTION EQUATIONS WITH
NEUMANN TYPE BOUNDARY CONDITIONS

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Let us for given in a Hilbert space $N^{(2)}(\Omega; \mathbb{R})$ the following evolution equation

$$u_t = K(t, x, \partial)u + f(t, x)$$

where $K(t; x, d) : N^{(2)} \rightarrow N^{(2)}$ – a linear second order differential expression, continuous with respect to the evolution variable $t \in \mathbb{R}^+$, $f(t, x) \in N^{(2)}$, and is continuous for all $t \in \mathbb{R}^+$, $\Omega \subset \mathbb{R}^n$, is bounded domain with the smooth boundary $\partial\Omega$, and $N^{(2)} = \{u \in H^{(2)} : \frac{\partial u}{\partial \nu}|_{\partial\Omega} = 0\} \rightarrow \varphi, \nu$ – a unit normal vector to $\partial\Omega$. We are interested in the solution to (1) a Cauchy problem $u|_{t=0} = \varphi \in N^{(2)}$, which is suggested to be found via a discrete approximation scheme [1] based on Lie algebraic properties of the Heisenberg algebra $\wp := \bigoplus_{j=1}^n \{x_j, \frac{\partial}{\partial x_j}, 1\}$. Namely we first construct a matrix quasirepresentation $\wp_N$ of this Heisenberg algebra $\wp$ on a finite–dimensional interpolation space $N^{(2)}_N$, $N := \text{dim } N^{(2)}_N \in \mathbb{Z}^+$, with conditions formulated in [2], and next we define an associated sequence of linear homomorphisms $\Phi_N : N^{(2)} \rightarrow N^{(2)}_N$ approximating the space $N^{(2)}$ and correspondingly the evolution equation (1) as

$$u_{N,t} = K_N(t)u_N + f_N(t), \quad u_N|_{t=0} = \varphi_N$$

for $u_N, \varphi_N$ and $f_N \in N^{(2)}_N$, $K_N(t) : N^{(2)}_N \rightarrow N^{(2)}_N$ satisfying the condition $\lim_{N \to \infty} \|K_N\Phi_N u - \Phi_N u\|_N = 0$, where $\|\cdot\|_N$ – any equivalent norm in $N^{(2)}_N$, $N \in \mathbb{Z}^+$.

As a result of functional analysis of the equation (2) one can state the following proposition.

**PROPOSITION.** If linear homomorphisms $\Phi_N : N^{(2)} \rightarrow N^{(2)}_N$ generate a compatible hierarchy of quasirepresentations $\wp_N : N^{(2)} \rightarrow N^{(2)}_N$, i.e. $\Phi_N : \Phi_N^{-1}N^{(2)}_N \rightarrow N^{(2)}_N$ are Lie algebra quasi–homomorphisms, then solutions to the finite–dimensional discretized evolution equation (2) as $N \to \infty$, approximate that to Eqn. (1) which accuracy at $n = 1$ is equal to $\left(\frac{C}{N}\right)^{N-1}, C > 0$.


STRUCTURE OF DISTRIBUTION NULL-SOLUTIONS TO FUCHSIAN PARTIAL DIFFERENTIAL EQUATIONS

Takeshi Mandai

AMS Class.: 35D05 (35A07, 35C20)

Consider a Fuchsian partial differential operator with weight \(m - k\) in a neighborhood of \((0, 0) \in \mathbb{R} \times \mathbb{R}^n\) in the sense of M. S. Baouendi and C. Goulaouic ([1])

\[
P = t^k \partial_t^m + \sum_{j=1}^{m} a_j(x) t^{k-j} \partial_t^{m-j} + \sum_{j<m} \left( \sum_{|\alpha| \leq m-j} b_{j,\alpha}(t, x) t^{d(j)} \partial_t^j \partial_x^\alpha \right),
\]

where \((t, x) = (t, x_1, \ldots, x_n)\) are variables in \(\mathbb{R} \times \mathbb{R}^n\), \(k\) and \(m\) are integers satisfying \(0 \leq k \leq m\), and \(d(j) := \max\{0, j - m + k + 1\}\). When \(k = m\), M. Kashiwara and T. Oshima ([4]) called such an operator “to have regular singularity in a weak sense along \(\Sigma_0 := \{t = 0\}\)”.

In the following two categories (coefficients, data, solutions)

(a) functions real-analytic in \((t, x)\),

(b) functions real-analytic in \(x\) and of class \(C^\infty\) in \(t\),

Baouendi and Goulaouic ([1]) showed the following results:

A. the unique solvability of the Cauchy problems (Cauchy-Kowalevsky-type theorem, Nagumo-type theorem),

B. the uniqueness in a wider class of solutions (Holmgren-type theorem).

In the category of

(c) functions of class \(C^\infty\) in \((t, x)\),

H. Tahara ([6], and so on) showed similar results to A and B for “Fuchsian hyperbolic operators”, which are Fuchsian operators satisfying the hyperbolicity and Levi conditions.

In all these cases, it easily follows that there exists no sufficiently smooth null-solutions of \(P\). Here, a distribution \(u\) near \((t, x) = (0, 0)\) is called a null-solution of \(P\), if \(Pu = 0\) near \((0, 0)\) and \((0, 0) \in \text{supp} u \subset \Sigma_+ := \{t \geq 0\}\). If \(k = 0\), then there exist no distribution null-solutions, since \(\Sigma_0\) is noncharacteristic for \(P\). From now on, we assume \(k \geq 1\).

K. Igari ([3]) showed the existence of a distribution null-solution under a weak additional condition in Case (a). This solution is real-analytic in \(x\), that is, \(u \in \mathcal{D}'_+(-T, T; \mathcal{O}(\Omega))\). Here, \(T > 0\), \(\Omega\) is a domain in \(\mathbb{C}^n\) including 0, \(\mathcal{O}(\Omega)\) denote the space of holomorphic functions on \(\Omega\), \(\mathcal{D}'(-T, T; X)\) denotes the space of \(X\)-valued distributions on \((-T, T)\), and \(\mathcal{D}'_+(-T, T; X) := \{u \in \mathcal{D}'(-T, T; X) | u = 0\text{ on }(-T, 0)\}\).

T. Mandai ([5]) showed the existence of distribution null-solutions with no additional conditions in Case (a),(b),(c). These solutions also belong to \(\mathcal{D}'_+(-T, T; \mathcal{O}(\Omega))\) (Case (a),(b)), or \(\mathcal{D}'_+(-T, T; C^\infty(\Omega))\) (Case (c)).
In this contribution, we study the structure of the (germs of) solutions belonging to $\mathcal{D}'_+(-T,T;\mathcal{O}(\Omega))$ in Case (b). The readers might feel the space unnatural. We can not, however, expect similar results in other spaces without any additional assumptions. The author believes that we will be able to show a similar structure theorem of the solutions belonging to $\mathcal{D}'_+(-T,T;\mathcal{D}'(U))$ for Fuchsian hyperbolic operators in Case (c), where $U$ is a domain in $\mathbb{R}^n$ including 0, by a similar method based on the same idea.

Now, let $\mathcal{O}_0 := \text{ind lim}_{0 \in \Omega} \mathcal{O}(\Omega)$ be the space of all germs of holomorphic functions at $0 \in \mathbb{C}^n$, and $(\text{Ker} \mathcal{D}'_+ P)_{(0,0)} := \text{ind lim}_{0 < T, 0 \in \Omega} \{ u \in \mathcal{D}'_+(-T,T;\mathcal{O}(\Omega)) \mid Pu = 0 \}$, $(\text{Ker} \mathcal{D}' P)_{(0,0)} := \text{ind lim}_{0 < T, 0 \in \Omega} \{ u \in \mathcal{D}'(-T,T;\mathcal{O}(\Omega)) \mid Pu = 0 \}$ be the spaces of all germs of solutions of $Pu = 0$ belonging to the respective spaces.

**Theorem.** Let $P$ be a Fuchsian operator (1), with the coefficients belonging to $C^\infty(-T_0,T_0;\mathcal{O}(\Omega_0))$, where $T_0 > 0$ and $\Omega_0$ is a domain in $\mathbb{C}^n$ including 0. Then, there holds

$$\mathcal{O}_0^k \cong (\text{Ker} \mathcal{D}'_+ P)_{(0,0)}, \quad (\mathcal{O}_0)^{m+k} \cong (\text{Ker} \mathcal{D}' P)_{(0,0)}.$$

Further, we can construct the isomorphisms (invertible linear maps) rather concretely, by expressing the solutions in (generalized) asymptotic series.

Note that the non-zero elements in $(\text{Ker} \mathcal{D}'_+ P)_{(0,0)}$ are the null-solutions of $P$.

We would like to emphasize the fact that the theorem includes the case when the characteristic indices may differ by integers, which the already known results exclude. It is also new to consider solutions with supports in $\Sigma_+$. 


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NONLINEAR POTENTIAL THEORY
OF SUBELLIPTIC EQUATIONS AND APPLICATIONS

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AMS Class.: 31C45 (31C12)

We obtain some results that relate to the nonlinear potential theory that is associated with solutions to the second order subelliptic equation

$$-\text{div}_x A(x, \nabla u) = 0,$$

where $\nabla u = (X_1 u, \ldots, X_k u)$ is the subgradient defined by $C^\infty$-vector fields $(X_1, \ldots, X_k)$ satisfying the Hörmander condition. Recall that the Hörmander condition means that, at each point of the domain, the successive commutators of given vector fields span the tangent space and the length of the commutators does not exceed a fixed number independent of the point. In particular, the sum of squares of the vector fields can serve as a model equation that generalizes the Laplace equation.

A peculiarity of our approach to the potential theory consists in studying the boundary behavior of harmonic and superharmonic functions from the viewpoint of intrinsic geometry of the domain. The boundary of the domain is obtained as a result of the Hausdorff completion of the metric space $D_1 = (D, d_D)$ with respect to the intrinsic metric $d_D$.

Note that $A$-superharmonic functions can be defined by comparison with continuous solutions to equation (1). The latter are called $A$-harmonic functions. In the framework of this approach, we need to prove that $A$-superharmonic functions, whose definition does not require any a priori regularity, possess some additional properties that allow us, under some conditions, to consider these functions as supersolutions to equation (1).

Regularity of supersolutions to equation (1), as well as interrelations between the capacity in Sobolev spaces and the geometry of vector fields, is studied. In addition, metric and analytic conditions are obtained for removing singularities of bounded solutions to general equations, in particular, to the equations of the form (1).

Note that, as in the theory of elliptic equations, studying $A$-superharmonic functions possess the following basic properties: the comparison principle, the Harnack inequality for $A$-harmonic functions, the convergence theorems for monotone sequences, etc. This fact allows us to develop a similar theory. Particular attention is focused on the questions connected with the geometry of vector fields.

As an application of non-linear potential theory we consider the classification of sub-riemannian manifolds.


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MULTIPLICITY RESULT FOR BUCKLING-BENDING PROBLEM

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AMS Class.: 34B15 (58C27, 73C50)

The equilibrium states of combined buckling-bending problem for thin clamped elastic circular plates can be found by solving the associated nonlinear von Kármán equations, which can be written in the form:

\[
\begin{align*}
[x^3u'(x)]' + \lambda x^3w(x) &= x^3w(x)f(x) + x^3g(x), \quad x \in (0, 1) \\
[x^3f'(x)]' &= -x^3w^2(x), \quad x \in (0, 1)
\end{align*}
\]  

where \( w(x) \) is the dimensionless radial derivative of transverse displacement, \( f(x) \) is the dimensionless radial derivative of the radial stress \( \lambda \) is the dimensionless load parameter and \( x \) is the dimensionless radial distance, with boundary conditions

\[
\lim_{x \to 0^+} xw(x) = 0, \quad w(1) = 0, \quad \lim_{x \to 0^+} xf(x) = 0, \quad f(1) = 0
\]

We denote \((1) + (2) + (3) = (CP)\). Let \( W^{1,2}((0, 1), x^3) \) be the real Sobolev space with the weight \( x^3 \) equipped with the following scalar product and norm:

\[
(u, v)_{1,2,x^3} = \int_0^1 x^3u(x)v(x)dx + \int_0^1 x^3u'(x)v'(x)dx, \quad \|u\|_{1,2,x^3} = \left( (u, u)_{1,2,x^3} \right)^{\frac{1}{2}}
\]

We denote

\[
M = \{ u \in C^\infty([0, 1]); \ u(1) = 0 \}
\]

and let \( V \) be the closure of \( M \) in the norm \( \|\cdot\|_{1,2,x^3} \). Then \( V \) equipped with the scalar product

\[
(u, v) = \int_0^1 x^3u'(x)v'(x)dx
\]

and the corresponding norm \( \|u\| = \left( (u, u) \right)^{\frac{1}{2}} \)

be the subspace of \( W^{1,2}((0, 1), x^3) \). The norms \( \|\cdot\| \) and \( \|\cdot\|_{1,2,x^3} \) are equivalent on the Sobolev space \( V \).

Using the standard approach we define the classical and the generalized solutions of the problem (CP). Then we can prove the following theorem:

**Theorem 1.** Any generalized solution of the problem (CP) is classical solution.

**Theorem 2.** The generalized solutions of the problem (CP) are identical with the solutions of a pair of operator equations

\[
w - \lambda Lw + N(w) = q, \quad f = B(w, w)
\]

defined on a Sobolev space \( V \), where \( N(w) = B(B(w, w), w) \) and \( B : V \times V \rightarrow V \) is bounded, bilinear, symmetric compact operator, \( L : V \rightarrow V \) is bounded, linear,
selfadjoint compact operator, \( N : V \to V \) is bounded, nonlinear (cubic) compact operator.

The nonlinear operator on the left hand side of (4) \( F_\lambda(w) = w - \lambda Lw + N(w) \) has the following property:

**Theorem 3.** For \( \lambda \in \mathbb{R} \) fixed, the mapping \( F_\lambda : V \to V, w \mapsto w - \lambda Lw + N(w) \) is the \( C^\infty \) nonlinear proper Fredholm operator of index zero.

Now we describe the solution structure of the equation (4). Let \( 0 < \lambda_1 < \lambda_2 < \lambda_3 < \ldots \) denote the eigenvalues of the linearized problem:

\[
F_\lambda'(0)(u) = u - \lambda Lu = 0 \quad \text{in} \quad V,
\]

where \( F_\lambda'(0)(u) \) is the Fréchet derivative of \( F_\lambda(w) \) at the point 0 in the direction \( u \).

**Definition 2.** Let \( X, Y \) be a real Banach spaces and \( \Omega \subset X \) be open. The point \( x_0 \in \Omega \) is called singular point of the mapping \( f : X \to Y \) if \( f'(x_0) : X \to Y \) is not an isomorphism. The value \( f(x_0) \in Y \) is called a singular value.

Let \( S \) be the set of singular points of \( F_\lambda \).

**Theorem 4.** If \( \lambda \leq \lambda_1 \), then the set of singular points of \( F_\lambda \) denoted by \( S \) is empty set.

Then for \( \lambda \leq \lambda_1 \) we have the following uniqueness result:

**Theorem 5.** For \( \lambda \leq \lambda_1 \) the equation (4) has unique solution in \( V \) for any \( q \in V \).

Now we suppose \( \lambda > \lambda_1 \).

**Theorem 6.** Let \( S_1 \) be the unit sphere in \( V \). Then there exists a radial diffeomorphism \( g : S_1 \to R \), such that \( u \mapsto g(u)u \in S \).

**Theorem 7.** If \( \lambda \in (\lambda_1, \frac{\lambda_1 + \epsilon}{2}) \), then the set of singular points \( S \) contains only fold points and cusp points. (\( \lambda_1 < \frac{\lambda_1 + \epsilon}{2} < \lambda_2 \))

**Theorem 8.** There exists one to one correspondence between \( S \) and \( W = F_\lambda(S) \).

**Theorem 9.** If \( \lambda \in (\lambda_1, \frac{\lambda_1 + \epsilon}{2}) \), then there exists in \( V \) a manifold of codimension 1 denoted by \( W \) such that \( V \setminus W = V_1 \cup V_3 \) consists of two open components \( V_1, V_3 \) with \( 0 \in V_3 \) and if \( q \in V_3 \) then the equation (4) has exactly three solutions, if \( q \in V_1 \), then the equation (4) has exactly one solution.


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STABILITY OF DIFFERENCE EQUATIONS

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AMS Class.: 39A10

Theory of stability in the sense of Lyapunov is well known and is used in concret problems of the real life. It is obvious that, in applications, asymptotic stability is more important than stability, because the desirable feature is to know the size of the region of asymptotic stability. However, in the study of asymptotical stability it is difficult to work with non-exponential types of stability. In [1,2] the study of exponential stability was extended to a variety of reasonable systems called h-systems. It is shown in this way that the typical theorems related to perturbations remain valid for h-systems.

We remark that the new concept of stability (called h-stability) has several implications in difference equations:

a) It permits to obtain a uniform treatment for the concept of stability in difference equations.

b) For non-autonomous systems, we can estimate the solutions not only in exponential form, but for more general functions. Thus, we can handle stabilities where the solutions are weakly stables.

c) The notion of h-systems permits to obtain asymptotic formulae for weakly stable difference equations.

In the present work we continue the research initiated in [1,2] addressing our study to characterize the h-stability for non-autonomous difference equations. As a consequence, we establish some results on asymptotic behavior of perturbed difference equations.

Consider the non-linear difference system

\[ x(n+1) = f(n, x(n)), \]  

along with its associated variational system

\[ z(n+1) = f_x(n, x(n, n_0, x_0))z(n), \]  

where \( f : \mathbb{N} \times \mathbb{R}^m \rightarrow \mathbb{R}^m \) and \( \Phi(n, n_0, x_0) = \frac{\partial x(n,n_0,x_0)}{\partial x_0} \) is the solution of equation (2).

Our main result is the following. **Theorem.** Let there exists a positive function \( h : \mathbb{N} \rightarrow \mathbb{R} \) and a constant \( c \geq 1 \). Then, \( ||\Phi(n+1, n_0, x_0)|| \leq ch(n)h(n_0)^{-1} \), for all \( n \geq n_0 \) and \( x_0 \in \mathbb{R}^m \), if and only if, there exists a Lyapunov function \( V(n, z) \) which satisfies the following properties:

a) \( V(n, z) \) is defined on \( \mathbb{N} \times \mathbb{R}^m \) and continuous with respect to the second argument.
b) \[ ||x - y|| \leq V(n, x - y) \leq c||x - y||, \text{ for } (n, x, y) \in \mathbb{N} \times \mathbb{R}^m \times \mathbb{R}^m. \]

c) \[ |V(n, z_1) - V(n, z_2)| \leq c||z_1 - z_2||, \text{ for } n \in \mathbb{N}, \ z_1, z_2 \in \mathbb{R}^m \]

d) \[ \Delta V_1(n, x - y) \leq -\left(1 - \frac{h(n+1)}{h(n)}\right)V(n, x - y), \text{ for } (n, x, y) \in \mathbb{N} \times \mathbb{R}^m \times \mathbb{R}^m. \]


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ALGEBRAS OF MONOGENIC FUNCTIONS
AND AXIAL-SYMMETRICAL POTENTIALS

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AMS Class.: 35J99 (30H05)

System of equations $y \frac{\partial \varphi}{\partial x} = \frac{\partial \psi}{\partial y}, \ x \frac{\partial \varphi}{\partial y} = - \frac{\partial \psi}{\partial x}$ describe the space potential
solenoid field which is symmetrical in relation to the axis $Ox$ in its meridional plane $xOy$. Here $\varphi$ is potential and $\psi$ is flow Stokes function.

Construction of methods for description of space potential solenoid fields with
axial symmetry which are analogous to the analytic functions methods in the
complex plane for the description of plane potential fields is the problem which
was formulated by M.A. Lavrentieff.

We have constructed analytic functions of vector variable with values in in-
finitely dimensional Banach algebra of even Fourier series and have proved that
components of these functions generate the axial-symmetrical potential functions
and Stokes flow functions both in bounded and non bounded domains of meriodi-
anal plane.

Let $R$ be algebra of real numbers and $C$ be algebra of complex numbers.

Let $H_C := \{a(\tau) = \sum_{k=1}^{\infty} a_k e_k, \text{ where } a_k \in C, e_k = i^{k-1} \cos(k-1)\tau, i$ is the
complex imaginary unit, $\sum_{k=1}^{\infty} |a_k| < \infty\}$ be Banach algebra of complex even
Fourier series with the norm $|a(\tau)|_{H_C} := \sum_{k=1}^{\infty} |a_k|$ and with the basis $\{e_k\}_{k=1}^{\infty}$.

Let $I_0 := \{a(\tau) \in H_C : a(0) = 0\}$ be the maximum ideal of algebra $H_C$ and
$f_{I_0} : H_C \rightarrow C$ be the linear functional such that $I_0$ is its kernel. Let $A$ be the linear operator which assigns to every function $\Phi : D_\zeta \rightarrow H_C$ the function
$F : D_z \rightarrow C$ by the formula $F(z) := f_{I_0}(\Phi(\zeta))$.

Theorem 1. Every monogenic function $\Phi : D_\zeta \rightarrow H_C$ can be expressed in
the form $\Phi(\zeta) = \frac{1}{2\pi i} \int_\gamma (te_1 - \zeta)^{-1}(A\Phi)(t)dt + \Phi_0(\zeta) \quad \forall \zeta \in D_\zeta$.

Here $\gamma$ is arbitrary closed rectifiable Jordan curve in $C$ which restrict a right
domain $D'_z$ such that $D'_z \subset D_z$ and $z = f_{I_0}(\zeta) \in D'_z$, the function $\Phi_0 : D_\zeta \rightarrow I_0$
is a monogenic function with values from $I_0$.

In this case the first and the second components of integral

\[
\frac{1}{2\pi i} \int_\gamma (te_1 - \zeta)^{-1}(A\Phi)(t)dt = \sum_{k=1}^{\infty} U_k(x,y)e_k, \quad \zeta = xe_1 + ye_2 \in D_\zeta,
\]
generate the axial-symmetrical potential and Stokes flow functions $\varphi, \psi$ in the
domain $D$ by the formulas ( when $y \neq 0$ )

\[
\varphi(x,y) = U_1(x,y) = \frac{1}{2\pi i} \int_\gamma \frac{(A\Phi)(t)}{\sqrt{(t-z)(t-\bar{z})}} dt, \quad z = x + iy,
\]

\[
\psi(x,y) = \frac{1}{2} U_2(x,y) = -\frac{1}{2\pi i} \int_\gamma \frac{(A\Phi)(t)}{\sqrt{(t-z)(t-\bar{z})}} dt, \quad \zeta = xe_1 + ye_2 \in D_\zeta.
\]
where \( \sqrt{(t - z)(t - \bar{z})} \) is a continuous branch of this function with break \( \{x + i\eta : |\eta| \leq |y|\} \).

These formulas are new expressions of functions \( \varphi, \psi \) in domain \( D \).

The element \( e_2 \) have not the inverse element in algebra \( H_C \).

Introduce the element \( e_0 \notin H_C \) which satisfies the rules for the multiplication:
\[
e_0 e_1 = e_0, e_0 e_2 = -e_1, e_0 e_2k+1 = e_0 - 2\sum_{m=1}^{k} e_2m, e_0 e_2k+2 = -e_1 - 2\sum_{m=1}^{k} e_2m+1
\]
for \( k = 1, 2, \ldots \).

Include the algebra \( H_C \) into the Banach space \( \tilde{H}_C := \{c = \sum_{k=0}^{\infty} c_k e_k : c_k \in \mathbb{C}, \sum_{k=0}^{\infty} |c_k| < \infty \} \) with the norm \( |c|_{\tilde{H}_C} := \sum_{k=0}^{\infty} |c_k| \). Notice that \( \tilde{H}_C \) is only an extension of linear space of algebra \( H_C \) and is not an algebra.

Consider the plane \( \tilde{\mu} := \{ \tilde{\zeta} = xe_1 + ye_0 : x, y \in \mathbb{R} \} \) in \( \tilde{H}_C \). Denote
\[D_{\tilde{\zeta}} := \{ \tilde{\zeta} = xe_1 + ye_0 : (x, y) \in D \} \subset \tilde{\mu}.
\]
Let \( \tilde{f}_{l_0} : \tilde{H}_C \to \mathbb{C} \) be the linear functional such that \( \tilde{f}_{l_0}(e_k) = f_{l_0}(e_k) \) for \( k = 1, 2, \ldots \) and \( f_{l_0}(e_0) = i \). Let \( B \) be the linear operator which assigns to every function \( \Psi : \tilde{\mu} \setminus D_{\tilde{\zeta}} \to H_C \) the function \( G : C \setminus \overline{D_z} \to \mathbb{C} \) by the formula
\[G(z) := \tilde{f}_{l_0}(\Psi(\tilde{\zeta})).\]

We have proved an analogue of the theorem 1 for monogenic functions \( \Psi : \tilde{\mu} \setminus D_{\tilde{\zeta}} \to H_C \).

In this case components of integral
\[\frac{e_0}{2\pi \i} \int_{\Gamma} ((te_1 - \tilde{\zeta})^{-1} (B\Psi)(t)) dt = \sum_{k=1}^{\infty} V_k(x, y)e_k, \quad \tilde{\zeta} = xe_1 + ye_0 \in \tilde{\mu} \setminus D_{\tilde{\zeta}}\]
generate the axial-symmetrical potential and Stokes flow functions \( \varphi, \psi \) in \( \mathbb{R}^2 \setminus \overline{D} \) by the formulas (when \( y \neq 0 \))
\[\varphi(x, y) = \pm V_1(x, y) \equiv \frac{1}{2\pi \i} \int_{\Gamma} \frac{(B\Psi)(t)}{\sqrt{(t-z)(t-\bar{z})}} dt, \quad z = x + iy, \pm y > 0,\]
where \( \sqrt{(t-z)(t-\bar{z})} \) is a continuous branch of this function with break \( \{x + i\eta : |\eta| \geq |y|\} \) (the expressions of functions \( \varphi, \psi \) in \( \mathbb{R}^2 \setminus \overline{D} \) was obtained for the first time). Here \( \Gamma \) is arbitrary closed (bounded in that is possible or no bounded) rectifiable or local rectifiable Jordan curve in \( \mathbb{C} \) which restrict a right domain \( D''_z \) such that \( \overline{D_z} \subset D''_z \) and \( z = \tilde{f}_{l_0}(\tilde{\zeta}) \in \mathbb{C} \setminus D''_z \).

