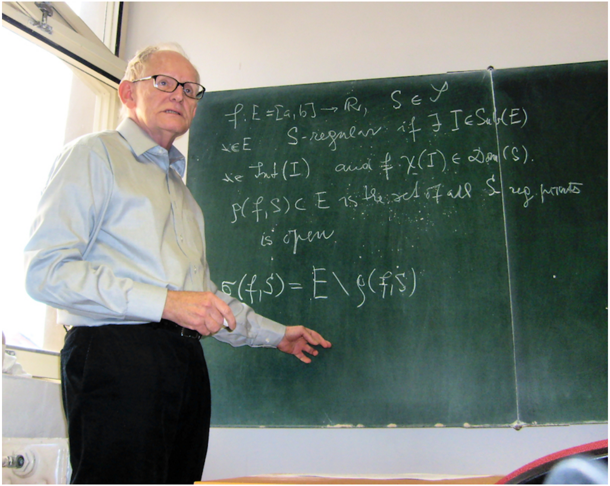
Štefan Schwabik

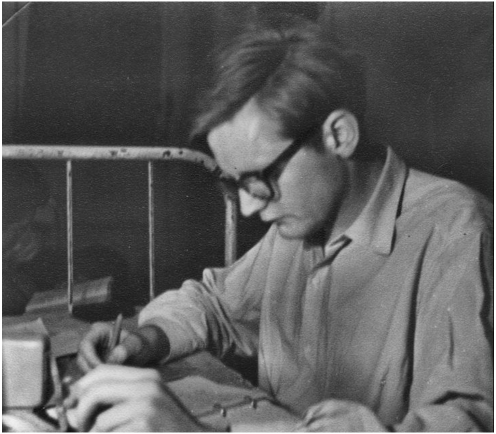


- <u>BORN</u>: March 15, 1941 (Gelnica, Slovakia)

- Born: March 15, 1941 (Gelnica, Slovakia)
- Education:
  - 1956-59 High School, Košice (Slovakia)
  - 1959-64 Charles University, Praha (Mathematical Analysis)

    (supervisor of the degree work: Jaroslav Kurzweil)

- Born: March 15, 1941 (Gelnica, Slovakia)
- Education:
  - 1956-59 High School, Košice (Slovakia),
  - 1959-64 Charles University, Praha (Mathematical Analysis)
    (supervisor of the degree work: Jaroslav Kurzweil),
  - 1964-69 Mathematical Institute of the Czechoslovak Academy of Sciences
    (postgraduate study) thesis defended: 1969, title acknowledged 1977
    (supervisor of the thesis: Jaroslav Kurzweil).



- Academic titles:
  - 1991 Doctor of Sciences, Czechoslovak Academy of Sciences,
  - 1993 Associate Professor (Habilitation Charles University),
  - 2000 Full Professor (Silesian University, Opava).

- Professional career:
  - since 1964: Mathematical Inst. of the Czechoslovak Acad.Sci. (now Institute of Mathematics, Acad.Sci. of the Czech Republic),
  - since 1991: Senior Research Worker,
  - since 2008: Emeritus Senior Research Worker,
  - 1996-2001: Head of the Dept.of Real and Probabilistic Analysis and of the Seminar on Differential Equations and Integration Theory.

- Teaching
  - 1963-1977 Czech Technical University (Electrotechnical faculty),
  - 1967 Šafárik University Košice (Slovakia) - control theory,
  - 1967-1993 Charles University (Faculty of mathematics and physics), integral transforms, ODE’s, control theory, special courses,
  - 1988 Universidade de Sao Paulo (IME) (Brazil), PhD course on generalized ODE’s.

- STUDENTS
  - Jiří Hnilica, Fac.of Electrical Eng., Czech Technical University Prague, now outside math,
  - Dana Fraňková, essentially contributed to the analysis of regulated functions, now succesful "businesslady", having math as a hobby,
  - Pavel Kindlmann, now professor of the South Bohemian University, mathematical models in ecology and biology,
  - Lenka Kozáková (Čelechovská), now Institute of Mathematics, Tomáš Baťa University, Zlín,
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  - Antonín Slavík, now member of Department of Mathematics Education, Faculty of Mathematics and Physics, Charles University, author of a beautiful booklet of product integration.
  - Petra Šarmanová, coauthor of the beautiful booklet on the history of integration, coauthor of the publication devoted to Otakar Borůvka, now Technical University of Ostrava, Faculty of Electrical Eng. and Computer Science, Dept.of Appl.Math.,
  - Ye GuoJu, coauthor of the monograph on integration in Banach spaces, now Professor of the Hohai university in Nanjing, China,
  - Marcia Federson, coauthor of several papers on functional differential equations, now Professor of the São Paulo University, Brazil.

- Other activities
  - Editorial board of Mathematica Bohemica (Editor in Chief 1989–2007),
  - Editorial board of Archivum Mathematicum,
  - Advisory board of Mathematica Slovaca,
  - Contributing editor of Real Analysis Exchange,
  - Scientific Boards of the Mathematical Institute of the Silesian University and of the Silesian University, Opava,
  - Scientific Board of the Faculty of Science, Masaryk University, Brno,
  - Grant Agency of the Acad.Sci. of the Czech Republic (vicepresident),
  - Council of Foreign Relations Academy of Sci. of the Czech Republic,

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  - Grant Agency of the Acad.Sci. of the Czech Republic (vicepresident),
  - Council of Foreign Relations Academy of Sci. of the Czech Republic,
  - Chairman of the Mathematical Society of the Czechoslovak Union of Mathematicians and Physicists (1974-??),
  - Vice Chairman of the Czech Union of Mathematicians and Physicists.

- **Control theory and Integral transforms** (1964-68)

Degree work and first 2 papers dealt with linear control problem of the form:

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with constant matrices A and B and where controls u belong to some convex set U whose interior is nonempty.

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• Extremale Regelungen für die lineare Zeitoptimale Regelungsaufgabe mit einem konvexen Regelungsbereich. *Časopis pro pěstování matematiky*, 91, 80–88 (1966).
• On the Linear Control Problem x'= Ax+Bu. *Časopis pro pěstování matematiky*, 93, 141–144 (1968).

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