We also have proved that all the axial-symmetrical potential and Stokes flow functions in \( D \) and in \( \mathbb{R}^2 \setminus \overline{D} \) can be expressed by the formulas given above if certain natural conditions are satisfied.

Proofs of results given above are contained in the work [1].


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QUALITATIVE THEORY OF INTEGRODIFFERENTIAL SYSTEMS

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In the present paper the $L^p$-boundedness ($p \geq 1$) for solutions and its derivatives of integrodifferential systems are investigated. We will find the conditions implying that all $j$-th derivatives $0 \leq j \leq n-1$ of solutions of a perturbed linear or nonlinear integrodifferential equations belong to $L^p(0, \infty)$ for some $p \geq 1$ provided that all derivatives of solutions of unperturbed equations possess the same property. Analogous results for differential equations have been considered [2].

The purpose of this paper is to extend above results to the case of $n-\text{th}$ order integrodifferential systems

$$T[x] \equiv x^{(n)}(t) + \sum_{k=0}^{n-1}[A_{k0}(t)x^{(k)}(t) + \int_{t_0}^{t} K_{k0}(t,s)x^{(k)}(s)ds] = f(t) \quad (T)$$

and corresponding linear and nonlinear perturbation

$$T_1[x] \equiv x^{(n)}(t) + \sum_{k=0}^{n-1}[A_{k0}(t) + B_{k0}(t)]x^{(k)}(t) + \int_{t_0}^{t} [K_{k0}(t,s) + R_{k0}(t,s)]x^{(k)}(s)ds + f(t), \quad (T1)$$

$$T_2[x] = T[x] - G(x)(t) = 0 \quad (T2)$$

with initial conditions $x^{(k)}(t_0) = \Phi_k(t_0), k = 0, 1, \ldots, n-1$, where $A_{k0}, B_{k0}, K_{k0}, R_{k0}$ are $r \times r$ matrices, $x, f$ are $r$-dimensional vectors,

$$G(x)(t) = G(t, x(t), x'(t), \ldots, x^{(n-1)}(t)).$$

**Definition.** If $Y_1$ and $Y_2$ are Frechet spaces we say that $\rho$ is an admissible map [1] from $Y_1$ into $Y_2$ if $\rho(Y_1)$ is contained in $Y_2$ and $\rho$ is continuous as a map from $Y_1$ into $Y_2$. The set of admissible maps from $Y_1$ into $Y_2$ will be denoted by $A(Y_1, Y_2)$. Examples of Frechet subspace of Frechet space $LL^1$ are $C, BC, BC_1$.

$BC_l$ - the set of all functions in $BC$ having a limit at infinity and $L^p \cap BC_0$ – set of functions in both $L^p$ and $BC_0$.

Consider the equation (T1) with $A_{k0}(t) = A_{k0} = const$ and $K_{k0}(t,s) = K_{k0}(t-s)$ and $\Psi(t) = B(t) + \int_{t_0}^{t} R(t,s), where B(t) = \max_{k} |B_{k0}(t)|, R(t,s) = \max_{k} |K_{k0}(t,s)|, k = 0, 1, \ldots, n-1$. $\Psi(s)$ is small. It means that $\Psi(s)$ is such that $||\Psi|| < (n|X_0|_1)^{-1}, X_0(t) = \max_{k} |X^{(k)}_T(t)|$.

**Theorem.** Suppose in equation (T1) that $X^{(k)}_T$ are in $L^1$ for every $k = 0, 1, \ldots, n-1$, $f(t)$ is in $BC$ and $y^{(k)}_0(t)$ are in $BC$ where $y(t)$ is the solution of (T) ($f = 0$) with initial conditions $y^{(k)}_0(t_0) = y^{(k)}_0, k = 0, 1, \ldots, n-1$. If $\Psi(t)$ is in $BC$ and $\Psi(t)$ is small, then $x^{(k)}(t), k = 0, 1, \ldots, n-1$ are in $BC$ and $\rho^{(k)}_{T_1}(f)$ are in $A(BC, BC)$, where $x(t)$ is the solution of (T1) and $\rho^{(k)}_{T_1}(f)(t) = \int_{t_0}^{t} X^{(k)}_{T_1}(t,s)f(s)ds$. 
**Corollary.** In equation (T1) suppose $y^{(k)}(t)$ are in $BC_0$, $X^{(k)}_T(t)$ are in $L^1$, $g(t)$ is in $BC_0$, $g(t) = - \sum_{k=0}^{n-1} [B_k(t)x^{(k)}(t) + \int_{t_0}^{t} R_k(t,s)x^{(k)}(s)ds]$. If $f(t)$ is in $BC_I(BC_0)$ then $\lim_{t \to \infty} x^{(k)}(t) = \int_{t_0}^{\infty} X^{(k)}_T(s)ds f(\infty)$. Furthermore, $\rho^{(k)}_{T_1}$ are in $A(BC_I, BC_1)[A(BC_0, BC_0)]$.

**Theorem.** Let $X^{(k)}_T(t)$ are in $L^1 \cap BC$, $f(t)$ is in $L^1 \cap BC$, $y^{(k)}(t)$ are in $L^1 \cap BC$. If $\Psi(t)$ is in $L^1 \cap BC$, then $x^{(k)}(t)$ are in $L^p \cap BC$ and $\rho^{(k)}_{T_1}$ are in $A(L^p \cap BC, L^p \cap BC)$.


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A FUNCTIONAL DIFFERENTIAL EQUATION
IN BANACH SPACES

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AMS Class.: 34G20

The study of the Cauchy problem for differential and functional equations in a Banach space relative to the strong topology has attracted much attention in recent years. However a similar study relative to the weak topology was studied by many authors, for example, Szep [8], Mitchell and Smith [6], Szufa [9], Kubiaczyk [3, 4], Kubiaczyk and Szufa [5], Cichoń [1], Cichoń and Kubiaczyk [2], and others.

Let \( E \) be a Banach space, \( E^* \) the dual space. We set \( B = \{ x \in E: \| x-x_0 \| \leq b \} \).

We denote by \( C(I, w) \) the space of all continuous function from \( I \) to \( E \), and by \((C(I, w), w) \) the space \( C(I, E) \) with the weak topology. Put

\[
\tilde{B} = \{ x \in C(J, E): x(J) \subset B, \| x(t) - x(s) \| \leq M|t-s|, \text{ for } t, s \in J \},
\]

where \( J = [0, h], \ h = \min \{ a, \frac{b}{37} \} \) and \( M > 0 \) is a constant.

We deal with the Cauchy problem:

\[
x' = Fx, \quad x(0) = x_0, \quad t \in I = [0, a]. \tag{1}
\]

in the case of \( F \) being an bounded operator of Volterra type from \( \tilde{B} \) into \( P(I, E) \) (the space of all Pettis integrable functions on \( I \)).

Now fix \( x^* \in E^* \), and consider

\[
(x^*x)'(t) = x^*((Fx)(t)), \quad t \in I. \tag{1'}
\]

**Definition 1.** A function \( x: I \to E \) is said to be a pseudo-solution of the Cauchy problem (1) if it satisfies the following conditions:

(i) \( x(\cdot) \) is absolutely continuous,

(ii) \( x(0) = x_0 \),

(iii) for each \( x^* \in E^* \) there exists a negligible set \( A(x^*) \) (i.e., \( \text{mes}(A(x^*)) = 0 \)) such that for each \( t \notin A(x^*) \),

\[
x^*(x'(t)) = x^*((Fx)(t)).
\]

Here ‘ denotes a pseudoderivative (see Pettis [7]).

In other words, by a pseudo-solution of (1) we will mean an absolutely continuous function \( x(\cdot) \), with \( x(0) = x_0 \), satisfying (1') a.e. for each \( x^* \in E^* \).

Now suppose that

(*) for each strongly absolutely continuous function \( x: J \to E \), \( (Fx)(\cdot) \) is Pettis integrable, \( F(\cdot) \) is weakly-weakly sequentially continuous, then the existence of a pseudo-solution of (1) is equivalent to the existence of a solution for

\[
x(t) = x_0 + \int_0^t (Fx)(s)ds, \tag{2}
\]
Theorem 1. If $F$ is a bounded continuous operator of Volterra type from $\tilde{B}$ into $P(I,E)$ and the assumptions (*) and

$$\beta\left(\bigcup\{(Fx)\mid J: x \in \tilde{X}\}\right) \leq r(\beta(X))$$

holds for every subset $X$ of $B$, where $r$ is a non-decreasing Kamke function and $\beta$ is the measure of weak noncompactness. Then the set $S$ of all pseudo-solutions of the Cauchy problem (1) on $J$ is a nonempty and compact in $(C(J,E), \omega)$.

Remark 1. In the proof of Theorem 1, we use the extension of Ambrosetti Lemma (see [6]) and the fixed point theorem of Kubiaczyk [4].

Remark 2. One can easily prove that the integral of a weakly continuous function is weakly differentiable with respect to the right endpoint of the integration interval and its derivative equals the integral at the same point (see [6] Lemma 2.3). In this case a pseudo-solution is, actually, a weak solution. Moreover, in some classes of spaces our pseudo-solutions are also strong $C$-solutions (in separable Banach spaces, for instance).


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ON A TURNING POINT PROBLEM AND THE WKB METHOD

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We consider the differential equation containing a small positive parameter $\varepsilon$

$$\varepsilon^{nh}y^{(n)} = \sum_{k=1}^{n} \varepsilon^{(n-k)h} p_k \cdot (x^m - \varepsilon^lx^{-r})^k y^{(n-k)}, \quad (1)$$

where $h$ is a positive constant, $l$ and $m$ are positive integers and $r$ is zero or a positive integer, which satisfy the singular perturbation condition:

$$h - (m + 1)l/(m + r) > 0.$$

The complex $x$-region is $0 < |x| \leq x_0$ for $r \neq 0$ or $|x| \leq x_0$ for $r = 0$. We suppose the characteristic equation of (1) is given by

$$L(x, \lambda) := \lambda^n - \sum_{k=1}^{n} p_k \cdot x^{km} \lambda^{n-k} = \prod_{k=1}^{n} (\lambda - a_k x^m) = 0, \quad (2)$$

where $\forall a_k \neq 0$ and $a_{k-1} < a_k$ ($k = 2, 3, \cdots, n$). Since the characteristic roots coincide at the origin, the origin is called a turning point of (1).

Then the differential equation (1) has $n$ outer WKB solutions

$$\tilde{y}_j^{\text{out}}(x, \varepsilon) := x^m \sum_{k \neq j} \frac{-a_j}{aj-a_k} \exp \left( \frac{a_j x^{m+1}}{\varepsilon^h (m + 1)} \right) \quad (3)$$

in the outer region $K\varepsilon^{1/(m+r)} \leq |x| \leq x_0$ and $n$ inner WKB solutions

$$\tilde{y}_j^{\text{in}}(t, \varepsilon) := p(t)^{\sum_{k \neq j} \frac{-a_j}{aj-a_k}} \exp \left( \frac{a_j}{\varepsilon^h} \int_{t_0}^{t} p(s)ds \right) \quad (4)$$

in the inner region $0 < |t| < \infty$ ($r \neq 0$) or $|t| < \infty$ ($r = 0$), where $p(t) := t^m - t^{-r}$, $t = x\varepsilon^{-1/(m+r)}$. Those WKB solutions are connected by the matching matrix.


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ON SPECIAL HYPERBOLIC SYSTEMS OF CONSERVATION LAWS

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Let’s $X$ be a space consisting of symmetric $2 \times 2$-matrixes $U = \begin{pmatrix} a & c \\ c & b \end{pmatrix}$, $a, b, c \in \mathbb{R}$; so dim $X = 3$. We consider the next Cauchy problem for a system of conservation laws:

$$
U_t + \tilde{f}(U)x = 0, \\
U(0, x) = U_0(x) \in L^\infty(\mathbb{R}, X),
$$

$$U = U(t, x) \in X; (t, x) \in \Pi = (0, +\infty) \times \mathbb{R}; \tilde{f} : X \mapsto X, U \mapsto \tilde{f}(U)$$ is a function of a matrix $U$ defined by some scalar real function $f(u) \in C^1(\mathbb{R})$. The system (1) is proved to be nonstrictly hyperbolic. Moreover for all $U \in X$ the operator $A(U) = df(U)$ is symmetric on $X$ with respect to a scalar product $(U, V) = \text{Tr}(UV)$. The system (1) isn’t genuinely nonlinear because one of characteristic fields is linearly degenerate. The distinctive property of the system (1) is presence of a large family of entropies. Remind that entropy $p(U)$ and correspondent entropy flux $q(U)$ are functions on $X$ connected by the relation $\forall U \in X \nabla q(U) = A(U) \nabla p(U)$ (remark that the operator $A(U)$ is symmetric). Let $\lambda_1 \leq \lambda_2$ be eigenvalues of $U$. Then the next pairs $(p, q)$ are entropy pairs

$$
p(U) = \varphi_1(\lambda_1) + \varphi_2(\lambda_2) + h(b - a, -2c), \tag{3}
$$

where $\varphi_1(u), \varphi_2(u) \in C^1(\mathbb{R})$, $h(y) \in C^1(\mathbb{R}^2 \setminus \{0\})$ is a homogeneous (of the first degree) function; $\psi'_i(u) = \varphi'_i(u)f'(u)$, $i = 1, 2$; $\mu = \frac{f(\lambda_1) - f(\lambda_2)}{\lambda_1 - \lambda_2}$. Besides in the case then the function $f$ is not affine on nondegenerate intervals any entropy has the form (3). Remark that smoothness of $p$, $q$ can be broken on the line generated unit matrix ( then $\lambda_1 = \lambda_2$ ) but the next important properties (the same as for smooth entropy pairs) still remain:

P1) If $U$ is a $C^1$-solution of (1) then $p(U)_t + q(U)_x = 0$ in the sense of distributions ( in $D'(\Pi)$);

P2) If $U$ can be obtained by vanishing viscosity method that is $U$ is a limit point in $L_{loc}^1(\Pi, X)$ for a sequence $U_\varepsilon \in C^2(\Pi, X)$ of solutions to parabolic systems $U_t + \tilde{f}(U)x = \varepsilon U_{xx}$ as $\varepsilon \to 0$ then $p(U)_t + q(U)_x \leq 0$ in $D'(\Pi)$ for all convex entropy $p(U)$.

It’s easy to show that the entropy $p(U)$ is smooth on $X$ if and only if

$$p(U) = \text{Tr}\tilde{\varphi}(U), \varphi(u) \in C^1(\mathbb{R}) \text{ with } q(U) = \text{Tr}\tilde{\psi}(U), \psi'(u) = \varphi'(u)f'(u).$$

We define a generalized entropy solution (briefly - g.e.s.) of the problem (1), (2) as a function $U \in L^\infty(\Pi, X)$ satisfying the Kruzhkov-Lax entropy condition (see [1,2]) $p(U)_t + q(U)_x \leq 0$ in $D'(\Pi)$ for all convex entropy of the form (3), and the initial condition (2).
It’s easy to see that the set of g.e.s. is invariant with respect to transforms $U \mapsto QUQ^{-1}$ for constant orthogonal matrices $Q$. Let a norm $\| U \|_p$ on $X$ coincide the operator norm if $p = \infty$ and for $1 \leq p < \infty$ it equals $\left( \text{Tr} f_p(U) \right)^{1/p}$, $f_p(u) = |u|^p$. Then any g.e.s. $U(t, x)$ satisfies the estimates:

1) for a.a. $t > 0 \quad \int \| U(t, x) \|^p dx \leq \int \| U_0(x) \|^p dx$;

2) for a.a. $(t, x) \in \Pi \quad \| U(t, x) \|_\infty \leq \text{ess sup}_{x \in \mathbb{R}} \| U_0(x) \|_\infty$ (maximum principle).

In spite of plenty of entropy conditions a g.e.s. to (1), (2) may be nonunique. In this connection we introduce a notion of a strong generalized entropy solution (briefly - s.g.e.s.) to (1), (2). It is a $X$-valued function $U \in L^\infty(\Pi, X)$ that satisfies (1) in $D'(\Pi, X)$ and (2) and such that its eigenvalues $\lambda_1 \leq \lambda_2$ are generalized solutions (in Kruzhkov sense, see [1]) to a scalar Cauchy problem

$$\lambda_t + f(\lambda)_x = 0, \quad \lambda(0, x) = \lambda_0(x) \tag{4}$$

with correspondent initial data $\lambda^0_1$, $\lambda^0_2$ where $\lambda^0_1 \leq \lambda^0_2$ are eigenvalues of the initial matrix $U_0$. If $U$ is a smooth solution of (1), (2) then it’s proved to be a s.g.e.s. of this problem too. We can represent a symmetric matrix $U$ as $U = T(\theta)\text{diag} \{ \lambda_1, \lambda_2 \} T(-\theta)$, where $\lambda_1 \leq \lambda_2$ and $T(\theta)$ is a rotation operator with an angle $\theta$. If $U = U(t, x)$ is a s.g.e.s. to the problem (1), (2) then the parameters $\lambda_1, \lambda_2$ are uniquely defined by existence and uniqueness of a generalized solution to the scalar problem (4). The remaining parameter $\theta = \theta(t, x)$ is proved to be a solution (in the sense of distributions) of a linear problem

$$((\lambda_2 - \lambda_1)\theta)_t + ((f(\lambda_2) - f(\lambda_1))\theta)_x = 0, \quad (\lambda_2 - \lambda_1)(\theta - \theta_0)|_{t=0} = 0 \tag{5}$$

here the function $\theta_0$ corresponds to the initial data $U_0$. We establish that the problem (5) is resolved and the function $(\lambda_2 - \lambda_1)\theta$ is uniquely defined by the initial data. So we can conclude that there exists an unique s.g.e.s. to the problem (1), (2). It’s also proved that any s.g.e.s. is a g.e.s. too (that is natural). If (may be after transform $U_0 \mapsto QU_0Q^{-1}$ with a constant orthogonal matrix $Q$) for a.a. $x \in \mathbb{R}$ $c_0 > 0$ or $a_0 - b_0 = c_0 = 0$ ($a_0, b_0, c_0$ are components of $U_0$) then any g.e.s. coincides with the s.g.e.s. and therefore is unique. As it is shown by example in this assertion we can’t replace the condition $c_0 > 0$ to $c_0 \geq 0$. For initial data satisfying the above condition we also prove that the sequence $U_\epsilon$ arising from the vanishing viscosity method converges strongly to the unique s.g.e.s. $U$ of the problem (1), (2). In the proof we use the compensated compactness method.

In conclusion note that the main results are generalized to the case of only continuous function $f$.


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GLOBAL BIFURCATIONS AND AN EXISTENCE RESULT FOR QUASILINEAR ELLIPTIC EQUATIONS

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In this paper we announce the results of the note [3]. It is devoted to the Rabinowitz type bifurcations from the first eigenvalue for the pseudo-Laplacian and the nonlinear Laplace operator. We also present an existence result.

Let $\Omega$ be a bounded domain in $\mathbb{R}^N$ ($N \geq 1$). Given any $p \in (1, \infty)$, we shall be concerned with the boundary value problem

\[
\begin{cases}
- \mathcal{A}_p(u)(x) = \lambda m(x)|u(x)|^{p-2}u(x) + f(x, u(x), \lambda), & \forall x \in \Omega, \\
u(x) = 0, & \forall x \in \partial\Omega,
\end{cases}
\]

where $\mathcal{A}_p$ is the pseudo-Laplacian, i.e.,

\[
\mathcal{A}_p(u) = \text{div}(|\nabla u|^{p-2}\nabla u)
\]

or $\mathcal{A}_p$ is the nonlinear Laplace operator defined by

\[
\mathcal{A}_p(u) = \sum_{i=1}^{N} D_i(|D_i u|^{p-2}D_i u).
\]

We assume that $m \in L^\infty(\Omega)$, $m_+ := \max(m, 0) \neq 0$ and the function $f \in C(\overline{\Omega}) \times \mathbb{R} \times \mathbb{R}$ satisfies some appropriate growth conditions.

Our results are closely related to those of M. del Pino and R. Manásevich [1]. Similar results have been obtained independently by P. Drábek in [2]. In both papers the domain $\Omega$ is supposed to be of class $C^{2,\alpha}$ for some $\alpha \in (0, 1)$ and only the case when the weight function $m \equiv 1$ in $\Omega$ is investigated. The main purpose of the note [3] is to establish analogous results under possibly weak assumptions on the set $\Omega$.

Let us consider the eigenvalue problem

\[
\begin{cases}
- \mathcal{A}_p(u)(x) = \lambda m(x)|u(x)|^{p-2}u(x), & \forall x \in \Omega, \\
u(x) = 0, & \forall x \in \partial\Omega.
\end{cases}
\]

It is well known that there exists a unique positive eigenvalue $\lambda_1(p)$ of the problem $(E^\lambda_p)$ with a positive eigenfunction. It is called the first eigenvalue of the operator $\mathcal{A}_p$. It will be used in Theorem 2 below.

Let us note that for every $\lambda \in \mathbb{R}$ the pair $(\lambda, 0)$ is a solution of the problem (1). It is called a trivial solution. We say that $P := (\lambda, 0)$ is a bifurcation point of (1) iff in any neighbourhood of $P$ in $\mathbb{R} \times W^{1,p}_0(\Omega)$ there is a nontrivial solution of (1).

Our first result on bifurcations of the problem (1) is the following:

**Theorem 1.** If $(\lambda, 0)$ is a bifurcation point of the problem (1), then $\lambda$ is an eigenvalue of the operator $\mathcal{A}_p$, i.e., the problem $(E^\lambda_p)$ admits an eigenfunction.
We are now in a position to state our main result. For some technical reasons, we impose the Lipschitz property on the set $\Omega$.

**Theorem 2.** Suppose that the domain $\Omega$ is of class $C^{0,1}$. Then the pair $(\lambda_1(p), 0)$ is a bifurcation point of the problem (1). Moreover, the component of the closure of the set of nontrivial solutions of the problem (1) in $\mathbb{R} \times W^{1,p}_0(\Omega)$, containing the point $(\lambda_1(p), 0)$, either is unbounded or contains the point $(\lambda, 0)$, where $\lambda \neq \lambda_1(p)$ is another eigenvalue of the operator $A_p$.

The proof of Theorem 2 is a modification of that of [1] Theorem 1.1. Because of the relatively weak assumptions about the regularity of the set $\Omega$, we need some refinements in comparison with the case when $\Omega$ is of class $C^{2,\alpha}$.

We conclude this paper with a corollary concerning the nonlinear problem

$$
\begin{align*}
- A_p(u)(x) &= g(u(x)), & \forall x \in \Omega, \\
   u(x) &= 0, & \forall x \in \partial\Omega,
\end{align*}
$$

where the domain $\Omega$ is of class $C^{0,1}$ and $g : \mathbb{R} \rightarrow \mathbb{R}$ is a continuous function such that $g(0) = 0$. Since $u \equiv 0$ is a trivial solution of the problem (4), we are interested in the existence of nontrivial solutions to this problem.

**Theorem 3.** Assume that $g(s)/\varphi_p(s)$ is bounded and

$$
\lambda := \lim_{s \to 0} \frac{g(s)}{\varphi_p(s)} < \lambda_1(p) < \liminf_{|s| \to \infty} \frac{g(s)}{\varphi_p(s)}.
$$

Then the problem (4) has at least one nontrivial solution $u \in C(\Omega)$ which does not change sign in $\Omega$.


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VERSAL DEFORMATIONS OF A DIRAC TYPE DIFFERENTIAL EXPRESSION AND ASSOCIATED WITH THEM DIFFEOMOORPHISMS GROUP Diff(S^1) ACTION MOMENTUM MAPPING

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AMS Class.: 58F08 (70H35, 35B17)

1. If are given some differential expression periodically depending on the variable \( x \in \mathbb{R}/\mathbb{Z} \), its normal form as well known, is such a simplest one which could be obtained by means of the associated Diff(S^1)-group action on the space of all such expressions. Correspondingly, a versal deformation [1] of this differential expression is a normal form for some many parametric infinitesimal smooth family of the given expression. Our report is devoted to analysis of a Dirac type differential expression with a spectral parameter \( \lambda \in \mathbb{C} \), basing on the theory of Diff(S^1)-actions on Poisson manifolds [2] endowed with centrally extended Lie-Poisson brackets, and on the theory of Cartan’ integrable fibered distributions.

2. Having built a general explicit expression for versal deformations of a Dirac type differential operation, we have succeeded in interpreting it via the Lie-algebraic theory of its generic momentum mapping. Making use of the Marsden - Weinstein reduction procedure with respect to some Casimir’s generated distributions, we have proved the following theorem.

**Theorem.** Versal deformations of a \( 2\pi \)-periodic Dirac type differential operation are described completely by generic elements of the associated to a Diff(S^1)-group action momentum mapping on reduced with respect to Casimir’s generated distributions functional manifolds.


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ON THE WEIGHTED NON–LOCAL BOUNDARY PROBLEM FOR A DEGENERATED EQUATION OF ELLIPTIC TYPE

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In the domain \( D \), bounded by the straight lines \( OO_\infty \ (x = 0) \), \( AA_\infty \ (x = 1) \) and the interval \( 0 \leq x \leq 1, \ y \geq 0 \), we consider the equation

\[
L(U) \equiv U_{xx} + U_{yy} + \frac{2p}{y} U_y = 0, \quad p = \text{const}. \tag{1}
\]

For equation (1) the following boundary problem \( \Omega_1 \) is posed and studied. Problem \( \Omega_1 \): To find the function \( U(x, y) \) with properties:

1) \( L(U) \equiv 0 \) in the domain \( D \);
2) \( U(x, y) \in C^0(\overline{D}) \cap C^2(D \cup OO_\infty \cup AA_\infty) \cap C^2(D) \);
3) \( U(0, y) - U(1, y) = \varphi_1(y), \ y \geq 0; \)
4) \( U_x(0, y) - U_x(1, y) = \varphi_2(y), \ y \geq 0; \)
5) \( U(x, 0) = \tau_1(x), \ x \in [0, 1], \ 2p < 1; \)
6) \( \lim_{y \to 0} (\ln y)^{-1} U(x, y) = \tau_2(x), \ x \in [0, 1], \ 2p = 1; \)
7) \( \lim_{y \to 0} y^{2p-1} U(x, y) = \tau_3(x), \ x \in [0, 1], \ 2p > 1; \)
8) \( \lim_{y \to +\infty} U(x, y) = 0 \) uniformly with respect to \( x \in [0, 1], \)

where \( \varphi_1(y), \ \varphi_2(y) \in C^0[0, +\infty), \ y^p \varphi_1(y), \ y^p \varphi_2(y) \in L(0, +\infty), \ \tau_i(x) \in C^0[0, 1]\), and \( \tau'_i(x) \) — are absolutely integrable on the interval \([0, 1]\) functions such that \( \varphi_1(0) = \varphi_2(0) = \tau_i(0) = \tau_i(1) = 0 \) \( \ i = 1, 2, 3. \)

The uniqueness of the solution of the problem \( \Omega_1 \) is proved on the basis of the extremum principle for elliptic equations [1]. The proof of existence of the solution is carried out in the following way [2]. The solution is sought in the form of the sum of two functions \( U(x, y) = U_1(x, y) + U_2(x, y) \), which are written out explicitly

\[
U_1(x, y) = y^{\frac{1-2p}{2}} \int_0^{\infty} J_{1-2p}^{-}\lambda y \left[ a_1(\lambda) e^{\lambda x} + a_2(\lambda) e^{-\lambda x} \right] d\lambda, \quad 2p < 1,
\]

\[
U_1(x, y) = \int_0^{\infty} J_0(\lambda y) \left[ a_1(\lambda) e^{\lambda x} + a_2(\lambda) e^{-\lambda x} \right] d\lambda, \quad 2p = 1,
\]

\[
U_1(x, y) = y^{\frac{1-2p}{2}} \int_0^{\infty} J_{2p-1}^{-}\lambda y \left[ a_1(\lambda) e^{\lambda x} + a_2(\lambda) e^{-\lambda x} \right] d\lambda, \quad 2p > 1,
\]
where the sum \( \left[ a_1(\lambda) e^{\lambda x} + a_2(\lambda) e^{-\lambda x} \right] \) using the Hankel transform is expressed via the given functions \( \varphi_1(y) \) and \( \varphi_2(y) \).

\[
U_2(x, y) = y^{\frac{1-2p}{2}} \sum_{n=1}^{\infty} a_n \sin \pi n x K_{\frac{1-2p}{2}} (\pi ny),
\]

where

\[
a_n = \frac{4 \left( \frac{\pi n}{2} \right)^{1-2p} \Gamma \left( \frac{1-2p}{2} \right)}{\Gamma \left( \frac{1-2p}{2} \right)} \int_0^1 \tau_1(x) \sin \pi n x \, dx, \quad 2p < 1,
\]

\[
a_n = -2 \int_0^1 \tau_2(x) \sin \pi n x \, dx, \quad 2p = 1,
\]

\[
a_n = \frac{4 \left( \frac{\pi n}{2} \right)^{2p-1} \Gamma \left( \frac{2p-1}{2} \right)}{\Gamma \left( \frac{2p-1}{2} \right)} \int_0^1 \tau_3(x) \sin \pi n x \, dx, \quad 2p > 1.
\]


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ON A PROBLEM OF NONLINEAR OSCILLATIONS OF A PLATE

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The basic model on which we carrying out our investigations is a class of retarded quasilinear partial differential equations which arises in the Berger approach [1] of a problem of nonlinear oscillations of an elastic plate in potential supersonic gas flow:

\[ \mu \ddot{u} + \gamma \dot{u} + \Delta^2 u - f \left( \int_{\Omega} |\nabla u(x,t)|^2 dx \right) \Delta u + \rho \frac{\partial u}{\partial x_1} - q(u_t) = p_0, \quad x \in \Omega, \quad t > 0 \]  

(1)

with the boundary conditions

\[ u|_{\partial \Omega} = \Delta u|_{\partial \Omega} = 0. \]  

(2)

Here \( \Omega \) is a bounded domain in \( R^2 \), \( x = (x_1, x_2) \), \( \mu, \gamma, \rho \) are positive parameters, \( \dot{u} = \frac{\partial u}{\partial t} \). Assumptions on the scalar function \( f(s) \) are given below and depend on statement we prove. Here and below \( u_t = u_t(\theta) = u(t + \theta), \theta \in (-t^*, 0) \), where \( t^* \) is a time retardation. The analysis carried out in [8, 5] shows that \( t^* = l(\nu - 1)^{-1} \), where \( \nu \) is a gas velocity, \( l \) is the size of \( \Omega \) along the flow. For convenience we do not give here a complicated structure of the retarded term \( q \) but describe several simple essential properties. In fact we use these properties only without appellation to the exact stucture. The properties are [5]:

1) \( q(u_t) \) linearly depends on \( u_t \);
2) If \( u(t) \in L^2(-t^*, T; H^2 \cap H^1_0(\Omega)) \) then

\[ \| q(u_t) \|^2 \leq C t^* \int_{t-t^*}^t \| \Delta u(\tau) \|^2 d\tau \]  

(3)

and the mapping \( u \rightarrow q(u_t) \) is a continuous linear operator from \( L^2(-t^*, T; H^2 \cap H^1_0(\Omega)) \) to \( L^2(0, T; L^2(\Omega)) \). Here \( H^s \) is the Sobolev space of order \( s \), \( \| \cdot \| \) is the norm in \( L^2(\Omega) \). Sometimes we use a bit stronger estimate than (3) of the same nature.

Retarded character of system (1),(2) demands initial conditions (cf.[6]) in the form

\[ u|_{t=0^+} = u_o; \quad \dot{u}|_{t=0^+} = u_1; \quad u|_{t \in (-t^*, 0)} = \varphi(x, t) \]  

(4)

and means that for definition of system’s state in time moment \( t > t^* \) we have to know the states during the time interval \( [t-t^*, t] \). Taking it in a mind we have to use together methods developed for infinite dimensional nonretarded and finite dimensional retarded problems. Our investigation is based on methods developed for dissipative dynamical systems [2, 9, 7, 5]. First of all we prove the existence and uniqueness theorem for solution. To do this we use Galerkin approximate solutions and compactness method. Note that taking into account retarded character of the
system we have to use the idea of the proof of Th 2.2.1 from [6] to show the existence of approximate solutions. After that we construct by the solution $u(t)$ the evolution semigroup $S_t$ which maps in the space of initial data (see (4)):

$$S_t(u_0; u_1; \varphi(s)) = (u(t); \dot{u}(t); u(t + s)), s \in (-t_*, 0).$$  \hspace{1cm} (5)

It is well known that in studying long-time behavior of evolution operator $S_t$ the important role is played by the notion of an attractor. Our main result is the existence of a compact global and an exponential attractors of finite fractal dimension. It shows that asymptotic behavior of the system is essentially finite dimensional. Applying the same general scheme to so-called quasistatic case of the problem ($\mu = 0$), we show the existence of a finite dimensional global attractor for this case. We also obtain the continuity of exponential attractors with respect to the Hausdorff metric in the domain of parameters $D_{(\mu_0, \nu_0, \rho_0)} \equiv (0, \mu_0] \times [\nu_0, +\infty] \times [0, \rho_0]$ and upper semicontinuity of global attractors in the closed domain $\overline{D}_{(\mu_0, \nu_0, \rho_0)}$.

The results were obtained in cooperation with Professor I.D.Chueshov.


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NEW CONFORMALLY INVARIANT NONLINEAR WAVE EQUATIONS

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We report on a new nonlinear representation of the conformal algebra admitted by a nonlinear relativistic wave equation

$$\Box u = \frac{\Box |u|}{|u|} u + \lambda u, \quad \lambda \neq 0$$

for complex scalar field, $u = u(x_0, x_1, ..., x_n)$ [1].

It is shown, that the above equation is invariant under the conformal algebra $AC(1, n+1)$ when $\lambda > 0$ and $AC(2, n)$ when $\lambda < 0$. In these representations the phase function $\theta = (i/2) \ln(u^*/u)$ is considered on the same footing as the geometrical variables, in the other words, dependent variable $\theta$ is added to the $(n+1)$–dimensional geometric space of the independent variables $(x_0, x_1, ..., x_n)$. This is the same effect we see for the eikonal equation [2]. It should be noted that the dilation and conformal operators obtained generate the nonlinear transformations of the variables.

If we put $\lambda = 0$ then equation (1) will be invariant under the infinite–dimensional algebra containing the conformal algebra as a subalgebra.

We also examine a more general nonlinear wave equation having the following structure:

$$\Box u = F(u, u^*, \nabla u, \nabla u^*, \Box |u|)$$

and find new nonlinear wave equations

$$\Box u = |u|^{4/(n-1)} R \left(|u|^{(3+n)/(1-n)} \Box |u| \right) u, \quad n \neq 1,$$

$$\Box u = \Box |u| R \left(\frac{\Box |u|}{(\nabla |u|)^2}, |u| \right) u, \quad n = 1,$$

which are invariant under the standard representation of the conformal algebra. Let us note that the second one is invariant under the infinite–dimensional algebra.

Making use of the results obtained, the multiparametrical exact solutions of the conformally invariant equations are constructed. Some solutions contain arbitrary functions.


SOME ASPECTS OF APPLICATION OF NUMERICAL-ANALYTIC METHOD FOR BVPs

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AMS Class.: 34B15

The boundary-value problem (BVP) with parameters

\[
\begin{cases}
\frac{dx}{dt} = f(t, x, \lambda_1), \quad t \in [0, T], \quad x \in \mathbb{R}^n, \quad \lambda_1 \in [a_1, b_1] \\
Ax(0) + Cx(\lambda_2) = d, \quad \lambda_2 \in (0, T), \\
x_2(0) = x_{10}, \quad x_2(0) = x_{20}
\end{cases}
\]

will be investigated by the so called numerical-analytic method [1], [2].

We require that in the domain

\[
\Omega = [0, T] \times D \times [a_1, b_1], \quad D \subset \mathbb{R}^n:
\]

i) \( f(t, x, \lambda_1) \in C(\Omega); \quad |f(t, x, \lambda_1)| \leq M, \quad M \in \mathbb{R}_+; \)

ii) \(|f(t, x', \lambda_1') - f(t, x'', \lambda_1'')| \leq K |x' - x''| + |\lambda_1' - \lambda_1''| M_1; \)

\(K \in \mathbb{R}_+^{n \times n}, \quad M_1 \in \mathbb{R}_+;\)

iii) the spectral radius \( r(Q) = r\left(\frac{I}{2}K\right) < 1; \)

iv) there exist such constants \( k_1, k_2 \in \mathbb{R}, \quad (k_1 \neq k_2) \) for which

\[
det (k_1 A + k_2 C) = det B \neq 0 ;
\]

v) \( D_\beta \neq \emptyset, \quad D_\beta = \{z \in \mathbb{R}^n : B(z, \beta) \subset D\}; \)

\(\beta = \frac{T}{2}M + |(k_2 - k_1)B^{-1}d(z)|, \quad d(z) = d - (A + C)z.\)

The problem is to find such values of the parameter \( \lambda = (\lambda_1, \lambda_2) = (\lambda_1^*, \lambda_2^*) \) for which there exist a solution \( x = x^*(t) \)

of BVP(1).

Some new results for this problem are submitted.

Let us introduce the sequence of functions

\[
x_m(t, z, \lambda) = x_m(t, z, \lambda_1, \lambda_2) = z + k_1 B^{-1}d(z) + \\
+ \int_0^t \left[ f(t, x_{m-1}(t, z, \lambda), \lambda_1) - \frac{1}{\lambda_2} \int_0^{\lambda_2} f(s, x_{m-1}(s, z, \lambda), \lambda_1) ds \right] dt + \\
+ \frac{k_2 - k_1}{\lambda_2} B^{-1}d(z), \quad m = 1, 2, \ldots; \quad x_0(t, z) = z + k_1 B^{-1}d(z).
\]
The following statement is valid.

**THEOREM.** Assume that i)-v) hold.

Then:
1. \( x_m(t, z, \lambda) \xrightarrow{m\rightarrow\infty} x^*(t, z, \lambda) \); \((t, z, \lambda) \in [0, T] \times D_\beta \times [a_1, b_1] \times (0, T)\);
2. the limit function is a solution of the "perturbed" BVP
   \[
   \begin{cases}
   \frac{dx}{dt} = f(t, x, \lambda_1) + \Delta(z, \lambda), \\
   Ax(0) + Cx(\lambda_2) = d,
   \end{cases}
   \]
   where
   \[
   \Delta(z, \lambda) = \frac{1}{T}(k_2 - k_1)B^{-1}d(z) - \frac{1}{\lambda_2}f_0^{\lambda_2}f(t, x^*(t, z, \lambda), \lambda_1)dt.
   \]
3. \(|x_m(t, z, \lambda) - x^*(t, z, \lambda)| \leq \frac{\pi T}{6} \cdot |Q^m(E - Q)M + KQ(E - Q)^{-1} \cdot |(k_2 - k_1)B^{-1}d(z)|\);
4. the pair \((x^*(t, z^*, \lambda^*), \lambda^*)\) is a solution of the BVP (1) if and only if
   \[
   \begin{cases}
   \Delta(z^*, \lambda^*) = 0, \\
   x_1^*(0, z^*, \lambda^*) = x_{10}, \\
   x_2^*(0, z^*, \lambda^*) = x_{20}.
   \end{cases}
   \]


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A MULTIPLICITY RESULT
FOR A PERIODIC BOUNDARY VALUE PROBLEM

Boris Rudolf

AMS Class.: 34C25

We consider the periodic boundary value problem

\[ x'' + f(x) \text{sgn}(x')|x'|^m + g(t, x) = s(t), \]
\[ x(0) = x(2\pi), \quad x'(0) = x'(2\pi). \]

(1)

(2)

Functions \( f : \mathbb{R} \to \mathbb{R}, \ g : I \times \mathbb{R} \to \mathbb{R} \) and \( s : I \to \mathbb{R} \) where \( I = [0, 2\pi] \) are assumed to be continuous.

This contribution gives a multiplicity result of Ambrosetti-Prodi type. We seek conditions under which the problem (1), (2) has at least 3 solutions. The method is based on the existence of lower and upper solutions and the topological degree. Similar multiplicity results can be found in [1], [3], [4] and [6].

**Theorem 1.** Assume that \( 0 < m \leq 2, \lim_{x \to -\infty} g(t, x) = -\infty, \lim_{x \to +\infty} g(t, x) = +\infty, \) uniformly for \( t \in I, \) and there are constants \( x_1, x_2, x_1 < x_2 \) such that

\[ g(t, x_1) < g(t, x_2) \quad \text{for } t \in I. \]

Then for every \( s(t) \in C(I) \) there is a solution of the problem (1), (2), there are functions \( a(t), b(t), a(t) < b(t) \) such that the problem (1), (2) has at least two solutions for \( s(t) \in S(a, b), \) and at least 3 solutions for every \( s(t) \in S(a, b), \) where \( S(a, b) = \{ s(t) \in C(I), \ a(t) < s(t) < b(t) \ \text{for } t \in I \}. \)

**Example.** Theorem 1 implies that the problem

\[ x'' + cx' - (x^3 - 3x) = s(t), \]
\[ x(0) = x(2\pi), \quad x'(0) = x'(2\pi), \]

has at least three solutions for every \( s(t) \in S(-2, 2), \) at least two solutions for \( s(t) \in \partial S(-2, 2) \) and there is a solution for every \( s(t) \in C(I). \)

Moreover for \( c \neq 0 \) and for a constant function \( s(t) = s \) the problem

\[ x'' + cx' - (x^3 - 3x) = s, \]
\[ x(0) = x(2\pi), \quad x'(0) = x'(2\pi). \]

admits only constant solutions and it has exactly three solutions for \( s \in (-2, 2), \) two solutions for \( s = -2, \ s = 2 \) and unique solution for \( s \in (-\infty, -2) \cup (2, \infty). \)

Theorem 1 describes the situation when the function \( g \) is decreasing in \( x \) at infinity. The case when \( g \) is increasing in \( x \) at infinity seems to be more complicated and the situation differs from [1], [3], [4], where dual versions of multiplicity results hold.
Theorem 2. Assume that $m \leq 1$, \( \lim_{x \to -\infty} g(t, x) = -\infty \), \( \lim_{x \to \infty} g(t, x) = \infty \), uniformly for $t \in I$, and there are constants $x_1$, $x_2$, $x_1 < x_2$ such that 

\[ g(t, x_1) > g(t, x_2) \]

for $t \in I$.

Then there are functions $a(t)$, $b(t)$, $a(t) < b(t)$ such that there is a solution of the problem (1), (2) for $s(t) \in \partial S(a, b)$, and there are at least 3 solutions for every $s(t) \in S(a, b)$.

Our following example illustates that the situation is really different from the case of the Theorem 1.

**Example.** We consider the problem

\[ x'' + g(x) = s(t), \quad x(0) = x(2\pi), \quad x'(0) = x'(2\pi), \]

where \[ g(x) = \begin{cases} 
  x + 2 & \text{for } x < -1 \\
  -x & \text{for } -1 \leq x \leq 1 \\
  x - 2 & \text{for } x > 1.
\end{cases} \]

Theorem 2 implies that for every $s(t) \in S(-1, 1)$ there are at least three solutions, and for every $s(t) \in \partial S(-1, 1)$ there is a solution of our problem.

Moreover there is no solution for $s(t)$ such that

\[ \left| \int_0^{2\pi} s(t) \sin t \, dt \right| > 8. \]


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ON THE INTEGRABILITY OF 2-D PRESSURE FREE GAS DYNAMICS SYSTEM

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AMS Class.: 35L65 (35Q58)

One considers the following system

\[
\begin{align*}
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_1)}{\partial x_1} + \frac{\partial (\rho u_2)}{\partial x_2} &= 0 \\
\frac{\partial (\rho u_1)}{\partial t} + \frac{\partial (\rho u_1^2)}{\partial x_1} + \frac{\partial (\rho u_1 u_2)}{\partial x_2} &= 0 \\
\frac{\partial (\rho u_2)}{\partial t} + \frac{\partial (\rho u_1 u_2)}{\partial x_1} + \frac{\partial (\rho u_2^2)}{\partial x_2} &= 0,
\end{align*}
\]

where \( \rho > 0 \) is density, \((u_1, u_2)\) is velocity; with initial data

\[
\begin{align*}
\rho(0, x_1, x_2) &= \rho_0(x_1, x_2) > 0 \\
u_1(0, x_1, x_2) &= u_1^{(0)}(x_1, x_2) \\
u_2(0, x_1, x_2) &= u_2^{(0)}(x_1, x_2).
\end{align*}
\]

Let us formulate the restrictions on initial data (2). Suppose there exist such continuously differentiable functions \((x_1(A, B), x_2(A, B)) : G \subset R^2 = (A, B) \rightarrow R^2 = (x_1, x_2), G \subset R^2\) is some domain, that

\[
\frac{\partial (x_1, x_2)}{\partial (A, B)} = \frac{1}{\rho_0(x_1, x_2)} > 0.
\]

Suppose there also exist the functions of initial velocity potential \(S_0(A, B) \in C^4(R^2)\) and initial mass potential \(\Phi_0(A, B) \in C^4(R^2)\) such that

\[
\begin{align*}
\frac{\partial S_0}{\partial A} &= u_1^{(0)}(x_1(A, B), x_2(A, B)) \\
\frac{\partial S_0}{\partial B} &= u_2^{(0)}(x_1(A, B), x_2(A, B)).
\end{align*}
\]

and

\[
\frac{\partial \Phi_0}{\partial A} = x_1(A, B) \quad \frac{\partial \Phi_0}{\partial B} = x_2(A, B),
\]

\(\Phi_0\) is the convex function. Let us note that from (3) it follows that the function \(\Phi_0\) is strictly convex.

Denote through \(A \subset R^2 = (A, B)\) the set of solutions of the following system of equations

\[
\begin{align*}
\frac{\partial^2 S_0(A, B)}{\partial B^2} \frac{\partial^2 \Phi_0(A, B)}{\partial A^2} - \frac{\partial^2 S_0(A, B)}{\partial A^2} \frac{\partial^2 \Phi_0(A, B)}{\partial B^2} &= 0 \\
2 \left( \frac{\partial^2 \Phi_0(A, B)}{\partial A^2} \frac{\partial^2 \Phi_0(A, B)}{\partial B^2} \frac{\partial^2 S_0(A, B)}{\partial \partial B} + \frac{\partial^2 S_0(A, B)}{\partial A^2} \frac{\partial^2 \Phi_0(A, B)}{\partial B^2} \right) &= 0.
\end{align*}
\]
Denote also through $B_R$ the disk of radius $R$ with the center $(A_s, B_s) \in G$.

Suppose the following two conditions hold

I) if the domain $G$ is unbounded, then

$$\inf_{G \cap B_R} S_0(A, B) \geq S(R), \quad \inf_{G \cap B_R} \Phi_0(A, B) \geq \Phi(R),$$

\[\Phi\) is convex and \(\frac{S(R)}{\Phi(R)} \to 0 \text{ as } R \to +\infty;\]

II) for every compact set $K \subset R^2$ the set $A \cap K$ consists of finite number of isolated points.

The system (1) is the generalization of the system of equations consisting of 2-D inviscid Burgers equation and continuity equation. It is natural to seek the generalized solutions of (1) in the space of Radon measures $(P_t, (I_1)_t, (I_2)_t)$ (the mass measure and vector momentum measure respectively), see [1]. As far as the characteristics of the system (1) are known the solution $(P_t, (I_1)_t, (I_2)_t)$ will be uniquely defined if for every $t \in [0, T]$ one determines the partition $\xi_t$ of the plane $R^2$. The elements $P_t$ of such partition $\xi_t$ are the connected convex subsets of $R^2$ and determine the unknown measures. To construct $P_t$ let us consider the convex hull $C_{\Omega}(t)$ of the function $\Omega(A, B; t) \equiv \Phi_0(A, B) + tS_0(A, B)$. Then $C_{\Omega}(t)$ generates the partition $\xi_t$ of the plane $(A, B)$. The elements of $\xi_t$ will be the sets $P_t(\alpha, \beta)$ on which $C_{\Omega}(t)$ coincides with appropriate linear function $\alpha A + \beta B + \gamma$. The set $P_t(\alpha, \beta)$ defines the measures $P_t, (I_1)_t, (I_2)_t$ at point $x_1 = \alpha, x_2 = \beta$ at moment $t$. This gives the variational representation of generalized solutions which was formulated in [2].

**THEOREM 1.** Suppose the conditions I), II) hold. Then constructed family of partitions $\xi_t$ generates the generalized solution of the problem (1), (2) – (5).

Thus the system (1) occurs to be integrable for smooth enough initial data.


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ON THE STABILITY OF NONAUTONOMOUS FUNCTIONAL DIFFERENTIAL EQUATION VIA LIMITING EQUATIONS

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AMS Class.: 34K20

Let $R^p$ be the real $p$-vector space with a norm $|x|$, $R^+ = [0, +\infty)$, $0 < h \leq \infty$. Let $B$ be an admissible strong fading memory space of functions $\varphi : [-h, 0] \to R^p$ with a norm $|\varphi|_B$ [2], $B_H = \{\varphi \in B : |\varphi|_B < H\}$. For any $t \in R^+$ and continuous function $x : [-h, A] \to R^p$ with $A > 0$ define $x_t \in C$ by the equality $x_t(s) = x(t+s)$ for $-h \leq s \leq 0$.

Consider a functional differential equation with delay

$$
\dot{x} = X(t, x_t),
$$

(1)

where $X : R^+ \times B_H \to R^p$ is a completely continuous mapping. Then for any initial condition $(\alpha, \varphi) \in R^+ \times B_H$ there exists the unique solution of (1) $x(t) = x(t; \alpha, \varphi)$ defined on $[\alpha, \beta]$ with $x_\alpha(\alpha, \varphi) = \varphi$.

Let $X = X(t, \varphi)$ satisfy the following assumption:

For every compact set $K \subset B_H$ $X = X(t, \varphi)$ is bounded and uniformly continuous with respect to $(t, \varphi) \in R^+ \times K$.

Under this assumption the collection of translations $\{X_t(t, \varphi) = X(\tau + t, \varphi), \tau \in R^+\}$ is precompact [1]. A function $X^* : R^+ \times \Gamma \to R^p$ is said to be limiting to $X$ if there exists a sequence $t_n \to \infty$ such that $X^{(n)}(t, \varphi) = X(t_n + t, \varphi) \to X^*(t, \varphi)$ uniformly in $K \subset R^+ \times B_H$ ($\Gamma$ is compact in $B_H$). An equation

$$
\dot{x} = X^*(t, x_t),
$$

(2)

is called the limiting one to (1).

Let $V = V(t, x), V \in C^1(R^+ \times G_H, R^+)$ be a Liapunov function, where $G_H = \{x \in R^n : |x| < H\}$. Its derivative with respect to (1) is a functional $V' : R^+ \times B_H \to R$, defined by

$$
V'(t, x_t) = \frac{\partial V(t, x)}{\partial t} + \sum_{i=1}^{n} \frac{\partial V(t, x)}{\partial x_i} x_i(t, x_t).
$$

Let $W : B_H \times R^+$ be a continuous functional.

A pair $(V, W)$ is said to be a Liapunov-Razumikhin pair for (1) if for every $\rho > 0$, $t \geq \rho$ and $\varphi \in B_H$ such that $\varphi - \rho \in B_H$ and $\varphi$ is continuous on $[-\rho, 0]$, we have $V(t, \varphi(0)) \leq W(t, \varphi) \leq \max\{\max_{-\rho \leq s \leq 0} V(t + s, \varphi(s)), W(t - \rho, \varphi - \rho)\}$; if $0 < V(t, \varphi(0)) = W(t, \varphi)$, then $V'(t, \varphi) \leq U(t, \varphi) \leq 0$.

Assume $V(t, x), W(t, x_t), U(t, x_t)$ are bounded and uniformly continuous with respect to compact subsets of corresponding spaces. Under these assumptions the
collections of translations \( \{ V_\tau(t, x) = V(\tau + t, x), \tau \in R^+ \} \) and \( \{ W_\tau(t, \varphi) = W(\tau + t, \varphi), \tau \in R^+ \} \) are precompact.

For every \( c_0 \in R \) and \( T > 0 \) we define
\[
N(t, c_0, T) = \{ \varphi \in B_H : \sup_{-T \leq s \leq 0} V^*(t + s, \varphi(s)) = c_0 \},
\]
\[
M(t, c_0) = \{ \varphi \in N(t, c_0) : V^*(t, \varphi(0)) = c_0 \},
\]
\[
L(t, 0) = \{ \varphi \in B_H : U^*(t, \varphi) = 0 \}.
\]

Suppose there is a Liapunov-Razumikhin pair \((V, W)\) for the equation (1) satisfying the assumptions above. We shall now assume that for \( V \) and \( W \) a property below holds [2]:

For every \( c > 0 \) there exists \( T = T(c) > 0 \) such that for an uniformly continuous \( \varphi \in B_\delta \) and \( t \in R^+ \sup_{-T \leq s \leq 0} V^*(t + s, \varphi(s)) \leq W^*(t, \varphi) = c \) implies \( s \leq 0 \)
\[
W^*(t, \varphi) = \sup_{-T \leq s \leq 0} V^*(t + s, \varphi(s)).
\]

We also set \( V(t, 0) = 0 \) and \( V(t, x) \geq a(|x|) \) for \( (t, x) \in R^+ \times G_\delta \), where \( a(u) \) is strictly increasing, \( a(0) = 0, \delta > 0 \).

Under these conditions by exploiting properties of the sets defined above, theorems on asymptotic stability and nonstability are proved. In particular, the following result is proved.

**Theorem.** Suppose for any sequence \( t_k \to \infty \) the set \( M(t, c_0) \cap L(t, 0) \) does not contain solutions of the corresponding equation \( \dot{x} = X^*(t, x) \) for \( c_0 \geq 0 \) except the zero solution.

Then the zero solution of (1) is asymptotically stable.

The uniform and equiasymptotic stability results for \( h < \infty \) with \( W(t, x) = \max_{-h \leq s \leq 0} V(t + s, x(t + s)) \) are also obtained.

\[1\] G. R. Sell, *Nonautonomous differential equations and topological dynamics.*


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EXISTENCE AND CONSTRUCTION
OF THE UNIQUE SOLUTION
OF THE ONE CLASS OF THE I KIND
VOLterra INTEGRAL EQUATION

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AMS Class.: 65R20

At mathematical modelling of dynamic systems of an input - exit type integro-
power Volterra series are used [1]. The identification of the kernels of these series
can be carried out with the help of specially selected test signals [2]. It results in
the so-called dual integral Volterra equation of the I kind:

\[
\int_0^{\omega_1} \int_0^{\omega_1} K_{12}(t, \nu_1, \nu_2)d\nu_1d\nu_2 - \int_0^{\omega_1+\omega_2} \left\{ \int_0^{\omega_1} K_{12}(t, \nu_1, \nu_2)d\nu_1 \right\}d\nu_2 =
\]

\(= f_{12} (t, \omega_1, \omega_2, ) \), \(0 \leq \omega_1 + \omega_2 \leq t \leq T; \ \omega_1, \omega_2 \geq 0, \)

\[
\int_0^{\omega_1} \int_0^{\omega_1} K_{12}(t, \nu_1, \nu_2)d\nu_1d\nu_2 - \int_0^{\omega_1+\omega_2} \left\{ \int_0^{\omega_1} K_{12}(t, \nu_1, \nu_2)d\nu_1 \right\}d\nu_2 =
\]

\(= f_{12} (t, \omega_1, \omega_2) \), \(0 \leq \omega_1 + \omega_2 \leq t \leq T; \ \omega_1, \omega_2 \geq 0. \)

In the present paper the necessary and sufficient conditions of existence of the
unique continuous solution of this equation are given.

Energoizdat, 1979, 240.

[2] Apartsin A. S., About the new classes of linear many-dimensional the equa-
tions of the I kind of type Volterra – Izvesiya Vishei Shcoli. Mathematics,

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DIFFERENCE-DIFFERENTIAL EQUATIONS WITH
THE FREDHOLM OPERATOR IN THE MAIN PART

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AMS Class.: 35R20 (35C99)

We consider the reduction of the degenerate difference-differential equations
with the Fredholm operator in the main expression to the regular problems. The
connection between the problem of choice of boundary conditions and the Jordan
structure of operator coefficients of the equations is shown. The theorems of
existence and uniqueness of boundary value problems are proved. These general
theorems are used for statement and investigation of boundary value problems for
concrete partial differential and difference equations with degeneration. Let us
consider the following equation

\[ \Lambda u \equiv L_0 Bu + L_1 A_1 u + \cdots + L_q A_q u = f(x), \] (1)

where \( B \) and \( A_i, i = 1, q \) are closed linear operators with the dense domains from
\( E_1 \) to \( E_2 \); \( E_1, E_2 \) are Banach spaces and \( D(B) \subseteq D(A_i), i = 1, q, x \in \Omega \subseteq R^r \), \( B \)
is Fredholm operator with \( \dim N(B) = \dim N(B^*) = n, R(B) = R(B), f(x) : \Omega \subseteq R^r \to E_2 \) is a sufficiently smooth function;

\[ L_i(\frac{\partial}{\partial x}, \Delta) = \sum_{|k| \leq q_i} a^i_k(x) D^k + \sum_{|k| \leq q_i} b^i_k(x) \Delta^k, \]

where \( D^k = \frac{\partial}{\partial x_{i_1}^{k_1} \cdots \partial x_{i_r}^{k_r}} \),

\[ \Delta^k u = \sum_{k_1} \cdots \sum_{k_r} (-1)^{|k| - |i|} C_{k_1}^i \cdots C_{k_r}^i u(x_1 + i_1 h_1, \ldots, x_r + i_r h_r), q_0 > q_1 > q_2 \geq \cdots \geq q_q, a^i_k(x), b^i_k(x) : \Omega \subseteq R^r \to R^1. \]

Suppose the following conditions are satisfied:

1. The operator \( B \) has a complete \( A_1 \)-Jordan set \( \phi^{(j)}_i, i = 1, n, j = 1, p_i, B^* \)
has a complete \( A_1^* \)-Jordan set \( \psi^{(j)}_i, i = 1, n, j = 1, p_i, \) and the systems \( \gamma^{(j)}_i, z^{(j)}_i, \)
where \( i = 1, n, j = 1, p_i, \) corresponding to them are biorthogonal.

Let us introduce the projectors

\[ P_k = \sum_{i=1}^{n} \sum_{j=1}^{p_i} \langle \gamma^{(j)}_i, \phi^{(j)}_i \rangle \equiv \langle \gamma, \Phi \rangle, \]

\[ Q_k = \sum_{i=1}^{n} \sum_{j=1}^{p_i} \langle \psi^{(j)}_i, z^{(j)}_i \rangle \equiv \langle \psi, Z \rangle, \]

where \( k = p_1 + \cdots + p_n \)-root number.

2. The operators \( A_2, \ldots, A_q \) \((P_k, Q_k)\)-commute (see [2]). Then there are matrices \( A_i, i = 2, q \), such that

\[ A_i \Phi = A_i Z, A_i^* \Psi = A_i^* \Gamma. \]
The projection operators $P_k, Q_k$ generate the direct decompositions

$$E_1 = E_{1k} \oplus E_{1\infty-k}, E_2 = E_{2k} \oplus E_{2\infty-k}.$$ 

Then we search for the solution of equation (1) in the following form

$$u(x) = \Gamma v(x) + (C(x), \Phi)$$

where $\Gamma = (B + \sum_{i=1}^{n} <, \gamma_i^{(1)} >, \gamma_i^{(1)})^{-1}$, $v \in E_{2\infty-k}$, $C(x) = (C_1(x), \ldots, C_n(x))$, $\Phi = (\Phi_1, \ldots, \Phi_n)$, $i = \overline{1,n}$. Projecting equation (1), where $u$ is defined from (2), onto $E_{2\infty-k}$ and $E_{2k}$, we obtain the regular equations

$$L_0v + \sum_{i=1}^{q} L_iA_i\Gamma v = (I - Q_k)f(x) \quad (3)$$

$$L_0A_B C + \sum_{i=1}^{q} L_iA_i'C = < f(x), \Psi >, \quad (4)$$

where $A_B, A_i$ are symmetrical cell-diagonal matrices.

**Theorem.** Suppose conditions 1 and 2 are satisfied, $f : \Omega \subset R^n \to E_2$ is a sufficiently smooth function. Then any solution of equation (1) can be represented in the form

$$u = \Gamma v + (C, \Phi),$$

where $v$ satisfies regular equation (3), the vector $C(x)$ is defined by system (4).


EXISTENCE THEOREMS
FOR THE EQUATION \( x^{(m)} = f(t, x) \)
IN BANACH SPACES

Stanisław Szufla Aldona Szukała

AMS Class.: 34G20

Assume that \( I = [0, a] \), \( E \) is a Banach space, \( B = \{ x \in E : \| x \| \leq b \} \) and \( f : I \times B \to E \) is a bounded continuous function. In this paper we shall give sufficient conditions for the existence of solutions of the Cauchy problem

\[
\begin{align*}
x^{(m)} &= f(t, x) \\
x(0) &= 0, x'(0) = \eta_1, \ldots, x^{(m-1)}(0) = \eta_{m-1},
\end{align*}
\]

where \( \eta_1, \ldots, \eta_{m-1} \in E \).

Let \( \alpha \) denote the Kuratowski measure of noncompactness or the ball measure of noncompactness.

Put

\[
\varphi(t, X) = \lim_{r \to 0^+} \alpha(f(I_{tr} \times X)) \text{ for } t \in (0, a) \text{ and } X \subset B,
\]

where \( I_{tr} = (t - r, t + r) \cap I \). Denote by \( B(0, r) \) the ball with center 0 and radius \( r \).

**Theorem 1.** Suppose that there is a continuous function \( u \) on \( [0, a] \) such that \( u(t) > 0 \) for \( t > 0 \), \( u(0) = \ldots u^{(m-1)}(0) = 0 \) and \( u^{(m)}(t) \) is positive and Lebesque integrable. If

\[
\varphi(t, X) \leq \frac{u^{(m)}(t)}{u(t)} \alpha(X) \text{ for } X \subset B \text{ and } t \in (0, a)
\]

and

\[
\lim_{t \to 0^+} \lim_{r \to 0^+} \frac{\alpha(f(t, B(0, r)))}{u^{(m)}(t)} = 0,
\]

then there exists an interval \( J = [0, d] \) such that problem (1) – (2) has at least one solution defined on \( J \).

**Theorem 2.** If \( E \) is a weakly compact generated space, then the conclusion of Theorem 1 remains true if we replace the assumption (3) by

\[
(3') \quad \alpha(f(t, X)) \leq \frac{u^{(m)}(t)}{u(t)} \alpha(X) \text{ for } X \subset B \text{ and } t \in (0, a).
\]

**Theorem 3.** Let \( w : \mathbb{R}_+ \to \mathbb{R}_+ \) be a continuous nondecreasing function such that \( w(0) = 0 \) and

\[
\int_{0^+}^{\infty} \frac{dr}{w^{m-1}(r)} = \infty.
\]
If 
\[ \alpha(\mathbf{f}(t,X)) \leq w(\alpha(X)) \text{ for } t \in I \text{ and } X \subset B, \]
then there exists an interval \( J = [0, d] \) such that the problem (1) − (2) has at least one solution defined on \( J \).

**Example.**
Consider the function \( w(r) = r|\ln r|^m \) for \( 0 < r \leq e^{-m} \), \( w(0) = 0 \). It can be easily verified that \( w \) is continuous, nondecreasing and

\[ |w(\xi) - w(\eta)| \leq w(|\xi - \eta|) \text{ for } 0 \leq \xi, \eta \leq e^{-m}. \] (4)

Moreover,

\[ \int_{0^+} \frac{dr}{\sqrt[\nu]{r^{m-1}w(r)}} = \int_{0^+} \frac{dr}{r|\ln r|} = \infty. \]

Let \( E = C(0, 1) \) and \( B = \{ x \in E : \|x\| \leq \frac{1}{2}e^{-m} \} \). We define the function \( f_1 : B \mapsto E \) by

\[ f_1(x)(\tau) = w(\|x(\tau)\|) \text{ for } \tau \in [0, 1] \text{ and } x \in B. \]

By (4) we get \( \|f_1(x) - f_1(y)\| \leq w(\|x - y\|) \) for \( x, y \in B \). From this we deduce that for a given completely continuous function \( f_2 : B \mapsto E \) the function \( f = f_1 + f_2 \) satisfies the inequality

\[ \alpha(f(X)) \leq w(\alpha(X)) \text{ for } X \subset B. \]


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ON A METHOD FOR INVESTIGATING SOME THREE-POINT BVPs

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AMS Class.: 34B10

Let a nonlinear differential equation be given with a three-point linear boundary condition (BVP)

\[
\frac{dx}{dt} = f(t, x), \quad x, f \in \mathbb{R}^n, \quad t \in [0, T], \\
Ax(0) + A_1x(t_1) + Cx(T) = d, \quad t_1 \in (0, T),
\]

(1)

where \( A, A_1, C \in \mathbb{R}^{n \times n} \) and these matrices are allowed to be singular. We are going to outline an approach for finding higher polynomial approximations for the solutions with numerical-analytic method [1,2].

We shall denote by \( \tau_i = \frac{T}{2} \left[ \cos \left( \frac{2(i-1)\pi}{2(p+1)} + 1 \right) \right], \quad i = 1, 2, ..., p + 1 \)
in the interval \([0, T]\) the nodes corresponding to the zeroes of the Chebyshev polynomials and by \( f^p(t, y_k(t)) \), for arbitrary \( y_k(t) \in D \), the Lagrange interpolation polynomial with respect the points \( \tau_i \)

\[
f^p(t, y_k(t)) = (f^p_1(t, y_k(t)), f^p_2(t, y_k(t)), ..., f^p_n(t, y_k(t))),
\]

\[
f^p_j(t, y_k(t)) = a_{k0}^j + a_{k1}^jt + a_{k2}^jt^2 + ... + a_{kp}^jt^p, \quad j = 1, 2, ..., n,
\]

\[
f^p_j(t, y_k(t)) = f^p_j(t, y_k(t)), \quad i = 1, 2, ..., p + 1.
\]

For the proof of the existence and approximate construction of the solutions of the given BVP (1) it is possible to use the following sequence of functions:

\[
x_{m+1}^{p+1}(t, x_0) =
\]

\[
= z_0(x_0, x_{m-1}^{p+1}) + \int_0^t \left[ f^p(t, x_{m-1}^{p+1}(t, x_0)) - \frac{1}{T} \int_0^T f^p(s, x_{m-1}^{p+1}(s, x_0)) \, ds \right] dt + 
\]

\[
+ \frac{t}{l_1}(k_2 - k_1)Hd(x_0, x_{m-1}^{p+1}), \quad x_0^{p+1}(t, x_0) = x_0, \quad m = 1, 2, ..., (2)
\]

where \( z_0(x_0, x_{m-1}^{p+1}) = x_0 + k_1Hd(x_0, x_{m-1}^{p+1}) \), \( d(x_0, x_{m-1}^{p+1}) = d- \)

\[-(A + A_1 + C)x_0 - A_1 \int_0^t \left[ f^p(t, x_{m-1}^{p+1}(t, x_0)) - \frac{1}{T} \int_0^T f^p(s, x_{m-1}^{p+1}(s, x_0)) \, ds \right] dt.
\]

\(^1\)The work was supported by OTKA, Grant TO19095
Let the given BVP satisfy the following conditions:

i) \( f(t,x) \in C([0,T] \times D) \), where \( D \subset R^n \) is a closed domain;

ii) \( |f(t,x)| < M, M \in R^+ \) and \( |f(t,x') - f(t,x'')| \leq K |x' - x''|, K \in R^{n \times n} \);

iii) \( D^p \neq \emptyset \), where \( D^p = \{ x_0 \in R^n : B(x_0, \beta^p) \subset D \} \),

\[
\beta^p(x_0) = \frac{T}{2} M + \frac{T}{2} L_p + [k_1 + \left( \frac{k_2 - k_1}{t_1} \right) T] \left[ H(d - (A + A_1 + C) x_0) + \frac{T}{2} |HA_1| (M + L_p) \right],
\]

\[
L_p = (5 + \lg p) \max E_p \left( f(t, x_j^{p+1}(t, x_0)) \right);
\]

iv) there exist constants \( k_1, k_2 \in R, (k_1 \neq k_2) \) such that

\[
\det \left[ k_1 A + k_2 A_1 + \left[ k_1 + \frac{T}{t_1} (k_2 - k_1) \right] C \right] = \det B \neq 0;
\]

v) the largest eigenvalue of the matrix \( Q = \frac{T}{2} (K + G) \) less than to one, where

\[
G = \left[ |k_1| + \left( \frac{k_2 - k_1}{t_1} \right) T \right] |HA_1| K, H = B^{-1}.
\]

**Theorem.** Let the given BVP (1) satisfy the conditions i) - v) then

1) The sequence (2) is uniformly convergent

\[
\lim_{m,p \to \infty} x^{p}_m(t, x_0) = x^*(t, x_0), (t, x_0) \in [0,T] \times D^p;
\]

2) The limit function \( x^*(t, x_0) \) is a solution of the perturbed BVP

\[
\begin{cases}
\frac{dx}{dt} = f(t,x) + \Delta(x_0) \\
Ax(t_1) + A_1 x(t_1) + Cx(T) = d,
\end{cases}
\]

where \( \Delta(x_0) = \frac{1}{t_1} (k_2 - k_1) Hd(x_0, x^*(t, x_0)) - \frac{1}{T} \int_0^T f(t, x^*(t, x_0)) dt.\)

3) The function \( x^*(t, x_0) \) is a solution to the given BVP (1) if and only if the parameter \( x_0 = x^*_0 \) is a solution to the determining equation \( \Delta(x_0) = 0.\)


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PARAMETER CONTINUATION METHOD
AND BOUNDARY VALUE PROBLEMS FOR
NONLINEAR INTEGRODIFFERENTIAL EQUATIONS

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AMS Class.: 47G20, 58E09

1. On the base of general theorems [1,2] about global and local invertibility of
nonlinear operators, mapping weakly metric spaces into Banach spaces a series of
new existence theorems for classical and generalized solutions of boundary value
problems for ordinary differential and integrodifferential equations are established.
For instance the conditions on the functions \( g \) and \( K \) are obtained at which the
boundary value problem

\[
x'' - g(t, x) + \int_0^1 K(t, s, x(s))ds = y(t)
\]

\[
x'(0) = x'(1) = 0
\]

for any right-hand side \( y \in C[0, 1] \) has classical solution \( x(t) \) with values on a given
finite or infinite interval \( (c,d) \).

2. In admitting some discrete group \( G \) of symmetries domain \( D \in \mathbb{R}^k \) the
integrodifferential equation

\[
x''(M, t) + \lambda g(x) + \int_D K(t, M, M', x(M'))dM
\]

\[
x'(0) = x'(1) = 0
\]

is considered. At natural restrictions on the functions \( g \) and \( K(t,M,M',x(M')) \)
existence theorem of local and global character for bifurcating solutions invariant
relative to subgroups of the group \( G \) are obtained. The corresponding abstract
results can be found in [3-8].

Investigations are supported RFFI,grant 96-01-000512.


determination of multiparameter solution families of nonlinear equations.*


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GLOBAL TRANSFORMATIONS 
OF FDE OF THE FIRST ORDER

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AMS Class.: 34K15, 39B40

The theory of global pointwise transformations $z(t) = L(t)y(\varphi(t))$ of homogeneous linear differential equations has been developed in the monograph [4] by F. Neuman. A general form

$$y''(x) = b(y(x))y'(x)^2 + p(x)y'(x)$$

($b, p$ are arbitrary functions) was derived by J. Aczél [1], [2] for the second order differential equations. This general form allows a transformation $z(t) = y(\varphi(t))$, $\varphi''(x) = p(x)\varphi'(x) - p(\varphi(x))\varphi'(x)^2$ and transforms the equation into itself on the whole interval of definition. Using analogous arguments as those given by J. Aczél and F. Neuman we can derive a general form

$$y'(x) = \sum_{i=0}^{n} a_i(x)b_i(y(x)) \prod_{j=1}^{m} \delta_{ij}(y(\xi_j(x))) + q(x)y(x)$$

for the functional differential equation in explicit form

$$y'(x) = f(x, y(x), y(\xi_1(x)), \ldots, y(\xi_m(x))), x \in I$$

with $m$ ($m \geq 1$) delays, such that allows transformations of the equation into itself on $I$. Here $b_i, \delta_{ij}$ are nontrivial solutions, in the class of functions continuous at a point, of Cauchy’s functional equation $b(uv) = b(u)b(v)$ ($u, v \in R - \{0\}$), $a_i, q$ are arbitrary functions. The transformation $z(t) = L(t)y(\varphi(t))$ is the most general pointwise transformation for this general form (1). We can investigate functional–differential equations considered in [3], such as

$$y'(x) + p(x) |y(\tau(x))|^\lambda \text{sign} y(\tau(x)) = 0, \lambda \geq 0;$$

$$y'(x) = \frac{(x-1)^3}{x^2(x-2)^2}y(x-1)^3, x \geq 3;$$

$$y'(x) = 2^{1-x}y(2x)^{1/3}y(3x)y(4x)^{1/3};$$

$$y'(x) = \frac{|y(x + \sin x)|^{\alpha_1} \text{sign} y(x + \sin x) |y(x + \cos x)|^{\alpha_2} \text{sign} y(x + \cos x)}{x^3 |\ln(x + \sin x)|^{\alpha_1} |\ln(x + \cos x)|^{\alpha_2}},$$

$x \geq 2\pi, \alpha_i > 0, \alpha = \alpha_1 + \alpha_2 > 1, \beta < 1$; for example.

We give an effective criterion for transformations converting any equation (1) into an equation of the same form and order with constant coefficients and deviations.
Assumptions: Each $\xi_j$ is a $C^1$ diffeomorphism of $I$ onto $I$ and $\xi_j'(x) > 0$ on $I$ ($j = 1, \ldots, m$); $\xi_0 = \text{id}_I$, $\xi_j(x) \neq \xi_k(x)$ for $j \neq k$ on $I = (a, b) \subseteq \mathbb{R}$; $j, k \in \{0, \ldots, m\}$, $m, n \in \mathbb{N} = \{1, 2, \ldots\}$;

$$\lim_{x \to a^+} \xi_j(x) = a, \quad \lim_{x \to b^-} \xi_j(x) = b$$

for $j = 1, \ldots, m$; $a \geq -\infty, b \leq \infty$.

**Theorem.** Let the Assumptions for the equation (1) be satisfied. Then the following assertions are equivalent

(a) The equation (1) is globally transformable into an equation with constant coefficients and discrete deviations.

(b) Each $\xi_j, \xi_k \in \{\xi_1, \ldots, \xi_m\}$ commute and to every function $\xi \in \{\xi_1, \ldots, \xi_m\}$ there exists a function $L \in C^1(I), L(x) \neq 0$ on $I$ such that

$$z(x) = L(x)y(\xi(x)), \quad \xi_j(\xi(x)) = \xi(\xi_j(x)), \quad x \in I,$$

is a stationary transformation of the equation (1).

(c) There exist functions $L_k \in C^1(I), L_k(x) \neq 0$ on $I$ such that the relations

$$a_i(\xi_k(x))\xi'_k(x)L_k(x) = a_i(x)b_i(L_k(x)) \prod_{j=1}^m \delta_{ij}(L_k(\xi_j(x))); \quad i = 1, \ldots, n;$$

$$\frac{L'_k(x)}{L_k(x)} = q(x) - q(\xi_k(x))\xi'_k(x)$$

hold on $I$ for functions $L_k, \xi_k; \xi_k \in \{\xi_1, \ldots, \xi_m\}, k \in \{1, \ldots, m\}$ and coefficients of (1). Moreover,

$$L_k(x) = \frac{f(x)}{f(\xi_k(x))}, \quad \text{where} \quad \frac{f'(x)}{f(x)} = q(x), \quad x \in I.$$


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ON INVESTIGATIONS OF HIGHER ORDER
PERIODIC IMPULSIVE SYSTEMS

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AMS Class.: 34B15

Our goal is to construct the approximate solution of a periodic impulsive
BVP system by a modified version of the matrix–vector technique based upon
the method of trigonometric collocation. The investigated system is
\[
\frac{d^n x}{dt^n} = \begin{cases} \frac{dy}{dt} = f(t, y, \dot{y}, \ddot{y}, \ldots, y^{(n-1)}), & t \in [0, \tau) \\ \frac{dz}{dt} = \tilde{f}(t, z, \dot{z}, \ddot{z}, \ldots, z^{(n-1)}), & t \in (\tau, T] \end{cases}
\]
under the boundary and impulsive conditions
\[
\begin{align*}
  y(0) &= z(T) \\
  \dot{y}(0) &= \dot{z}(T) \\
  \vdots \\
  y^{(n-1)}(0) &= z^{(n-1)}(T),
\end{align*}
\]
where \( x, f, \tilde{f}, g_i \in \mathbb{R}^m \), \( s_i, r_i \in \mathbb{R}^{m \times m} \), \( \det s_i, \det r_i \neq 0 \), \( g_i \)
are constant vectors, \( i = 0, 1, \ldots, n - 1 \). Let the functions
\( f(t, y, \dot{y}, \ddot{y}, \ldots, y^{(n-1)}) \), \( \tilde{f}(t, z, \dot{z}, \ddot{z}, \ldots, z^{(n-1)}) \)
be continuous in each component together with their Jacobian matrices in \([0, \tau] \times D \rightarrow \mathbb{R}^m \) and
\([\tau, T] \times D \rightarrow \mathbb{R}^m \), respectively, and \( f(0, y, \dot{y}, \ldots) = \tilde{f}(T, z, \dot{z}, \ldots) \). On the base
of the Theorem in [2] if the problem has a solution \( x^*(t) \) then that can be
approximated by trigonometric vector–polynomial of sufficiently large degree.
This solution has the form
\[
x_k(t) = \begin{cases} y_{k1}(t) = (y_{1,k1}(t), y_{2,k1}(t), \ldots, y_{m,k1}(t)), & t \in [0, \tau) \\ z_{k2}(t) = (z_{1,k2}(t), z_{2,k2}(t), \ldots, z_{m,k2}(t)), & t \in (\tau, T] \end{cases}
\]
where with \( \omega = \frac{2\pi}{T} \)
\[
y_{i,k1}(t) = a_{i,0} + \sum_{l=1}^{k_1} a_{i,l} \cos \omega t + b_{i,l} \sin \omega t, \quad i = 1, \ldots, m.
\]
\[
z_{i,k2}(t) = \alpha_{i,0} + \sum_{l=1}^{k_1} \alpha_{i,l} \cos \omega t + \beta_{i,l} \sin \omega t
\]
Introducing the vector of coefficients and the vector of values as
\[
\begin{align*}
  y_{i,k1}^* &= (a_{i,0}, a_{i,1}, b_{i,1}, \ldots, a_{i,k1}, b_{i,k1}) \\
  z_{i,k2}^* &= (\alpha_{i,0}, \alpha_{i,1}, \beta_{i,1}, \ldots, \alpha_{i,k2}, \beta_{i,k2}), \quad i = 1, \ldots, m
\end{align*}
\]
\[
\begin{align*}
  y_{i,k1}^M &= (y_{i,k1}(t_0), y_{i,k1}(t_1), \ldots, y_{i,k1}(t_{K_1-1})) \\
  z_{i,k2}^M &= (z_{i,k2}(t_0), z_{i,k2}(t_1), \ldots, z_{i,k2}(t_{K_2-1})), \quad i = 1, \ldots, m,
\end{align*}
\]
where \( t_0 = 0, t_1, \ldots, t_{K_1 - 1} < \tau \) and \( \tau < \bar{t}_0, \bar{t}_1, \ldots, \bar{t}_{K_2 - 1} = T \) (\( K_1 = 2k_1 + 1 \), \( K_2 = 2k_2 + 1 \), \( K_1 = K_1 - 2n \)) are the equidistant nodes of collocation in the \([0, \tau)\) and \((\tau, T]\) intervals, respectively. It can be shown on the basis of [1] and [3] that the following one-to-one correspondence holds between the vectors of values, the \( s^{th} \) order derivatives of the vectors of values and the vectors of coefficients

\[
\begin{align*}
\mathbf{y}^{M}_{i,k_1} &= \mathbf{M}_{K_1 \times K_1} \mathbf{y}^{\Gamma}_{i,k_1}, \\
\mathbf{z}^{M}_{i,k_2} &= \mathbf{M}_{K_2 \times K_2} \mathbf{z}^{\Gamma}_{i,k_2}, \\
M \left[ \frac{d^s \mathbf{y}_{i,k_1}(t)}{dt^s} \right] &= \Phi^s_{K_1 \times K_1} \mathbf{y}^{\Gamma}_{i,k_1}, \\
M \left[ \frac{d^s \mathbf{z}_{i,k_2}(t)}{dt^s} \right] &= \Phi^s_{K_2 \times K_2} \mathbf{z}^{\Gamma}_{i,k_2},
\end{align*}
\]

where the matrices \( \mathbf{M} \) and \( \Phi^s \) can be expressed in terms of trigonometric functions of \( \omega, K_1, K_1, K_2, \tau, T \). Thus the system of determining equations that consists of the boundary conditions, the conditions expressing the impulse effect and the DE fulfilled at the nodes is set up in terms of the vectors of coefficients. The numeric solution of the system of the determining equations (the number of which is \( 2m (k_1 + k_2 + 1) \)) we produce by using an ABS–method [4].


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SOME RESULTS ON SOLVABILITY
FOR SOME CLASSES OF BOUNDARY PROBLEMS
FOR PSEUDODIFFERENTIAL EQUATIONS
IN A PLANE ANGLE

Vladimir Vasil’ev

AMS Class.: 35C15 (35S15)

Let \( C_+ = \{(x_1, x_2) \in \mathbb{R}^2 : x_2 > |x_1|\} \) is the plane angle. One considers the following boundary problem

\[
(Au)(x) = 0, \quad x \in C_+, \tag{1}
\]

\[
(B_{1,j}u)(x) \bigg|_{x_2 - x_1 = 0} = g_1(x_2 + x_1), \quad (B_{2,j}u)(x) \bigg|_{x_1 > 0} = g_2(x_2 - x_1), \quad j = 1, 2, \ldots, m, \tag{2}
\]

\( A, B_{1,j}, B_{2,j} \) are pseudodifferential operators with symbols \( A(\xi), B_{1,j}(\xi), B_{2,j}(\xi) \), which are homogeneous of order \( \alpha, \beta_{1,j}, \beta_{2,j} \) respectively and are bounded on unit cycle. Additionally the symbol \( A(\xi) \) must have very specific property, more precisely it has admit the wave factorization with respect to \( C_+ \), i.e. the representation in the form

\[
A(\xi) = A_\neq(\xi)A_= (\xi), \tag{3}
\]

and the multipliers \( A_\neq(\xi), A_=(\xi) \) are defined generally speaking only on the set \( \{x \in \mathbb{R}^2 : x_2 \neq x_1^2\} \), \( A_\neq(\xi) \) admits analytical continuation into radial tube domain \( T(C_+) = \mathbb{R}^2 + iC_+ \) over the cone \( C_+ \) which satisfies the estimate

\[
|A_\neq(\xi + i\tau)| \leq c(1 + |\xi| + |\tau|)^{\pm \alpha}, \quad \tau \in C_+.
\]

The multiplier \( A_=(\xi) \) must have the analogical properties when we change the \( C_+ \) on \( -C_+ \) and \( \alpha \) on \( \alpha - \alpha \).

The number \( \alpha \) we call by index of wave factorization.

It’s shown the class of symbols which admit the wave factorization is enough large. So, particularly, the wave factorization for the Helmholtz operator with symbol \( A(\xi) = \xi_1^2 + \xi_2^2 + k^2, \quad k \in \mathbb{R} \setminus \{0\}, \) has the form

\[
A(\xi) = \left( \sqrt{2}\xi_2 + \sqrt{\xi_2^2 - \xi_1^2 - k^2} \right) \left( \sqrt{2}\xi_2 - \sqrt{\xi_2^2 - \xi_1^2 - k^2} \right)
\]

where by \( \sqrt{\xi_2^2 - \xi_1^2 - k^2} \) we mean the boundary value from \( T(C_+) \) of analytical function \( \sqrt{z_2^2 - z_1^2 - k^2} \).

Existence of wave factorization (3) gives possibility in the case \( \alpha - s = m + \delta, |\delta| < 1/2, \quad m \in \mathbb{Z}, \quad m > 0, \) to write out the general solution of equation (1) which depends on \( 2m \) arbitrary functions from appropriate classes of Sobolev –
Slobodetsky spaces on $\mathbb{R}_+[1]$. If we substitute the general solution into boundary conditions (2) we obtain a system of $2m$ linear integral equations with respect to $2m$ unknown functions on $\mathbb{R}$ (after applying the Fourier transform), and then this system can be easily reduced to $4m \times 4m$-system of linear integral equations on positive half-axis. Under satisfying of some assumptions (with respect to right side $g_1, g_2$) this system decomposes to $m \times 4 \times 4$-system of linear integral equations on half-axis with kernels which are homogeneous of order $-1$. By applying the Mellin transform every such $4 \times 4$-system reduces to $4 \times 4$-system of linear algebraic equations with parameter $\lambda$, $\Re \lambda = 1/2$. If every such determinant is non-vanishing then we obtain the existence and uniqueness theorem (under satisfying of above conditions) for the problem (1), (2) and a priori estimate of solution in weighted scale of Sobolev – Slobodetsky spaces $H^{s,\varphi}(C_+)$:

$$
||u||_{s,\varphi} \leq c([g_1]_{l_1,\varphi-1} + [g_2]_{l_2,\varphi-1}),
$$

$l_j = s - \alpha + \beta_j + 1/2, \ j = 1, 2$.


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EXACT AND APPROXIMATE ARTIFICIAL BOUNDARY CONDITIONS FOR 2D EXTERNAL TIME–DEPENDENT SCATTERING PROBLEMS

N. A. Zaitsev

AMS Class.: 35L05 (65M99)

1. One considers in $\mathbb{R}^2$ a time-dependent problem described by the scalar equation

$$Du = f$$

such that outside a bounded domain $G \subset \mathbb{R}^2$

1) the function $f$ vanishes;

2) the initial data $u|_{t=+0} = u_t|_{t=+0} = 0$;

3) the operator $D$ is the wave operator, i.e.,

$$Du \equiv u_{tt} - c^2 \Delta u,$$

where $c = \text{const} > 0$ is the propagation speed of waves, $\Delta$ is the Laplacian.

The structure of the operator $D$ inside the domain $G$ does not matter: $D$ may be any nonlinear operator, but the problem (1) has to have a unique solution.

Denote by $C$ a circle strictly containing the domain $G$. Let us consider the following initial–boundary problem concurrently with the original one:

$$\begin{cases}
Du = f & \text{inside } C \\
Lu = 0 & \text{on } C
\end{cases}$$

(2)

where $L$ is a linear operator.

The aim is to generate the operator $L$ so that these two problems would be equivalent, i.e., their solutions were identical inside $C$. Such condition $Lu = 0$, we call exact artificial boundary condition (ABC), was propose by I. Sofronov in [1]. The idea is in writing out exact conditions for each Fourier component of the unknown function on $C$ using Green’s function method. Exact ABCs are evidently non–local in both space and time, but Sofronov’s conditions admit the implementation such that calculation formulas are local in time, and therefore they are comparatively cheap.

Here we use Riemann’s function method to generate exact ABCs. It allows to obtain more general exact ABCs, which include Sofronov’s ABC as a special case and admit the local in time calculation formulas as well.

2. Besides the exact ABCs we suggest approximate ABC which is obtained by approximation the exact ABC. This ABC is local in space and non–local in time, but calculation formulas are local in time as well. For the circular boundary of calculation domain this ABC is the following

$$u_t + u_r + \frac{u}{2R} = \frac{1}{2R^2} \int_0^t (u_x + u/4) dt \quad \text{as } r = R,$$

(3)
where $R$ is a radius of the circle $C$ and $\varphi$ is a polar angle.

Unlike the exact ABCs the condition (3) doesn’t require the Fourier series expansion and is much more simple and cheap. Moreover, it is generalized to the case of arbitrary shape of boundary $C$.

One of the benchmark problems from [2], simulating a propeller sound scattering around the fuselage of an aircraft, was computed using ABC (3). Calculations showed that condition (3) is much better than known condition $u_t + u_r + \frac{u}{2R} = 0$.


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SHARP ERROR ESTIMATES
OF FINITE ELEMENT METHODS
FOR SECOND-ORDER HYPERBOLIC EQUATIONS

Alexander A. Zlotnik

AMS Class.: 65M

The initial-boundary value problem

\[ \rho \partial^2 u / \partial t^2 + Lu = \rho f \quad \text{in} \quad Q = \Omega \times (0, T), \]
\[ u|_{\partial \Omega \times (0, T)} = 0; \quad u|_{t=0} = u^{(0)}, \quad \partial u / \partial t|_{t=0} = u^{(1)} \quad \text{in} \quad \Omega \]

is solved. Here \( Lu = -\sum_{i,j=1}^{n} \partial / \partial x_i (a_{ij} \partial u / \partial x_j) + au \) is an self-adjoint elliptic operator, \( \Omega \) is a bounded domain in \( \mathbb{R}^n \) \((n \geq 1)\) with piecewise-smooth boundary \( \partial \Omega \). The coefficients \( \rho, a_{ij}, a \) depend on \( x = (x_1, \ldots, x_n) \).

Consider the three-level method with weight \( \sigma \) which is standard finite element method with respect to \( x \) (using a finite-dimensional subspace \( S_h \) in \( H^{(1)} = W^{1/2}(\Omega) \)). Its operator form is

\[
(B_h + \sigma \tau^2 L_h)\partial_t \overline{\partial_t} v_{h,m} + L_h v_{h,m} = f^{h,\tau}_m, \quad 1 \leq m \leq M - 1,
\]

\[
(B_h + \sigma \tau^2 L_h)\partial_t v_{h,0} + (\tau/2)L_h v_{h,0} = u^{(1),h} + (\tau/2)f^{h,\tau}_0,
\]

\[
(B_h + \sigma \tau^2 L_h)v_{h,0} = u^{(0),h} \quad \text{or} \quad L_h v_{h,0} = (Lu^{(0),h}) \]

where \( B_h \) is the mass matrix and \( L_h \) is the stiffness one, \((\cdot)^h\) and \((\cdot)^{h,\tau}\) are averaging operators, \( \tau = T/M \).

Define the norm of the data vector \( d = (u^{(0),h}, u^{(1),h}, f) \):

\[ ||d||_{\alpha_1, \alpha_2} = ||u^{(0),h}||_{H^{(\alpha)}} + ||u^{(1),h}||_{H^{(\alpha-1)}} + ||f||_{F^{\alpha_1, \alpha_2}}, \quad \alpha_1 + \alpha_2 = \alpha - 1. \]

The functions in \( H^{(\alpha)} \) have the smoothness of the order \( \alpha \) in the \( H^{(0)} = L_2(\Omega) \) norm (for \( \alpha < 0 \) they are generalized functions) whereas the functions in \( F^{\alpha_1, \alpha_2} \) have dominating mixed smoothness of the order \( \alpha_1 \) with respect to \( x \) and \( \alpha_2 \) with respect to \( t \) in the \( F^{0,0} = L_{2,1}(Q) \) norm \( ||| \cdot |||_{L_{2,1}(Q)} = ||| \cdot |||_{H^{(0)}} |||_{L_1(0,T)} \). Under natural assumptions the following error estimates are valid \([1]\)

\[
\max_{0 \leq m \leq M} \left( ||(u - v)_m||_{H^{(0)}} + \int_0^{m-1} (sL u - v)dt \right)_{H^{(1)}} \]

\[
\leq c \left[ \max (T, 1) |(h|^2 + \tau^2) \right]^{\alpha/3} ||d||_{\alpha_1, \alpha_2}, \quad 0 \leq \alpha \leq 3, \quad (3)
\]

\[
\max_{1 \leq m \leq M} \left( ||\overline{\partial_t}(u - v)_m||_{H^{(0)}} + ||(sL u - v)||_{H^{(1)}} \right) \]

\[
\leq c \left[ \max (T, 1) |(h|^2 + \tau^2) \right]^{(\alpha-1)/3} ||d||_{\alpha_1, \alpha_2}, \quad 1 \leq \alpha \leq 4 \quad (4)
\]
where $s_L u$ is the elliptic projection of $u$ onto $S_h$; the operator $s_L$ may be omitted in (3) for $\alpha \leq 3/2$ and in (4) for $\alpha \leq 5/2$. Note that the spaces $H^{(1/2)}$, $F^{(1/2,0)}$ include piecewise-smooth discontinuous functions. The sharpness of these estimates follows from [2].

The similar error estimates are proved also in the case of general form equation (with coefficients depending on $x$). For the problem (1), (2), a two-level method and three-level methods with splitting operators are studied as well. For $n = 1$, the case of discontinuous (measurable and bounded, only) coefficients is considered also [1,3]. Finally, there exists a version of the above results for the abstract second-order equation in a Hilbert space.

The work is partially supported by the RFBR (grant 97–01–00214).


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SUPPLEMENT
A UNIFORMLY CONVERGENT FINITE DIFFERENCE SCHEME FOR A SINGULARLY PERTURBED INITIAL VALUE PROBLEM

G. M. Amiraliyev, Fevzi Erdoğan, Van, Turkey

AMS Class.: 65K10 (65L60)

In this paper we construct a completely exponentially fitted finite difference scheme for the initial value problem with small parameter by first and second derivatives. We prove the first order uniform convergence of the scheme in the sense of discrete maximum norm. Numerical results are presented.

ON THE STABILITY OF NONAUTONOMOUS FUNCTIONAL DIFFERENTIAL EQUATION

Alexandre Andreev, Ulyanovsk, Russia

AMS Class.: 34K20

In the report the problem of nonautonomous functional differential equations asymptotic stability is considered under the supposition that the derivative of Liapunov functional is a nonpositive scalar function. By exploiting limiting equations a result on location of the positive limit set in n-vector space is obtained, theorems on asymptotic stability and nonstability are proved. Some examples are given.
NECESSARY AND SUFFICIENT CONDITIONS
OF REDUCIBILITY OF LINEAR SYSTEMS

Svetlana Artiemieva, Minsk, Belarus

AMS Class.: 34A30

Reducible systems are the simplest class of linear systems. A reducible system is known to have asymptotical characteristics identical with those of the corresponding system with constant coefficients. However, this does not include all difficulties associated with the investigation of reducible systems, because:

1) in many cases it is difficult to define if the given system is a reducible one;
2) even though the fact of reducibility is admitted both transformation and stationary system from this transformation remain as a rule, unknown.

N. P. Erugin has shown that the formula

\[ \int_{x_0}^{x} P(x) \, dx = Ax + H(x) \]  

(*) with limited matrix \( H(x) \) is sufficient for the reducibility of Lappo-Danilevsky’s system of the second order \( \dot{y} = P(x)y \) to the stationary system \( \dot{z} = Az \). Besides, Erugin has also shown that the given formula is not necessary. The question arises: how closely the condition (*) is related to the necessary condition?

It turns out that the condition (*) is necessary and sufficient for the linear systems with functionally-commutative coefficient matrices of unspecified order having only real characteristic values. Some other necessary and sufficient conditions of reducibility of linear systems have been also obtained.

THE CONDITIONS OF THE PASSAGE
TO THE LIMIT FOR A SEQUENCE
OF THE DIRICHLET PROBLEMS

Galina Balashova, Moscow, Russia

AMS Class.: 35B30 (35B37)

We have established the conditions of the passage to the limit for a sequence of nonhomogeneous boundary value problems:

\[ L_m(u_m) = \sum_{|\alpha|=0}^{t_m} (-1)^{|\alpha|} D^\alpha (A_{\alpha m}(x, \delta_k u_m(x))) = h^m(x), \quad (1) \]

\[ \partial_\Omega u_m(x) |_{\partial \Omega} = \psi_{\alpha m}(x'), x' \in \partial \Omega, |\omega| = 0, 1, \ldots, t_m - 1. \quad (2) \]

The equations (1) can be truncated of order \( 2m \) for the case of a problem with an equation of finite order or they can be random either of infinite or finite order. The vector \( \delta_k u(x) \) is

\[ \{D^\alpha u\}_|_{|\alpha| \leq k} = \left\{ \frac{\partial u}{\partial x_1}, \ldots, \frac{\partial u}{\partial x_n}, \frac{\partial^2 u}{\partial x_1^2}, \frac{\partial^2 u}{\partial x_1 \partial x_2}, \ldots, \frac{\partial^k u}{\partial x_n^k} \right\}. \]

It turns out that two principal distinct cases must be considered: the case of infinite order limit equation and the case of finite order limit equation. The analysis of the obtained conditions of the limit passage is shown in examples.
POSITIVE SOLUTIONS OF A THIRD ORDER BOUNDARY VALUE PROBLEM

Abdelkader Boucherif, Tlemcen, Algeria

AMS Class.: 34B15

Consider the following third order boundary value problem

\[
\begin{align*}
-\left(p_1(p_2y')'ight)' + hy &= f(x, y) \quad 0 < x < 1 \\
y(0) &= y'(0) = y'(1) = 0
\end{align*}
\]

We provide some sufficient conditions on the nonlinearity \(f\) that will insure the existence of at least one positive solution. This talk is based on a joint work with M. Dahmani.

REGULARITY OF THE INVERSION PROBLEM FOR THE STURM-LIOUVILLE EQUATION IN THE SPACE \(L_p\)

N. Chernyavskaya, L. Shuster, Beer-Sheva, Israel

AMS Class.: 34A30

Necessary and sufficient conditions for the equation

\[
-(r(x)y'(x))' + q(x)y(x) = f(x), \quad f(x) \in L_p(R), \ x \in R
\]

to have a unique solution \(y(x)\) of the form

\[
y(x) = \int_{-\infty}^{\infty} G(x, t) f(t) \, dt, \quad x \in R
\]

uniformly in \(p \in [1, \infty]\), with \(\|y\|_p \leq c\|f\|_{L_p(R)}\), where \(c\) is an absolute constant, are found. Here \(r(x) > 0, q(x) \geq 0\) for \(x \in R\),

\[
\frac{1}{r(x)} \in L^1_{\text{loc}}(R), \ q(x) \in L^1_{\text{loc}}(R), \ \lim_{|d| \to \infty} \left( \int_{x-d}^{x} \frac{dt}{r(t)} \cdot \int_{x-d}^{x} q(t) \, dt \right) = \infty.
\]

\(G(x, t)\) is the Green function corresponding to (1). Moreover, for \(G(x, t)\) we obtain two-sided estimates, sharp by order for \(x = t\).
GENERALIZED GREEN OPERATOR FOR IMPULSE BOUNDARY VALUE PROBLEM

Sergey Chujko, Slavyansk, Ukraine

AMS Class.: 34A37 (34C25)

We obtain necessary and sufficient conditions for the existence and a generalized Green operator for the construction of solutions of linear system

$$\frac{dz}{dt} = A(t)z + f(t), \quad t \neq \tau_i, \quad \ell_i z(\cdot) = a_i,$$

where $A(t)$ is an $n \times n$ matrix, $f(t)$ is a vector-function, $a_i$ is constant from $R^k$, $i = 1, 2, \ldots, p$; $\ell_i z(\cdot)$ are linear vector-functionals

$$\ell_i z(\cdot) : C[[a, \tau_{i+1} \setminus \{\tau_1, \ldots, \tau_i\}]_{1} \rightarrow R^k, \quad \ell_i z(\cdot) = \sum_{j=0}^{i} \ell_{i}^{(j)}(\cdot),$$

where $\ell_{i}^{(0)} z(\cdot) : C[[a, \tau_1] \rightarrow R^k, \ldots, \ell_{i}^{(i)} z(\cdot) : C[[\tau_i, \tau_{i+1}] \rightarrow R^k$ – linear bounded functionals.

THE FREDHOLM OPERATORS OF THE SECOND ORDER PARABOLIC PROBLEM

Vladimír Ďuríkovič, Bratislava, Slovak Republic

AMS Class.: 35K20 (35K60)

We consider the mixed problems of the first, second and third type for the general second order parabolic equations. Under rather general conditions for the nonlinearity of the equation, the generic properties, surjectivity and bifurcations of these problems are studied. We apply the Nikolskij’s decomposition theorem for the Fredholm operator of the zero index together with qualitative results of this operator.
SUPERPOSITION OF EMBEDDINGS
AND FEFFERMAN’S INEQUALITY

M. Krbec, Prague, Czech Republic

AMS Class.: 46E35 (35B60, 46E30)

This talk will concern some of recent results on imbeddings in Sobolev spaces, in particular, those employing the scale of Lorentz-Zygmund spaces, and applications to a local version of Fefferman’s inequality

\[ \int_\Omega (f(x))^2 V(x) \, dx \leq c(\Omega, V) \int_\Omega (\nabla f(x))^2 \, dx, \quad f \in W^{1,2}_0(\Omega), \]

where \( \Omega \) is a bounded domain with smooth boundary in \( \mathbb{R}^n \).

We shall tackle the problem of the strong unique continuation property for the inequality \( |\Delta u| \leq V(x)|u| \) and we show how the so called “smalness” condition, that is, a uniform convergence of \( c(B, V, \chi_B) \) to zero as the radii of balls \( B \subset \Omega \) tend to zero, is related with the imbeddings. We recover some of known results and obtain their generalizations.

ON SOME PROPERTIES OF THE SOLUTIONS
OF AN ELLIPTIC EQUATION WITH SPECTRAL PARAMETER

Alexander Makin, Moscow, Russia

AMS Class.: 35

In an arbitrary domain \( G \subset R^N \) we consider general nonselfadjoint elliptic operator \( L \) of the second order. If a function \( \tilde{u}(x) \) is a generalized eigenfunction of the operator \( L \) corresponding to an eigenvalue \( \lambda \) so that \( L \tilde{u} + \lambda \tilde{u} = 0 \) then a generalized associated function of order \( n \tilde{u}(x) \) corresponding to the same \( \lambda \) and the eigenfunction \( \tilde{u}(x) \) satisfies to the equation \( L \tilde{u} + \lambda \tilde{u} = \frac{n-1}{n} \).

Order - sharp estimates between \( L^2 \)-norms of root functions of the operator \( L \) have been established. We have also obtained an order - sharp relation between the \( L_\infty \) and \( L^2 \)-norms of an associated function.

Estimates of such type are employed to study convergence of spectral expansions when the general number of associated functions is infinite.
APPEL POLYNOMIALS OF SOME TYPE
AND THEIR ZEROS

Mariana Marčoková, Žilina, Slovak Republic

AMS Class.: 33C45

It is well-known that a polynomial system \( p_n(x) \) is Appell iff for \( n = 0, 1, 2, \ldots \)

\[(i)\]
\[p_n(x) = \sum_{k=0}^{n} a_{n-k} \frac{x^k}{k!}, \quad a_0 = 1\]

or

\[(ii)\]
\[p'_n(x) = p_{n-1}(x)\]

holds.

Examples of Appell polynomials are: the powers \( \sum_{k=0}^{n} x^k \), the Hermite polynomial system \( \sum_{k=0}^{n} H_k(x) \frac{x^k}{2^k k!} \) and the Bernoulli polynomial system \( \sum_{k=0}^{n} B_k(x) \frac{x^k}{k!} \), where \( H_k(x) \) are Hermite polynomials and \( B_k(x) \) are Bernoulli polynomials.

Polynomials which satisfy (ii) are the solutions of some first order linear differential equation by means of which we obtain some information about zeros of these polynomials.

SOME RESULTS OF NON-NEWTONIAN
COMPRESSIBLE FLUIDS

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AMS Class.: 76A05 (35Q30)

To describe certain non-linear effects of viscous fluids which cannot be explained by the classical theory of the Navier-Stokes equations, several models have been suggested. We deal with a problem of decay for weak solution for the certain class of non-Newtonian fluids. We obtain for arbitrary data bounded in \( L^1 \cap L^2 \cap V \) (\( V \) is a class of regularity of initial data) \( L^2 \)- decay in \( n \geq 3, ||u||_2 \leq c(t+1)^{-n/2+1} \).

The method was originally applied on parabolic conservation laws and then extended by M. E. Schonbeck on Navier-Stokes equations. The proof is based on theory of spectral analysis of nonlinear problem. The result, which will be presented, is completely new in area of non-Newtonian fluids.
LAPPO-DANILEVSKI SYSTEMS AND LIAPUNOV TRANSFORMATIONS

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AMS Class.: 34C20

We shall consider linear systems

\[
\frac{dx}{dt} = A(t)x, \quad t \in \mathbb{R}_+ = [0; +\infty], \quad x : \mathbb{R}_+ \to \mathbb{R}^n, \\
\frac{dy}{dt} = B(t)y, \quad t \in \mathbb{R}_+, \quad y : \mathbb{R}_+ \to \mathbb{R}^n,
\]

where elements of \( n \times n \) matrices \( A \) and \( B \) are continuous and bounded on \( \mathbb{R}_+ \). We shall say that (1) is equivalent to (2) if there exists a Liapunov matrix \( L(t) \) such that the transformation \( x = L(t)y \) transfers (1) to (2). The set of systems (1) can be separated into the classes of asymptotically equivalent systems. One of the problem of the theory of asymptotically equivalent systems is to choose the system-representative for every class. The asymptotic properties of the system-representative is the same as the asymptotic properties of all systems from the class. The triangle systems or the system with piece-wise constant coefficients can be used as the system-representatives. If (2) is a Lappo-Danilevski system, i.e. matrix \( B \) is commutative with its integral, then we can obtain the solution in the exponential form. Therefore, it would be efficient to use Lappo-Danilevski systems as the system-representatives. Some new results on asymptotic equivalence of linear systems and Lappo-Danilevski systems are submitted.

DYNAMICAL SYSTEMS WITH SEVERAL EQUILIBRIA AND NATURAL LIAPUNOV FUNCTIONS

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AMS Class.: 34O20 (39A11)

Dynamical systems with several equilibria occur in various fields of science and engineering: electrical machines, chemical reactions, economics, biology, neural networks. The development of qualitative concepts for such systems starts with the classical by now paper of J. Moser (1964) on non-oscillating electrical networks and has been contributed among others by V. M. Popov (with his concepts on mutability and gradient like systems) as well as by G. A. Leonov and his co-workers. As pointed out by many researchers good results may be obtained if a Liapunov function is available. Fortunately for (almost) all cited above physical systems the Liapunov function is associated in a natural way as an energy of a certain kind that is decreasing or at least nonincreasing along systems solution.

Acknowledgment. This work has been carried out while the author was Meyerhoff Visiting Professor at the Department of Theoretical Mathematics, the Weizmann Institute of Science, Israel.
A BOUNDARY VALUE PROBLEM FOR A SECOND ORDER DIFFERENTIAL EQUATION WITH OPERATORIAL COEFFICIENTS AND NON LOCAL BOUNDARY CONDITIONS

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AMS Class.: 35R20 (35R15, 35B45)

In this work we prove the existence and uniqueness of the strong solution for a boundary value problem generated by a differential equation of operatorial coefficients with non local boundary conditions.

We consider in $D = ]0, T_1[ \times ]0, T_2[\] the following boundary value problem:

\[ Lu = \frac{\partial^2 u}{\partial t_1 \partial t_2} + B \left[ \text{sign}(1 - |\mu_1|^2) \frac{\partial u}{\partial t_1} + \text{sign}(1 - |\mu_2|^2) \frac{\partial u}{\partial t_2} \right] + \\
+ A \left[ \text{sign} \left( (1 - |\mu_1|^2) \cdot (1 - |\mu_2|^2) \right) u \right] = f(t_1, t_2) \\
L_{\mu_1} u = u|_{t_1=0} - \mu_1 u|_{t_1=T_1} = \varphi(t_2) \; ; \; L_{\mu_2} u = u|_{t_2=0} - \mu_1 u|_{t_2=T_2} = \psi(t_1), \]

where $f$ and $u$ are functions on $D$ and take values in Hilbert space $H$. $A$ and $B$ are linear operators in $H$ satisfying certain conditions. The proofs are based on the establishment of an apriori estimate of the solution as well as on the density of the image of the operator generated by the equation and the boundary conditions in the data space.

PERIODIC SOLUTIONS OF NON-LINEAR MAY-LEONARD SYSTEMS

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There are well known methods for the description of long-term behavior of a system ([1],[2],[4]) (the Routh-Hurwitz criteria or the Poincaré-Andronov theorem for example), we encounter difficulties because of the algebraic problems — even in case of small dimension — if we haven’t got some assumptions for the symmetricity of the system. In this paper we study a more general, non-symmetric form of the May-Leonard system ([3],[5]):

\[ y' = -\frac{1}{\alpha + \beta + 1} \cdot \begin{bmatrix} 1 & \alpha & \beta \\ \beta & 1 & \alpha \\ \alpha & \beta & 1 \end{bmatrix} \cdot y. \]

Instead of the Routh-Hurwitz criteria we apply a classification by M. L. Zeeman [6] for the observation of long-term behavior of a system. This classification is founded upon the geometric analysis of the nullclines of a system and define a combinatorial equivalence relation on the space in terms of simple inequalities on the parameters. We give a simple necessary condition on the parameters in this non-symmetric May-Leonard system to predict the occurrence of a Hopf - bifurcation and consequently, of isolated periodic orbits.
ASYMPTOTIC STABILITY OF A DISCRETE CONTROL SYSTEM WITH COMPACT UNCERTAINTY

Boris A. Abramov

Wholly depends on reliable operation of automated control systems of compound technological complexes a realization of a posed problem of control.

Now in connection with the increased capabilities of modern computer facilities actual the development of a technique of automated research of dynamic properties of uncertain compound technological complexes and objects because of algebraic methods of the analysis is. These methods allow considerably to reduce evaluations connected to the analysis of dynamic properties of control systems by frequent methods and provide the solution of a problem of a construction of highly effective control systems possessing robust properties.

In the report the interval mathematical model of a discrete dynamic control system in the form of a system of stochastic difference equations of a type ITO ia adduced. The research problem of a property robust stability of discrete stochastic control systems with compact uncertainty is posed and the criterion robust stability because of uses of the algebraic approach is obtained. The computing algorithms of research of a property robust stability are developed and the example of particular use of an obtained criterion robust stability is adduced.


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Typical manufacturing processes in metallurgy, chemical and oil processing industry, notorious for undesirable environmental effects, are multivariable, spatially-distributed, nonstationary, dynamically uncertain processes with delays and high level of noise in the information and control channels. They are subjected to a wide variety of operational conditions, including emergency-type situations and transient regimes when pollutants can drastically exceed nominal levels.

The goal of this paper is the development of the methodology for the design of robust multi-target automatic control systems including the information technology and software design.

Specific scientific research tasks are:

- formulation of the heuristic procedure for structural synthesis of robust multi-target automatic control systems,
- definition of the existence conditions for task decomposition of the parametric synthesis problem for corresponding one- and multi-target control subsystems,
- definition of the aggregated comparison system for multi-target automatic control systems and investigation of its dynamic properties,
- development of the information technology of the software intended for robust multi-target automatic control systems.

ININVARIANCY OF BINARY SYSTEMS

Zhansulu A. Baykenova

The research concerned the dynamic properties of sufficiently wide classes of control systems such as linear, stationary and essentially non-stationary, nonlinear with nonlinearity of a type of a sector, determined and stochastic, is indissolubly connected to the availability at them of invariant sets. Knowledge of invariant sets allocated classes of control systems is the comprehensive characteristic, permitting evaluation of their dynamic accuracy.

The class of binary control systems establishes in solving the tasks of controlling essentially non-stationary dynamic processes in indefinite conditions, from the point of view of high efficiency, has acquired a reputation for itself. A methodological basis of construction of such systems forms the principle binarity. This principle allows automatic formation of controlled feedback connections giving a closed nonlinear control system required by set of dynamic properties.

The characteristic feature possessed of binary control systems is invariant, unlimited, closed sets of a conical type.

In the paper the method of properties of invariancy of systems binary of automatically controlled with unlimited closed sets of a conical type is suggested and computing algorithms of construction of such invariant sets are developed. Cases of continuous and discrete binary control systems are considered according to availability or absence of additive disturbance of “white noise” type.


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SYSTEM OF DIFFERENTIAL EQUATIONS
WITH UNSTABLE TURNING POINT AND
MULTIPLE ELEMENT OF SPECTRUM
OF DEGENERATE OPERATOR

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AMS Class.: 34E15 (34E20, 34C11, 34B15)

The systems of singular perturbation differential equations (SSPDE) play the
great role in many mathematical models of biological problems and in medicine.
One of the known problems for obtaining such problems is as follows: Is there
a biological process stable or not? If the spectrum of the degenerate operator
is stable, then such system is known pretty well. Remembering classical results
(Vasilieva’s method, Lomov’s method and others) which give the answer of the dis-
cussed problems concerning to bounded solutions of the adequate SSPDE. If the
system contains turning points i.e. some elements of the spectrum of the degener-
ate operator are nonstable, then the general theory of obtaining such problems is
not discovered yet, however some special problems have solution (see [1], [2], [3],
[4]).

In the present report we consider the problem

\[ L_\varepsilon w(x, \varepsilon) \equiv \varepsilon^2 w''(x, \varepsilon) - A(x)w(x, \varepsilon) = h(x), \]

\[ E_1 w(m, \varepsilon) = E_1 (\mu - 2\alpha_m + \hat{w}_m), \quad E_2 w'(m, \varepsilon) = E_2 (\mu - 3\alpha_m + \mu^{-3}\hat{w}_m), \]

where \( \varepsilon \to +0, x \in I = [0, 1], m = 1, 2; \mu = \sqrt{\varepsilon}. \) Here \( A \) denotes a linear operator
on \( R^n, \alpha_m \) and \( \hat{w}_m \) are given vectors, \( E_k \ (k = 1, 2) \) – diagonal matrices of the
n-th order of the form \( E_1 = \text{diag}\{1, 0, \ldots, 0\}, \ E_2 = \text{diag}\{0, 1, \ldots, 1\}, \ h(x) \) – given
vector-function, \( w(x, \varepsilon) \) – a sought vector-function.

Problem (1) we shall consider under the following conditions: 1) \( A(x), h(x) \in C^\infty[I], \) 2) Spectrum of the degererase operator \( A \) is real and fulfils the following condition

\[ 0 \leq \lambda_1(x) \equiv x\hat{\lambda}_1(x) < \lambda_2(x) < \ldots < \lambda_p(x) \equiv \ldots \equiv \lambda_n(x), \]

where \( \hat{\lambda}_1(x) > 0 \) for all \( x \) in \( I. \)

One can see from (2) that the point \( x = 0 \) is a turning point for equation
(1), and moreover if \( \lambda_1(x) \geq 0, \) then it is nonstable turning point. Thus it is
necessary to construct a solution of (1) in case when the simplified equation –
\( A\omega(x) = h(x) \) has, in general case, a point of discontinuity at 0. System (1) with
several conditions for the spectrum of degenerate operator \( A \) was considered in [2],
[3], [4] and in other articles of authors. In the present report we shall prove, that
ignoring the nonstability of the turning point some partial solutions of the vector equation (1) can be bounded in the domain.


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DENSE SETS AND FAR-FIELD PATTERNS
FOR THE VECTOR THERMOELASTIC EQUATION

Fioralba Cakoni

AMS Class.: 35J05 (73, 47)

In the theory of direct problems in thermoelasticity \[2\], \[3\], it is shown how
the solution of a boundary value problem and the related far-field corresponding
to an incident field and to a given obstacle can be calculated.

The direct scattering problem asks: given an open domain \(V \subset \mathbb{R}^3\) with con-
ected \(C^2\)-boundary \(S\) and \(V^e = \mathbb{R}^3 \setminus \overline{V}\), given a plane wave time harmonic
\(U^i(r, \hat{k}) : V^e \to \mathbb{R}^f\) (\(\hat{k}\) the direction of propagation), determine a solution
\(U(r, \hat{k}) = U^i(r, \hat{k}) + U^s(r, \hat{k})\) of the equation in \(V^e\)

\[
\tilde{L}(\partial_r) U(r, \hat{k}) = \left[ \frac{(\mu \Delta + \omega^2) \hat{I}_3 + (\lambda + \mu) \nabla |\gamma \nabla q|}{q \kappa \eta} \right] = 0
\]

such that \(\tilde{B}(\partial_r, \hat{n}) U(r, \hat{k}) = 0\) on \(S\), where boundary differential operator \(\tilde{B}(\partial_r, \hat{n})\)
is expressed via the surface traction operator, and \(U^s(r, \hat{k})\) satisfy the asymptotic
Kupradze relations as \(r \to \infty\). Let call \(U^\infty(\hat{r}, \hat{k}) \in [L^2(\Omega)]^3\) the set of the
far-field patterns related to a radiation solution \(U^s(r, \hat{k})\) of (1) \[3\]. We show
that the functional equation \(\mathfrak{S} U^s = U^\infty\) with the linear operator \(\mathfrak{S}\) mapping a
radiation solution \(U^s(r, \hat{k})\) onto its far-field \(U^\infty(\hat{r}, \hat{k})\) is ill-posed. The uniqueness
is proved in \[3\]. A further study provides that the rang of the operator \(\mathfrak{S}\) is not
\(\equiv [L^2(\Omega)]^3\). Using the vector spherical harmonics and the decomposition of the
solution \(U^s(r, \hat{k})\) into the irrotation part and the solenoidal part characterised
by five potentials (solution of the scalar Helmholtz equation), we prove that the
Fourier coefficients with respect to the vector spherical harmonics of the far-field
pattern \(U^\infty(\hat{r}, \hat{k}) \in [L^2(\Omega)]^3\) corresponding to a radiation solution \(U^\infty(\hat{r}, \hat{k})\) must
satisfy a growth condition (we do not represent here because of its complicated
analytic shape). Futhermore, if the solution does exist it will not continuously
depend on \(U^\infty\) in any reasonable norm of the corresponding spaces. Finally the
main result characterizes the range of the operator \(\mathfrak{S}\) in the case of the Dirichlet
boundary condition.

We define as a Herglotz thermoelastic function a classical entire solution of (1)
which satisfies the growth property

\[
\limsup_{r \to \infty} \frac{1}{r} \int_{B(0, r)} \| U(r') \|^2 dV(r') < +\infty.
\]

The representation theorem that allows any Herglotz function to be decomposed
into plane waves propagating in every direction, is very important for the following.
Let \((\hat{k}_n)\) be a sequence of unit vectors that is dense on \(\Omega\). We prove that 
\[
[L^2(\Omega)]^0 = \text{span} \tilde{U}_\infty(\mathbf{r}, \hat{k}_n)
\]
if and only if there does not exist a Herglotz thermoelastic function

\[
U = (u, \Theta) = (u^1 + u^2 + u^s, \Theta^1 + \Theta^2)
\]
such that

\[
\tilde{u} = u^1 \otimes \hat{k} + u^2 \otimes \hat{k} + u^s \otimes \hat{\theta} + u^s \otimes \hat{\varphi}
\]
is an eigenfunction of the interior eigenvalue problem

\[
(\Delta^* + \omega^2)\tilde{u} = 0, \quad \text{div} \tilde{u} = 0 \quad \text{in} \ V, \quad \tilde{u} = 0 \quad \text{on} \ S,
\]
where \(\Delta^*\) is elastostatic operator.

We raise by one the rank of the tensorial character of the fields in order to avoid the polarization effect of the thermoelastic waves.

The above results give the basic tool to construct an inverse algorithm of Colton-Monk type for the thermoelastic scattering problem corresponding to a rigid scatterer at zero temperature.

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STABILITY OF THE TRIVIAL INVARIANT TORUS
OF A CERTAIN CLASS OF DISCONTINUOUS
DYNAMICAL SYSTEMS

Sergey Doudziany

We have consider the impulsive system

\[ \dot{\varphi} = a(\varphi), \quad \dot{x} = f(\varphi, x), \quad \Delta x|_{\varphi \in \Gamma} = I(\varphi, x), \]

where \( \varphi \in \mathbb{S}_m \), \( x \in \mathbb{R}^n \); the functions \( a(\varphi) \), \( f(\varphi, x) \), \( I(\varphi, x) \) are \( 2\pi \)-periodic with respect to each of their variables \( \varphi_\nu \), \( \nu = 1, \ldots, m \). We assume that the set \( \Gamma \) is a subset of the torus \( \mathbb{S}_m \), which is a manifold of dimension \( m - 1 \) defined as

\[ \Gamma = \{ \varphi \in \mathbb{S}_m : \langle b, \varphi \rangle = 0 \} \]

where \( b = (b_1, \ldots, b_n) \) is a vector with positive integer coordinates and the following statement holds \( \langle a(\varphi), b \rangle|_{\varphi \in \Gamma} = 0 \).

Sufficient characteristics of stability, asymptotic stability, unstability of the trivial invariant torus of this impulsive system are formulated and proved. All conditions are given in the region \( Z_\Omega = \{ \varphi \in \Omega, x \in \bar{J}_h \} \subset Z \). We denote \( \omega \)-boundary set of positive semitriajcetory of the solutions \( \varphi_t(\varphi), \varphi \in \mathbb{S}_m \) of the first equation of this system by \( \Omega_\varphi \), hence \( \Omega = \bigcup_{\varphi \in \mathbb{S}_m} \Omega_\varphi \subset \mathbb{S}_m \).


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STABILITY OF THE LINEAR DIFFERENTIAL EQUATIONS WITH RANDOM PERIODIC COEFFICIENTS

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AMS Class.: 35B37

In present time some theoretical and applied research considering the behaviour of the dynamic system as random. On mathematical language this question reduce to learning of the condition stability in mean square of solution of the system differential equations with random coefficients.

1. We shall consider a simple mathematical model of the dynamic system with random periodic influence which describe a system of the linear differential equation

\[
\frac{dx(t, \mu)}{dt} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & -\mu \alpha(t, \xi(t)) \end{pmatrix} x(t, \mu),
\]

where \(\xi(t)\) – random Markovian process taking the state \(\theta_k (k = 1, n)\) with probability \(p_k (k = 1, n)\) which satisfied the following system of differential equations

\[
\frac{dp_k(t)}{dt} = \sum_{s=1}^{n} \alpha_{ks}(t) p_s(t), \quad (k = 1, n) \quad \alpha_{kk}(t) \leq 0, \quad \alpha_{ks}(t) \geq 0 \quad (k \neq s) \quad (k, s = 1, n),
\]

coefficients \(a(t, \xi(t)) = (a(t, \theta_1), a(t, \theta_2), \ldots, a(t, \theta_n)) = (a_1(t), a_2(t), \ldots, a_n(t))\) is a periodic function \(a_k(t + 2n) = a_k [k = 1, n]\).

We shall find and analyze the condition of the stability in the mean square of zero solution of the system (1) because investigating the stability the mean square is a most importand for engineering elaboration.

That when \(0 < \mu < \varepsilon_1, \vert p - 2\omega i \vert < \varepsilon_2\), where \(\varepsilon_1, \varepsilon_2\) – sufficient small numbers the system of differential equation (1) have nonstability solution if following conditions are valid:

\[
\frac{\nu}{\omega^2} \left[ \frac{1}{2\nu^2} + \frac{1}{2(\nu^2 + \omega^2)} - \frac{2}{\nu^2 + \omega^2} \right] < 8\beta + 0(\mu), \quad \frac{\alpha}{\omega} < \frac{\mu}{8(\nu^2 + 4\omega^2)} + \sqrt{\frac{1}{16}\nu^2} - \chi^2,
\]

\[
x \equiv \beta + \frac{\mu}{16\nu^2} - \frac{\mu\nu}{4\nu^2 - (\nu^2 + \omega^2)} + \frac{\mu\nu}{16\nu^2(\nu^2 + 4\omega^2)}.
\]


METHOD OF AVERAGING FOR THE SYSTEM OF FUNCTIONAL-DIFFERENTIAL INCLUSIONS

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AMS Class.: 34A60 (34K15, 34C29)

Let \( C_0 \) be the Banach space of all continuous functions of \([-r, 0]\) into \( R^n \) with the supremum norm. We denote by \( x_t \) the element \( C_0 \) defined by \( x_t(s) = x(t + s) \) for \( s \in [-r, 0] \) and for fixed \( t \in [0, T] \), \( r \geq 0, T > 0 \).

Let us denote by \((\text{comp. } R^n, H)\) and \((\text{conv } R^n, H)\) the metric space all nonempty compact and nonempty compact convex, respectively, subsets of \( n \)-dimensional Euclidean space \( R^n \) with Hausdorff metric \( H \). We study the existence of solutions to functional-differential inclusions of the form

\[
\begin{aligned}
\dot{x}(t) &\in F(t, x_t, y_t) \\
\dot{y}(t) &\in G(t, x_t, y_t)
\end{aligned}
\]

and we give theorem concerning the method of averaging for the system

\[
\begin{aligned}
\dot{x}(t) &\in \varepsilon F(t, x_t, y_t) \\
\dot{y}(t) &\in F(t, x_t, y_t)
\end{aligned}
\]

with the initial conditions \( x(t) = \varphi(t) \) and \( y(t) = \psi(t) \) for \( t \in [-r, 0] \) where \( \varepsilon > 0, \varphi, \psi : [-r, 0] \to R^n \) are given absolutely continuous functions.

Let’s assume that the multivalued mappings \( F, G : [0, T] \times C_0 \times C_0 \to \text{comp. } R^n \) satisfy the following conditions:

1° \( F \) and \( G \) are measurable for \( t \in [0, T] \) and for fixed \( (u, v) \in C_0 \times C_0 \)

2° \( F \) and \( G \) are Lipschitzian with respect to \( (u, v) \)

3° there exists a \( M > 0 \) such that \( H(F(t, u, v), \{0\}) \leq M \) for \( (t, u, v) \in [0, T] \times C_0 \times L_0 \)


STABILITY OF FUNCTIONAL-DIFFERENTIAL EQUATIONS OF NEUTRAL TYPE

Teresa Janiak Elżbieta Łucszak-Kumorek

AMS Class.: 34K20 (34A60)

The goal of this paper is to give some results concerning stability, uniform stability and asymptotic stability of the zero solution of functional-differential equations of the form

\[(*) \quad x(t) \in F(t,x_t,x_{t})\]

where $F$ is a multifunction with values that are nonempty compact convex subsets of $n$-dimensional $\mathbb{R}^n$.

Let $C_0$ and $L_0$ denote the Banach spaces of all continuous and Lebesgue integrable functions, respectively, of $[-r, 0]$ into $\mathbb{R}^n$ with the norms $||x||_0 = \sup_{-r \leq t \leq 0} |x(t)|$ and $|y|_0 = \int_{-r}^0 |y(t)| dt$ for $x \in C_0$ and $L_0$, respectively, where $| \cdot |$ denotes the Euclidean norm. For a given function $x : [-r, T] \to \mathbb{R}^n$ and fixed $t \in [0, T]$ we denote $x_t(s) = x(t+s)$ for $s \in [-r, 0]$ $r \geq 0, T > 0$. It will be assumed throughout of this paper that multivalued mapping $F : [0, \infty) \times S_H \times S^1_H \to \Omega(\mathbb{R}^n)$, where $S_H = \{ u \in C_0 : ||u||_0 \leq H \}$, $S^1_H = \{ \nu \in L_0 : |\nu|_0 \leq H \}$ and $\Omega(\mathbb{R}^n)$ denote a collection of all nonempty compact convex subsets of $\mathbb{R}^n$, satisfies the following conditions:

1° $F(\cdot, u, \nu) : [0, \infty) \to \Omega(\mathbb{R}^n)$ is measurable for fixed $(u, \nu) \in S_H \times S^1_H$

2° there exists a Lebesgue integrable function $m : [0, \infty) \to \mathbb{R}^+$ such that $g(F(t,u,\nu)\{0\}) \leq m(t)$ for a.e. $t \in [0, \infty)$ and $(u, \nu) \in S_H \times S^1_H$ where $g$ is the Hausdorff metric defined in $\Omega(\mathbb{R}^n)$

3° $F(t,\cdot, \cdot) : S_H \times S^1_H \to \Omega(\mathbb{R}^n)$ satisfies for fixed $t \in [0, \infty)$ the Lipschitz condition of the form $g(F(t,u,\nu), F(t,\hat{u},\hat{\nu})) \leq k(t) (||u - \hat{u}||_0 + |\nu - \hat{\nu}|_0)$ where $k : [0, \infty) \to \mathbb{R}^+$ is a Lebesgue integrable function;

4° $0 \in F(t,\hat{O},\hat{O})$ for a.e. $t \in [0, \infty)$ where $\hat{O}$ and $\hat{O}$ are the nulls of $C_0$ and $L_0$ respectively.

The basic idea of this paper is to use Lyapunov functionals to present sufficient conditions for stability, uniform stability and asymptotic stability of the trivial solution of inclusions $(*)$.


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GRADIENT METHOD IN SOBOLEV SPACES
FOR QUASILINEAR ELLIPTIC PROBLEMS

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AMS Class.: 46N20 (49M10, 35J65)

The gradient method is widely applied to the approximate solution of elliptic boundary value problems through discretization. The extension of the results on the gradient method from $\mathbb{R}^n$ to Hilbert spaces assumes Gateaux differentiability of the operator, a condition unsatisfied for PDE’s.

Our aim is to modify the Hilbert space method such that it applies directly to boundary value problems in Sobolev spaces. This is achieved by transforming the equation $T(u) = g$, which contains a non-differentiable operator $T$, by an auxiliary positive linear operator $B$. Namely, if $B^{-1}T$ has an extension $A$ on the energy space $E_B$ of $B$ which fulfils (besides some smoothness and structure conditions)

$$m(\|u\|) \|h\|^2 \leq \langle A'(u)h, h \rangle \leq M(\|u\|) \|h\|^2 \quad (u, h \in E_B),$$

then the gradient method yields linear convergence to the weak solution for any right-hand side $g \in R(B)$. The iterating sequence has the form

$$u_{k+1} = u_k - \alpha_k B^{-1}(T(u_k) - g) \quad (k \in \mathbb{N}),$$

i.e. the equation for $T$ is reduced to equations containing $B$.

The result applies to quasilinear elliptic boundary value problems if $-\Delta$ (or its power) is chosen as $B$, i.e. elliptic regularity is used to achieve differentiability of the transformed operator. Consider e.g. the following second order problem:

$$T(u) := -\text{div } p(x, \nabla u) + q(x, u) = g(x), \quad u|_{\partial \Omega} = 0$$

(1)

on a bounded domain $\Omega \subset \mathbb{R}^N$ with $\partial \Omega \in C^{2,\nu}$, $p \in C^{1,\nu}(\bar{\Omega} \times \mathbb{R}^n)$, $q \in C^1(\bar{\Omega} \times \mathbb{R})$, $g \in C^{0,\nu}(\bar{\Omega})$. Assume that

(i) the matrices $\{\partial_{\eta_i} p_i(x, \eta)\}_{i,k=1,...,N}$ $(x, \eta) \in \bar{\Omega} \times \mathbb{R}^n)$ are symmetric and have eigenvalues between positive constants $\lambda$ and $\Lambda > 0$;

(ii) $0 \leq \partial_\xi q(x, \xi) \leq \beta |\xi|^{p-2}$ $(x, \xi) \in \bar{\Omega} \times \mathbb{R})$ with some $\beta > 0$ where $p := \frac{2N}{N-2}$ for $N \geq 3$ and $2 \leq p < \infty$ is arbitrary for $N = 2$.

Then for every $u_0 \in C^{2,\nu}(\bar{\Omega})$ with $u_0|_{\partial \Omega} = 0$ there exist $M_0 \geq m_0 > 0$ such that the following sequence converges to the weak solution $u^*$ of (1):

$$u_{k+1} := u_k - \frac{2}{M_0 + m_0}(-\Delta)^{-1}(T(u_k) - g).$$

(2)
Further, convergence is linear with quotient $q = \frac{M_0 - m_0}{M_0 + m_0}$.

These results are found in [1], [2].

**Numerical aspects.** 1. This method reduces numerical problems to those which arise in the solution of the Poisson equations in (2), being considerably simpler than (1). Solving (2) by the finite element method, for example, has the advantage of providing an approximation for $u_{k+1}$ which is sufficiently smooth (at least $H^2$) for computing $T(u_{k+1}) - g$ for the next iteration.

2. Consider semilinear equations of the form

$$-\sum_{i=1}^{N} \partial_i(a_{ij}\partial_j u) + p(u) = g, \quad u|_{\partial \Omega} = 0$$

where $p$ is an algebraic polynomial with coefficients depending on $x$. If $\Omega$ is transformed to a cube (or a ball) and $a_{ij}, q, g$ and $u_0$ are approximated by trigonometric (algebraic) polynomials then the sequence $(u_k)$ will consist of trigonometric (algebraic) polynomials and the solution of (2) means linear combination (linear system of algebraic equations) for the coefficients, respectively.


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ON A NON-UNIFORMLY NONLINEAR ELLIPTIC SYSTEM BVP

A. Mamourian

AMS Class.: 35J70 (30G20)

Making use of function theoretic methods in partial differential equations, the nonlinear systems with uniformly ellipticity in the sense of Lavrentiev are studied extensively.

This work is aimed at the investigation of boundary value problems for the systems of nonlinear equations with degeneration of ellipticity in two dimensional domains. In particular, the boundary conditions with negative index are considered. Existence and uniqueness of the classical solution to the problem is established for natural class of boundary data. The proof involves an a priory estimate similar to the one used for treating nonlinear elliptic systems with the index equal to zero.

Let $L = L_0 + L_1 + \ldots + L_m$ be the boundary contours of an $m + 1$-connected Liapounoff region $D$, where $L_0$ contains all contours $L_j$, $j \geq 1$. Consider the equation

$$W_z = H(z, W_z) = \bar{H}(\theta(z) W_z) + F(z)$$

in $D$, $z = x + iy$ ($\bar{z} = x - iy$), $W = W(z) = u(x, y) + iv(x, y)$ and $W_z = \frac{\partial W}{\partial z} = (\frac{\partial W}{\partial x} - i \frac{\partial W}{\partial y})/2$, $W_{\bar{z}} = \frac{\partial W}{\partial \bar{z}} = (\frac{\partial W}{\partial x} + i \frac{\partial W}{\partial y})/2$. We assume the right hand side of equation (1) satisfies the conditions

$$|H(z, \eta_1) - H(z, \eta_2)| \leq Q(z, \eta_1, \eta_2)|\eta_1 - \eta_2|$$

$$Q(z, \eta_1, \eta_2) \leq \tilde{Q}(\eta_1 - \eta_2) \leq 1$$

with the boundary condition

$$\text{Re}[a(t) W(t)] = \gamma(t)$$

on $L$; $a$, $\gamma$ are given functions on $L$. In respect to $\tilde{Q}$, we assume:

(A1): $\tilde{Q}(\beta)$ as a real function of $\beta = |\eta_1 - \eta_2|$ is continuous in $[0, \infty]$; if $\beta \in (0, \infty)$, then $\tilde{Q}(\beta) < 1$; the function $\beta \tilde{Q}^2(\beta)$ is increasing and concave.

Concerning the coefficients of the boundary values (1)-(4), we shall make the usual assumptions for uniformly elliptic case, i.e.

(A2): The complex function $F(z)$ assumed to be measurable belonging to the class $L_p(D)$, for some $p > 2$, $\theta(z)$ is assumed to be measurable belonging to the class $L_\infty(D)$; the complex function $a(t)$ ($a(t) \neq 0$) and real function $\gamma(t)$ are
Hölder continuous on $L$, with the respect to $\alpha$, where $0 < \alpha \leq 1$ ($a, \gamma \in H_{\alpha}(L), 0 < \alpha \leq 1$). We shall also assume that $|\theta(z)| = 1$. The solution $w$ will be sought in the Sobolev space $W^{1}_{p}(D), p > 2$.

**Remark.** Let us recall that, in the classical boundary values problems (1)–(4), relative to the uniformly ellipticity of the nonlinear system of equations of Lavrentiev type, the solution $w$ is sought in the Sobolev space $W^{1}_{p}(D)$, for some $p > 2$. For equation (1) in the case of degeneration of ellipticity (3), we shall not apply the $L_{p}$-Theory directly for the proof of existence. Therefore formulation of boundary values problem (1)–(4), involves the weak boundary condition.

If $\tilde{H}$ in (1), the condition of solvability of (1)–(4) holds.

**Proposition 1.** Let conditions (A$_{1}$), (A$_{2}$) hold, if the index $n \leq 0$ (m arbitrary finite), then there exists a solution (in $W^{1}_{p}$, for some $p > 2$) of problem (1)–(4).

**Proposition 2.** Under the conditions (A$_{1}$), (A$_{2}$), if $n \leq 0$, problem (1)–(2) has a unique solution.


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INSURING OF SOLUTION STABILITY
OF EQUATION SYSTEMS

Vilen Roizman

It is well known that the solution of differential equation systems may be reduced to the solution of linear equation systems. The linear computational models of $Ax = b$ type, where $b$ - vector of experimentally measured parameters $b_j; j = 1, 2, ..., n$; $x$ - vector of missing parameters $x_j; j = 1, 2, ..., m$; $A$ - matrix $n \times m$ are widely spread in dynamics of systems and processes. However the models have practical importance only when the errors of experimental input information cannot cause intolerable large errors of the values being determined, i.e. when the models are stable.

In the report it is determined the meaning of the model stability with regard to all or to group of factors; it is introduced the value of the relative error of parameters identified with the help of linear model:

$$\frac{\|\Delta x\|}{\|x\|} \leq \frac{C(A)\Delta b}{b} + [C(A)^2] \frac{\Delta A}{A}.$$  (3)

This estimation, thus, is represented by the number of stipulation $C(A)$ and the faults of characteristics and elements $A$ being measured. Estimation (1) allows to explain the decreasing stability of the model while the degree of is growing. In other words, it states the necessity to search the compromise between the desire to give through description or the object using large number of factors and ensuring the stability of the model. The estimation (1) shows that the model can be regularized not only by way of influencing the $A$ operator, which is in real production environments can not always be available for the various reasons. Not less efficient regularization can achieved by way of influencing the $b$ vector of parameters being measured, which metod is based on the statistical nature of the vector. To achieve this, you have to carry out a great number of $b$ measurements, insert the value of the vector into the calculated model and count the realization of every one of identified parameters. Mathematical expectations of paramaters values calculated on the base of these realizations are assumed as true values of these parameters.

Estimation (1) places interest in pure practical aspect, since it states the functional interdependence of economical factors (accuracy of the method and accuracy of measuring facilities), and theoretical (accuracy models) thus making it possible to choose one of these requirements to provide the two others set apriory.

Control of effectiveness of statistic method for ensuring the stability solutions war applied to definition of rotors discs eccentricity of aviation gas-turbine engine compressor.

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ON SOME PROPERTIES OF SOLUTIONS TO ONE NONLINEAR P-TYPE EQUATION

Vladimir Tsegel’nik

AMS Class.: 34A

We investigate some analytic properties of solution to the equation

\[
\left( \frac{d^2y}{dz^2} \right)^2 - \left( \frac{dy}{dz} \right)^2 + 4(1 - \beta)^2 \left[ (1 - \beta)^2 - 2 \left( y - z \frac{dy}{dz} \right) \right]^2 = 16 \left[ (1 - \beta)^2 - 2 \left( y - z \frac{dy}{dz} \right) \right]^{\frac{3}{2}},
\]

(1)

which is satisfied by the function \( y = zH \), where \( H(z, u, w) = -u^2w^2(2z)^{-1} + (1 - \beta)uwz^{-1} + u + w \) is the Hamiltonian associated with the third Painleve equation

\[
zwd^2w = z \left( \frac{dw}{dz} \right)^2 - w \frac{dw}{dz} + w^3 + \beta w - z
\]

(2)

in the case \( \gamma = 0, \alpha = -\delta = 1 \).

1. Using the direct and the inverse Backlund transformations as well as the formulas determining a one-to-one correspondence between the solutions \( y_{\beta}, w_{\beta} \) to the equation (1) and (2) respectively, we obtain for the equation (1) the Backlund transformation

\[
y_{\beta} = y_{\beta - 2} - 2\beta zw_{\beta - 2}^{-1} + 2(\beta - 2),
\]

where

\[
w_{\beta - 2} = \frac{1}{2} \frac{dy_{\beta - 2}}{dz} + 2z \frac{d^2y_{\beta - 2}}{dz^2} \left\{ \left( z \frac{d^2y_{\beta - 2}}{dz^2} \right)^2 - \frac{1}{(3 - y_{\beta - 2})^2} + 4(3 - \beta) \right\}^{-1},
\]

and \( y_{\beta - 2} \) is the solution of the equation (1) in which \( \beta \rightarrow \beta - 2 \).

2. We obtain the nonlinear functional equation

\[
y_{\beta - 4} = y_{\beta - 2} - 2(\beta - 4) + 2zR^{-1},
\]

(3)

where \( R = -w_{\beta} + [2\beta w_{\beta} + w_{\beta}^3(w_{\beta - 2} + w_{\beta + 2}) + 2(2 - \beta)w_{\beta - 2}]w_{\beta - 2}^2 \), which connect solution of the equation (1) under the various values of the parameter \( \beta \).

The equation (1) can be considered as:

1) the principle of nonlinear superposition of solutions to the equation (1);
2) the alternative form of the Backlund transformation.
It should be mentioned that the equation (1) is a new one in the list of equation [1] for polynomial Hamiltonian associated with Painleve’s equations.

The research was supported by the International Soros Science Education Program.


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INTEGRAL INEQUALITIES IN MEASURE SPACES

László Horváth, Veszprém, Hungary

AMS Class.: 26D15

Suppose \((X, \mathcal{A}, \mu)\) is measure space. In this talk we consider integral inequalities of the form

\[
y(x) \leq f(x) + g(x) \int_{S(x)} y d\mu, \quad x \in X
\]

and the corresponding integral equations

\[
y(x) = f(x) + g(x) \int_{S(x)} y d\mu, \quad x \in X
\]

where \(S(x) \in \mathcal{A}\) for every \(x \in X\), \(f, g : X \to \mathbb{R}\) are \(\mu\)-integrable over \(S(x)\) for every \(x \in X\).

We examine the existence of the solutions of the equation (2), and we derive estimates for the solutions of (1) in terms of the solutions of (2). Finally, we obtain explicit bounds for the solutions of (1) and (2).

Lp SOLUTIONS OF NONLINEAR INTEGRAL EQUATIONS

Alejandro Omón, Manuel Pinto, Santiago, Chile

We study nonlinear integral equations on unbounded domains. The compactness of the nonlinear integral operator is considered using Schauder’s fixed point theorem. We obtain the existence of \(L^p\) solutions for \(1 \leq p < \infty\) and the case \(p = \infty\) is also studied. Moreover, we have some results about integral equations of first kind.


LOCAL CONTINUITY OF SOLUTIONS OF SINGULAR EVOLUTION EQUATIONS, RECENT RESULTS AND OPEN PROBLEMS

Emmanuele Di Benedetto, Evanston, USA

We will present new results and will survey the main issues concerning the local behaviour of solutions of singular evolution equations, of the type

$$\frac{\partial}{\partial t} \beta(u) - \Delta u = \{ \text{lower order terms} \},$$

in $\mathcal{D}'$ over some space–time domain. The key feature is that, the graph $\beta(\cdot)$ exhibits multiple discontinuities (as opposed to only one singularity as in the classical Stefan problem). The flow of two immiscible fluids is a porous medium (Buckley–Leverett models) and Stefan–type multiple transitions of phase, lead to evolutions equations of this type, and their quasi–linear versions. We will indicate recent techniques introduced in this context and point out to some open issues. Among these are the boundary behavior of the solutions, as well as their local behavior when one has lower order terms with critical (Hadamard) growth conditions. The latter typically arises in the Buckley–Leverett system.

ASYMPTOTIC PROPERTIES OF N-TH ORDER DELAY DIFFERENTIAL EQUATIONS

Jozef Džurina, Košice, Slovak Republic

AMS Class.: 34C10

We deduce oscillatory and asymptotic properties of solutions of n-th order delay differential equations

$$L_n y(t) \pm p(t) u(\tau(t)) = 0$$

from the oscillation of a set of the first order delay differential equations with larger deviating argument of the form

$$y'(t) + q_i(t) y(w(t)) = 0.$$
ON THE STEADY MOTION OF A NAVIER-STOKES FLUID PAST A SELF-PROPELLED BODY

Giovanni P. Galdi, Pittsburgh, USA

AMS Class.: 35Q, 76C

We say that a body \( B \), moving in an infinite viscous fluid \( F \), undergoes a self-propelled motion if the external force exerted on \( B \) is identically zero. Typical examples of self-propelled bodies are rockets, submarines, fishes, microorganisms, etc. This kind of movement is contrasted to the towed motion, where \( B \) moves because of the action of a nonzero external force \( F \). A characteristic example is the fall of \( B \) in the air, under the action of gravity.

In this talk, we shall consider steady translational flow of a self-propelled body, namely, \( B \) moves with a constant velocity \( \xi \neq 0 \) and the motion of \( F \), as seen by an observer attached to \( B \), is independent of time. In particular, for small Reynolds numbers, we shall give necessary and sufficient conditions for this kind of motion to occur, namely, a steady self-propelled motion can happen if and only if a certain thrust vector \( G \) is not zero. \( G \) depends only on \( \xi \) and on the geometric properties of the body \( B \) such as size or shape. Moreover \( G \) is one-to-one related with the force \( F \) of corresponding towed motions.

CONSISTENT CONSTRUCTION OF COARSE GRIDS

W. Hackbusch, Kiel, Germany

AMS Class.: 65N

In Finite Element applications one often would like to have a whole hierarchy of discretisation levels. In particular, for multi-grid iterations one needs also very coarse grids in order to solve the fine-grid problem. If the domain of the pde has a complicated shape, the construction of low-dimensional hierarchy level is impossible, when we use the standard FEM.

Another example ist the homogenisation process which replaces a complicated fine-grid situation by a simpler one in the coarse-level discretisation.

The aim of our construction is a low-dimension FE representation of higher-dimensional ones in such a way that the standard FE error estimates still remain valid for the new Finite Elements (called Composite Finite Elements) in the presence of complicated domains (microstructures, many small holes in the domain etc.).

The mathematical analysis requires a special stability consideration, since a coarsening technique can easily become unstable if the coarsening steps are repeated several times (multi-grid situation). Furthermore, nonstandard continuation results from function analysis are needed.
ON DIFFERENTIABILITY OF SOLUTIONS WITH RESPECT TO PARAMETERS IN STATE-DEPENDENT DELAY EQUATIONS

Ferenc Hartung, Veszprém, Hungary

AMS Class.: 34K05

In this talk we study differentiability of solutions with respect to parameters in state-dependent delay equations. In particular, we give sufficient conditions for differentiability of solutions in the $W^{1,p}$ norm ($1 \leq p < \infty$). In establishing our main results we make use of a version of the Uniform Contraction Principle for quasi-Banach spaces.

SPECTRAL AND OSCILLATORY PROPERTIES OF SINGULAR DIFFERENTIAL OPERATORS

Roman Hilscher, Brno, Czech Republic

AMS Class.: 34C10

We study spectral properties of self-adjoint differential operators

$$\ell(y) \equiv \frac{1}{w(t)} \sum_{k=0}^{n} (-1)^k \left( p_k(t)y^{(k)} \right)^{(k)}$$

in the Hilbert space $L^2_w(I)$, where $I = [a, +\infty)$, $p_n^{-1}, p_{n-1}, \ldots, p_0, w \in L_{loc}(I)$, $p_n, w > 0$. We are interested in necessary and sufficient conditions (for the weight function $w(t)$) which guarantee that all self-adjoint extensions of the minimal differential operator generated by $\ell$ have spectrum discrete and bounded below $\equiv$ property BD.

The spectrum of $\ell$ is studied via oscillatory behavior of a certain associated $(2n - 2)$-order equation. We give a particular interest to fourth order operators since then this equation is of the second order. Special examples of new weight functions $w(t)$ are given as well.
PARTIAL DIFFERENTIAL EQUATIONS
WITH HYSTERESIS IN THERMOPLASTICITY

Pavel Krejčí, Praha, Czech Republic, and Berlin, Germany

AMS Class.: 35G25 (73B30, 73E60, 73B05)

As an application of our recent thermodynamically consistent hysteresis model for rate-independent thermoplasticity we consider an initial and boundary value problem for the system of equations for the displacement \( u \) and the absolute temperature \( \theta \)

\[
    u_{tt} = \left( \frac{\gamma(u_x)}{x} + P[u_x, \theta] + \mu u_{xt} - \beta \theta \right) + f(\theta, x, t), \\
    (C_V \theta + V[u_x, \theta])_{t} - \theta_{xx} = (P[u_x, \theta] + \mu u_{xt} - \beta \theta) u_{xt} + g(\theta, x, t)
\]

with hysteresis operators \( P \) and \( V \), which represents the balance of momentum and energy in the uniaxial case, where also the effects of kinematic hardening, viscosity and thermic dilation are taken into account. We prove that a unique strong solution exists globally and depends continuously on the data. (Joint work with J. Sprekels.)

ON ZEROS OF NONOSCILLATORY SOLUTIONS
OF SECOND ORDER
QUASILINEAR DIFFERENTIAL EQUATIONS

Kusano Takaši, Fukuoka, Japan

AMS Class.: 34C10

We consider half-linear differential equations of the form

\[
    \left( |x'|^{\alpha-1} x' \right)' + \lambda q(t)|x|^{\alpha-1} x = 0, \quad t \geq a,
\]

where \( \alpha > 0 \) is a constant, \( q : [a, \infty) \to (0, \infty) \) is a continuous function and \( \lambda \) is a positive parameter. A nonoscillatory solution \( x(t; \lambda) \) of (A) is called minimal [resp. maximal] if it satisfies \( \lim_{t \to \infty} x(t; \lambda) = 1 \) [resp. \( \lim_{t \to \infty} x'(t; \lambda) = 1 \)].

We are interested in the problem of finding those values of \( \lambda \) for which (A) has minimal [resp. maximal] solutions \( x(t; \lambda) \) satisfying the boundary condition \( x(a; \lambda) = 0 \). Under suitable conditions on \( q(t) \) we can show that this problem is solved for a sequence of positive parameters, \( \{\lambda_n\}_{n=0}^\infty \), such that

\[
0 < \lambda_1 < \lambda_2 < \cdots < \lambda_n < \cdots, \quad \lim_{n \to \infty} \lambda_n = \infty,
\]

and that the minimal [resp. maximal] solution \( x(t; \lambda_n) \) corresponding to \( \lambda = \lambda_n \) has exactly \( n \) zeros in \((a, \infty)\), \( n = 0, 1, 2, \cdots \).
SINGULAR BOUNDARY VALUE PROBLEMS

Donal O’Regan, National University of Ireland, Galway, Ireland

AMS Class.: 34B15

We present some recent results for the second order equation

\[ \frac{1}{p} (py')' + f(t,y,py') = 0, \quad 0 < t < 1 \]

with Dirichlet or mixed boundary data. The nonlinearity \( f \) is allowed to change sign. Singularities at \( y = 0, t = 0 \) and \( t = 1 \) are discussed.

GLOBAL BIFURCATION IN VARIATIONAL INEQUALITIES

Klaus Schmitt, Salt Lake City, USA

AMS Class.: 73CSO (73Kxx, 49Rxx, 73Hxx, 3Bxx)

During the lecture a brief survey on global bifurcation results for variational inequalities will be presented. By means of several concrete examples we shall illustrate how abstract bifurcation results presented in Le/Schmitt: Global Bifurcation in Variational Inequalities: Applications to Obstacle and Unilateral Problems, vol. 123, Applied Math. Sciences, Springer, 1997 may be used to deduce the existence of nontrivial solutions of these problems.
ON PERIODIC SOLUTIONS OF NONLINEAR FUNCTIONAL-DIFFERENTIAL EQUATIONS

Bedřich Půža, Brno, Czech Republic

AMS Class.: 34K15

For the system of functional-differential equations

$$\frac{dx(t)}{dt} = p(x)(t),$$

where $p : C_\omega(R^n) \to L_\omega(R^n)$ is a regular operator and $C_\omega(R^n)$ and $L_\omega(R^n)$ are spaces of $n$-dimensional $\omega$-periodic vector functions with continuous and integrable components on $[0, \omega]$ respectively, sufficient conditions for existence and uniqueness of $\omega$-periodic solution are established. The effective criteria are illustrated by differential equations with deviating arguments.

LONG-TIME BEHAVIOUR OF SOLUTIONS TO STOCHASTIC PDE’s

Bohdan Maslowski, Prague, Czech Republic

AMS Class.: 60H15 (60H10)

Recent results on the asymptotic behaviour of solutions to stochastic semilinear equations of the form

$$dX(t) = (AX(t) + f(X(t)))dt + B(X(t))dW(t)$$

with an initial condition $X(0) = x \in H$ are reviewed, where $H$ is a separable Hilbert space, $W$ is an $H$-valued Wiener process (Brownian motion), $A$ stands for an infinitesimal generator of a strongly continuous semigroup on $H$ and $f$ and $B$ are nonlinear terms defined on a suitable subspace of $H$, with values in $H$ and $L(H)$, respectively. The results are applied to a wide range of stochastic PDE’s including stochastic reaction-diffusion equations, stochastic Burgers and Navier-Stokes equations, stochastic Cahn-Hilliard equation and others.
PERIODIC PROBLEM FOR ORDINARY DIFFERENTIAL EQUATIONS VIA THE MULTIVALUED POINCARÉ TRANSLATION OPERATOR

Lech Górniewicz, Toruń, Poland

We shall consider periodic problems for ordinary differential equations of the form:

\[
\begin{align*}
\{ & x'(t) = f(t, x(t)) \\
& x(0) = x(a),
\end{align*}
\]

where \( f : [0, a] \times \mathbb{R}^n \to \mathbb{R}^n \) satisfies suitable assumptions.

To study of the above problem we shall follow an approach based on the topological degree theory. Roughly speaking, if on some ball of \( \mathbb{R}^n \), the topological degree of, associated to (I), multivalued Poincaré operator \( P \) turns out to be different from zero, then problem (I) has solutions.

Next by using the multivalued version of the classical Liapunov-Krasnosielski guiding potential method we calculate the topological degree of the Poincaré operator \( P \). To do it we associate with \( f \) a guiding potential \( V \) which we assume locally Lipschitzean (instead of \( C^1 \)) and hence, by using Clarke generalized gradient calculus we are able to prove existence results for (I), of the classical type, obtained earlier under the assumption that \( V \) is \( C^1 \).

Note that using of the same type technique (adopting to the random case) we are able to obtain all of the above mentioned results for the following random periodic problem:

\[
\begin{align*}
\{ & x'(\xi, t) = f(\xi, t, x(\xi, t)) \\
& x(\xi, 0) = x(\xi, a),
\end{align*}
\]

where \( f : \Omega \times [0, a] \times \mathbb{R}^n \to \mathbb{R}^n \) is a random operator satisfying suitable assumption.

COMPARISON THEOREMS FOR HALF-LINEAR EQUATIONS OF FOURTH ORDER

Jaroslav Jaroš, Bratislava, Slovakia

AMS Class.: 34C15

In the paper a Picone type identity for fourth order nonlinear ordinary differential operators of the form

\[
l_\alpha[x] \equiv (p\varphi(x''))'' + q\varphi(x)
\]

and

\[
L_\alpha[y] \equiv (P\varphi(y''))'' + Q\varphi(y)
\]

where \( \varphi(u) := |u|^\alpha - 1 u \), \( \alpha > 0 \) being a constant, and \( p, q, P \) and \( Q \) are continuous functions on a given interval \( I \) with \( p(t) > 0 \) and \( P(t) > 0 \) on \( I \) is derived and then Sturmian comparison theory for the corresponding fourth order equations \( l_\alpha[x] = 0 \) and \( L_\alpha[y] = 0 \) based on this identity is developed.
GLOBAL EXISTENCE AND ASYMPTOTIC BEHAVIOR IN TIME OF SMALL SOLUTIONS TO THE ELLIPTIC-HYPERBOLIC DAVEY-STEWARTSON SYSTEM

Nakao Hayashi         Hitoshi Hirata

We study the initial value problem for the elliptic-hyperbolic Davey-Stewartson system (c.f. [2],[3],[5])

\[
\begin{cases}
  i\partial_t u + \Delta u = c_1|u|^2 u + c_2 u \partial_x \varphi, & (t, x) \in \mathbb{R}^3, \\
  (\partial_{x_1}^2 - \partial_{x_2}^2) \varphi = \partial_{x_1}|u|^2, & u = u(t, x), \quad \varphi = \varphi(t, x), \\
  u(0, x) = \phi(x),
\end{cases}
\]

where \(\Delta = \partial_{x_1}^2 + \partial_{x_2}^2\), \(c_1, c_2 \in \mathbb{R}\), \(u\) is a complex valued function and \(\varphi\) is a real valued function. When \((c_1, c_2) = (-1, 2)\) the system (*) is called DSI equation in the inverse scattering literature (for example [4]). Our purpose in this paper is to prove global existence of small solutions to (*) in the usual weighted Sobolev space \(H^{3,0} \cap H^{0,3}\), where

\[
H^{m,l} = \{ f \in L^2; \|(1 - \partial_{x_1}^2 - \partial_{x_2}^2)^{m/2}(1 + x_1^2 + x_2^2)^{l/2} f\|_{L^2} < \infty\}.
\]

Furthermore we prove \(L^\infty\) time decay estimates of solutions to (*) such that

\[
\|u(t)\|_{L^\infty} \leq C(1 + |t|)^{-1}.
\]

The key points in the proofs are the a-priori estimates for the time local solution:

\[
\sup_{-T \leq t \leq T} \|u(t)\|_{X^{2,2}(t)}^2 \leq 4\delta_3^2,
\]

and

\[
\sup_{-T \leq t \leq T} (1 + |t|)^{-C\delta_3} \|u(t)\|_{X^{2,3}(t)}^2 \leq 4\delta_3^2,
\]

where \(\| \cdot \|_{X^{m,l}(t)} = \sum_{|\alpha| \leq m} \|\partial^\alpha \cdot\| + \sum_{|\alpha| \leq l} \|J^\alpha \cdot\|\), which is obtained by making use of the structure of main nonlinear term such that

\[
u \int_{x_2}^\infty \partial_x |u|^2 dx_2' = u \frac{1}{2it} \int_{x_2}^\infty \bar{J}_x u - u \bar{J}_x u dx_2'.
\]

Above time local solution is obtained in for example [6],[7],[9], by using vector field methods and local smoothing effect of linear Schrödinger operator.


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ON CONTROLLABILITY OF LINEAR WITH 
RESPECT TO STATE DYNAMICAL SYSTEMS

S. M. Khryashchev

AMS Class.: 34, 93

Consider linear dynamical control system (DCS)

\[ \dot{x} = A(u)x, \quad x \in \mathbb{R}^{n+1} \setminus \{0\}, \quad u \in U \subset \mathbb{R}^1 \]

where \( x \) is the state and \( u \) is the control. Equation (1) can be rewritten in the form

\[ \dot{\xi} = \Xi(\xi, u), \quad \xi \in \mathbb{R}P^n, \]

\[ \dot{r} = \langle q, A^s(u)q \rangle r, \quad r \in \mathbb{R}_+^1 \]

where \( \mathbb{R}P^n \) is projective space, \( p : \mathbb{R}^{n+1} \setminus \{0\} \to \mathbb{R}P^n, \) \( \xi = p(x), \) \( r = |x|, \) \( q = x/|x|, \)
\( A_s = \frac{1}{2}(A + A^*), \) \( \langle ., . \rangle \) denotes the scalar product. \( \xi \) are angle coordinates of state and \( r \) is radial coordinate. The explicit expression for \( \Xi \) and controllability conditions for DCS (2) are given in [1]. These conditions in terms of a spectral characteristic of DCS were given. In this paper, we shall obtain controllability conditions for the control system (2)–(3), i.e. for DCS (1) in \( \mathbb{R}^{n+1} \setminus \{0\}. \)

Let \( U = \{ \hat{u} | \hat{u} : T \to U \} \) be a set of admissible controls. We shall use the piecewise constant controls \( \hat{u} \in U \).

We assume that the following conditions are fulfilled:
1. DCS (1) is locally controlled along any trajectory.
2. The spectral characteristic of DCS has a linking property [1].
3. For all \( q \) there exist \( u_1, u_2 \) such that

\[ \langle q, A^s(u_1)q \rangle > 0, \quad \langle q, A^s(u_2)q \rangle < 0. \]

**Theorem.** Let the Assumptions 1,2,3 be fulfilled. Then DCS (1) is controlled in \( \mathbb{R}^{n+1} \setminus \{0\}. \)

**Proof.** Let \( x_0 = (r_0, \varphi_0) \) and \( x_*= (r_*, \varphi_*) \) are any points in \( \mathbb{R}^{n+1} \setminus \{0\}. \) We shall show that the initial point \( x_0 \) can be transferred to the final point \( x_* \), i.e. there exist a control \( \hat{u} \in U \) and moments \( t_0, t_1 \) such that \( x_* = x(x_0, t_0, t_1, \hat{u}) \) where \( x(x_0, t_0, t_1, \hat{u}) \) is a trajectory of DCS (1) from the point \( x_0 \) corresponding to the control \( \hat{u} \). We shall denote this fact by \( x_0 \overset{t_0, \hat{u}, t_1}{\to} x_* \). We shall sometimes omit \( t_0, \hat{u}, t_1 \).

Assumptions 1,2 imply that for DCS (2) the point \( \varphi_0 \) can be transferred to the point \( \varphi_* \), i.e. \( \varphi_0 \overset{\hat{u}}{\to} \varphi_* \). The control process was described in [1]. It was realized in \( n \) steps by some control sequence \( u_k, k = 1, \ldots, n. \) If \( \hat{u}(t) = u_k, \) \( t_{k-1} < t \leq t_k, \)
\( t_n = t_*, \) \( u_k \in U \) we denote \( \hat{u}^n = u_1 \cdots u_n. \) Let \( T = t_n - t_0 \) be a time of the control. There exists \( T_0 = T_0^0(A, U) \) such that \( T \leq T_0 \) for any \( \varphi_0, \varphi_* \).

Consider the control \( u = u_k \) at \( k \)-th step for DCS (2) and DCS (2)–(3). The trajectories of DCS (2) are situated on cells \( C^k, \) \( \dim C^k = k, \) which are some pieces.
of the integral surfaces in $R^n$. The trajectories of DCS (2)–(3) are situated on cells $D^k$, $\dim D^k = k$, which are some pieces of the integral surfaces in $R^{n+1} \setminus \{0\}$. Consider the last step ($k = n$). Let $Re\lambda_k(u_n) \leq Re\lambda_{k+1}(u_n)$ where $\lambda_k(u_n)$, $k = 1, \ldots, n$ are eigenvalues for the matrix $A(u_n)$. The trajectories of DCS (2) start from the unstable limit cycle $s(u_n)$ which corresponds to $Re\lambda_{2}(u_n)$. The trajectories of DCS (2)–(3) start from the plane $(R^2 \setminus \{0\})(u_n)$ which corresponds to the cycle $s(u_n)$. If $Re\lambda_{1,2}(u_n) < 0$ then the stable focus is situated in $(R^2 \setminus \{0\})(u_n)$ and if $Re\lambda_{1,2}(u_n) > 0$ then the unstable focus is situated in $(R^2 \setminus \{0\})(u_n)$.

The condition (4) (Assumption 3) ensures controllability on $R^n_1$ for DCS (3). Beforehand the initial point $x_0 = (\varphi_0, r_0)$ can be transferred to the point $x_0' = (\varphi_0', r_0')$ which lies in some neighborhood $V_{R_0}(0)$ of $x = 0$ or in some neighborhood $V_{R_0}(\infty)$ of $x = \infty$ (here we take into account the sign $Re\lambda(u_n)$). Thus, $x_0 \to x_0'$. The number $R_0$ will be defined by the number $R$ below (see (5)). Choosing a suitable integral surface the point $x_0'$ can be transferred to the final point $x_*$. Let for $n - 1$ of previous steps the arbitrary point $x_0'$ be transferred to the point $x_{n-1}'$, i.e. there exists the control $\hat{u}^{n-1} = u_1 \ast \ldots \ast u_{n-1}$ such that

$$
\begin{align*}
x_0' &\to x_{n-1}', \\
\varphi_0' &\to \varphi_{n-1}', \\
r_0' &\to r_{n-1}'
\end{align*}
$$

where $\varphi_{n-1}' \in s(u_n)$, $x_{n-1}' \in (R^2 \setminus \{0\})(u_n)$, $r_{n-1}' < R$ for all $x_0'$ such that $|x_0'| = r_0' < R_0$. Here we use that $T = t_{n-1}' - t_0' < T_0$ and $|A(u)|$ is bounded on $U$. $T$ is the time of control (5). The number $R$ will be defined by the state $x_*$ below.

Let the final point $x_*$ lie on the integral surface $D^{n-1}(u_n)$. First, let $x_* \notin K(u_n)$. $K(u_n)$ is some invariant space for the matrix $A(u_n)$, $\dim K(u_n) < n$. Hence, there exists $x_{n-1} \in D^{n}(u_n)$ such that $x_{n-1} \to x_*$. Let us select $x_{n-1}, x_{n-1}$ such that $x_{n-1}' \to x_{n-1}'$ where $\Delta u_{n-1}'$ is the local control. Assumption 1 implies that if $|x_{n-1} - x_{n-1}'| < \rho$ then there exists such control. Here $\rho$ is radius of local controllability in the point $x_{n-1}'$ for DCS (1) and $x_{n-1}' \in (R^2 \setminus \{0\})(u_n)$. Let $x_{n-1}'$ belong to the accessibility set from $(R^2 \setminus \{0\})(u_n)$. Consider the case $Re\lambda_{1,2}(u_n) > 0$. Then $|x_0'| < |x_{n-1}'|/\cos \psi = R$, $\rho = |x_{n-1}'|/\tan \psi$ where $\psi$ is radius of local controllability for DCS (2).

Finally, if $x_* \in K(u_n)$ then there exists the local control $\Delta u_n''$ such that $x_n \to x_*$. Analogously, the case $Re\lambda_{1,2}(u_n) < 0$ is considered. The Theorem is proved.


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ALMOST PERIODICALLY FORCED LOCALY CONVEX LAGRANGIAN SYSTEM.

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AMS Class.: 34C27 (49J40)

Consider equation:

$$\frac{d^2x}{dt^2} + \nabla v(x) = e(t) \tag{1}$$

where $x(t) = (x_1, ..., x_n)^T, \ e = (e_1, ..., e_n)^T, e \in CAP(\mathbb{R}, \mathbb{R}^n)$, i.e., each of $e_i, i = \overline{1, n}$, belongs to the space $CAP(\mathbb{R})$ of Bohr-almost periodic functions; $v : \mathbb{R}^n \to \mathbb{R}, \ v \in C^2(\mathbb{R}^n, \mathbb{R})$. Our aim is to study the almost periodic (a. p.) solutions of equation (1).

Let $f : \mathbb{R} \to \mathbb{R}$ be locally integrable. Its mean value, when it exists (resp. upper mean value), is denoted by $M\{f(t)\}$ (resp. $\overline{M}\{f(t)\}$); it is the limit (resp. upper limit) of $\frac{1}{T} \int_{-T}^{T} f(t) dt$, when $T \to \infty$. The Hilbert space of Besicovitch-a.p. functions $B_2(\mathbb{R})$ is the closure of $CAP(\mathbb{R})$ for the semi-norm $f \to (\overline{M}\{|f(t)|^2\})^{1/2}$. Denote the $s$-order Sobolev derivative of $f \in B_2(\mathbb{R})$ by $f^{(s)}(t)$.

$$P(\mathbb{R}; \mathbb{R}^n) := \{p(t) = (p_1, ..., p_n)^T, p_j = \sum_{k=1}^{l} p_{jk}e^{i\lambda_{jk}}, f_{jk} = f(\lambda_{jk}) \in C; Im(f(\lambda)) = -Im(f(-\lambda)), \lambda_{jk} \in \mathbb{R}, l \in \mathbb{N}, j = \overline{1, n}\}.$$

The Hilbert space $B_2^m(\mathbb{R}; \mathbb{R}^n)$, $m \in \mathbb{N}$, is the closure of $P(\mathbb{R}; \mathbb{R}^n)$ for the norm $\|p\|^2_m = \sum_{k=1}^{l} \sum_{j=1}^{n} (1 + |\lambda_{jk}|^2)^m |p_{jk}|^2$. Scalar product in such a space has a form: $(f, g)_m = \left(\sum_{s=1}^{m} \sum_{j=1}^{n} M\{|f^{(s)}_j(t)|^2\} \right)^{1/2}, f = (f_1, ..., f_n)^T, g = (g_1, ..., g_n)^T$. The associated euclidean norm is denoted by $\|\|_m$.

For $f \in B_2^m(\mathbb{R}; \mathbb{R}^n)$ the $s$-order Sobolev generalized derivative is $f^{(s)} = (f_1^{(s)}, ..., f_n^{(s)})$. Note that $f = (f_1, ..., f_n)$ belongs to $B_2^m(\mathbb{R}; \mathbb{R}^n)$ if and only if $f_j \in B_2^m(\mathbb{R}; \mathbb{R}) \equiv B_2^m(\mathbb{R})$ for $j = \overline{1, n}$, and $\|f\|^2_m = \sum_{j=1}^{n} \|f_j\|^2_m$.

Denote by $LAP(\mathbb{R}; \mathbb{R}^n)$ the space of those functions $f = (f_1, ..., f_n)$ from $CAP(\mathbb{R}; \mathbb{R}^n)$, where $f_j, j = \overline{1, n}$ are 1-lipschitzian on $\mathbb{R}$.

For the finding of a. p. solutions of (1) a variational method is used. If there exists $E \in CAP(\mathbb{R}, \mathbb{R}^n)$, such that $\frac{d^2E}{dt^2} = e(t)$, then in setting $x(t) = u(t) + E(t)$ equation (1) is equivalent to: $\frac{d^2u}{dt^2} + \nabla v(u + E) = 0$.

A. p. solutions of this equation are exactly the critical points of the functional $J_0(u) = \lim_{T \to -\infty} \frac{1}{2T} \int_{-T}^{T} \frac{1}{2} (\frac{du}{dt})^2 - v(u + E) |dt|$ on $CAP^1(\mathbb{R}; \mathbb{R}^n)$. Compactness is very difficult to exhibit in such a space. For this reason we shall work on a Hilbert space $B_2^1(\mathbb{R}; \mathbb{R})$. Consider a functional on $B_2^1(\mathbb{R}; \mathbb{R})$: $J(u) = lim_{T \to -\infty} \frac{1}{2T} \int_{-T}^{T} [\frac{1}{2} (u(1)^2) - v(u + E)] |dt$.

Note that $J$ is defined correctly on $LAP(\mathbb{R}; \mathbb{R}^n)$, because $LAP(\mathbb{R}; \mathbb{R}^n) \subset B_2^1(\mathbb{R}; \mathbb{R}^n)$.

Theorem. Let the following conditions hold:
1) there exists $E \in CAP(\mathbb{R}, \mathbb{R}^n)$, such that $\frac{d^2E}{dt^2} = e(t)$;
2) for each $j = \overline{1, n}$ there exists $K_j \in \mathbb{R}: \ \bar{O}seE_j < K_j$;
3) $\frac{dv(a)}{dt} = 0$, \( a = (a_1, ..., a_n)^T \), \( l_j + K_j \leq a_j \leq m_j - K_j \) \( j = \overline{1,n} \);

4) $\frac{d^2v(x)}{dt^2} < 0$, \( x = (x_1, ..., x_n)^T \), \( x_j \in [l_j, m_j] \) \( j = \overline{1,n} \);

5) there exists $M \in \mathbb{R}$ : $\sum_{i=1}^{n} \left( \frac{\partial v(x)}{\partial x_j} \right)^2 + \sum_{i,j=1}^{n} \left( \frac{\partial^2 v(x)}{\partial x_i \partial x_j} \right)^2 < M$, \( x \in \mathbb{R} \).

Let $V = \{ u \in \text{LAP}(\mathbb{R}; \mathbb{R}^n), l_j \leq u_j \leq m_j, j = \overline{1,n} \}$; $G$ is the closure of $V$ into $B_{\infty}(\mathbb{R}; \mathbb{R}^n)$.

Then there exists a unique up to equivalence in $B_{\infty}(\mathbb{R}; \mathbb{R}^n)$ function $\hat{u} \in B_{\frac{3}{2}}(\mathbb{R}; \mathbb{R}^n) \cap G$, such that $\| \hat{u}^2 + \nabla v(\hat{u} + E) \|_0 = 0$. The function $\hat{u}$ minimizes $J$ on $G$.


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ON DIFFERENTIAL EQUATIONS ARISING IN ITERATION THEORY

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Let \((F_t)_{t \in \mathbb{C}}\) be analytic family of locally analytic functions \(F_t(x) = F(t, x)\), for \(|x| < R, t \in \mathbb{C}\), such that the translation equation

\[
(T) \quad F(t, F(s, x)) = F(t + s, x), \quad F(0, x) = x, \quad |x| < R, \quad t \in \mathbb{C}
\]

is fulfilled. Then as an easy consequence, we deduce the system of differential equations

\[
(1) \quad \frac{\partial F(x, t)}{\partial t} = G(x) \cdot \frac{\partial F(x, t)}{\partial x}
\]

\[
(2) \quad \frac{\partial F(x, t)}{\partial t} = G(F(x, t))
\]

\[
(3) \quad G(x) \cdot \frac{\partial F(x, t)}{\partial x} = G(F(x, t))
\]

where

\[
(4) \quad G(x) = \left. \frac{\partial F(x, t)}{\partial t} \right|_{t=0}.
\]

This system (1)–(4) is often called the system of Aczél-Jabotinsky equations. G. Targonski raised the question whether, conversely, this system, or part of it, implies the translation equation. We will treat this problem for the most interesting case of equation (3), now viewed as an ordinary differential equation

\[
G(\phi(x)) = G(x) \cdot \frac{d\phi}{dx}
\]

with respect to \(x\) in the analytic case, where \(G(x) = d_1 x + d_2 x^2 + \ldots, G \neq 0\), is given. We will relate this differential equation, in the neighbourhood of the singularity \(x = 0\), to a number of problems, such as:

1) Construction of analytic families of commuting power series \(F_t\), i.e. with \(F_t \circ F_s = F_s \circ F_t, t, s \in \mathbb{C}\),

2) Characterizing the solutions of (T) among these families,

3) analytic dependence of the solutions of (3) on internal parameters and analytic continuation. Furthermore, we will sketch the role of similar differential equations in the theory of linear functional equations and their solution by Schröder type expressions and also results of Aczél and Gronau about the system (1)–(4) in the nonanalytic case.
ON BOUNDARY VALUE PROBLEMS FOR FUNCTIONAL-DIFFERENTIAL EQUATIONS

Alexander Domoshnitsky

AMS Class.: 35K15

Consider the following equation

\[ x''(t) + (Tx)(t) = f(t), \quad t \in [0, \omega], \tag{1} \]

where \( T: C_{[0, \omega]} \to L_{[0, \omega]} \) is a linear bounded Volterra operator, \( C_{[0, \omega]} \) is the space of continuous functions \( x: [0, \omega] \to \mathbb{R}^1 \), \( L_{[0, \omega]} \) is the space of summable functions \( y: [0, \omega] \to \mathbb{R}^1 \). The following operators are the particular cases of this operator \( T \):

\[ (Tx)(t) = \int_0^t K(s, t)x(s) \, ds \]

\[ (Tx)(t) = \begin{cases} p(t)x(t - \tau(t)) & \text{if } t - \tau(t) \geq 0, \\ 0 & \text{if } t - \tau(t) < 0. \end{cases} \]

Introduce the linear bounded functionals \( \nu: C_{[\omega_\nu, \omega]} \to \mathbb{R}^1 \), \( \mu: C_{[\omega_\mu, \omega]} \to \mathbb{R}^1 \), where \( \omega_\nu < \omega \), \( \omega_\mu < \omega \). Consider the following boundary value problem

\[
\begin{aligned}
x''(t) + (Tx)(t) &= f(t), \quad t \in [0, \omega], \\
\nu x &= \alpha, \quad \mu x = \beta,
\end{aligned}
\tag{2}
\]

where \( \alpha, \beta \in \mathbb{R}^1 \).

Let \( T = T^+ - T^- \), where \( T^+ \) and \( T^- \) are positive operators acting from \( C_{[0, \omega]} \) to \( L_{[0, \omega]} \).

Let the functional \( \mu: C_{[\omega_\mu, \omega]} \to \mathbb{R}^1 \) have the following representation

\[
\mu(x) = \int_{\omega_\mu}^{\omega_k} x(s) \, dR_k(s) + \int_{\omega_k}^{\omega_{k-1}} x(s) \, dR_{k-1}(s) + \cdots + \int_{\omega_2}^{\omega_1} x(s) \, dR_1(s),
\]

where \( \omega_\mu = \omega_{k+1} < \omega_k < \ldots < \omega_2 < \omega_1 = \omega \) and the functions \((-1)^{j+1}R_j\) are nondecreasing for \( j = 1, \ldots, k \).

**Theorem:** Let the following conditions hold:

1) \( \int_0^{\omega} (T^- 1)(t) \, dt < \frac{1}{\omega} \),

2) \( \int_0^{\omega} (T^+ 1)(t) \, dt \leq \frac{1}{\omega} \),

3) \( \nu \) is a positive functional,

4) \( \omega_\nu < \omega_\mu \).
5) $\forall_{s=\omega_{j+2}}^{\omega_{j+1}} R_{j+1}(s) \leq \forall_{s=\omega_{j+1}}^{\omega_{j}} R_{j}(s)$ for all odd $j$.

Then there exists unique solution $x$ of problem (2) for each $\alpha \in \mathbb{R}^1$, $\beta \in \mathbb{R}^1$, $f \in L$.

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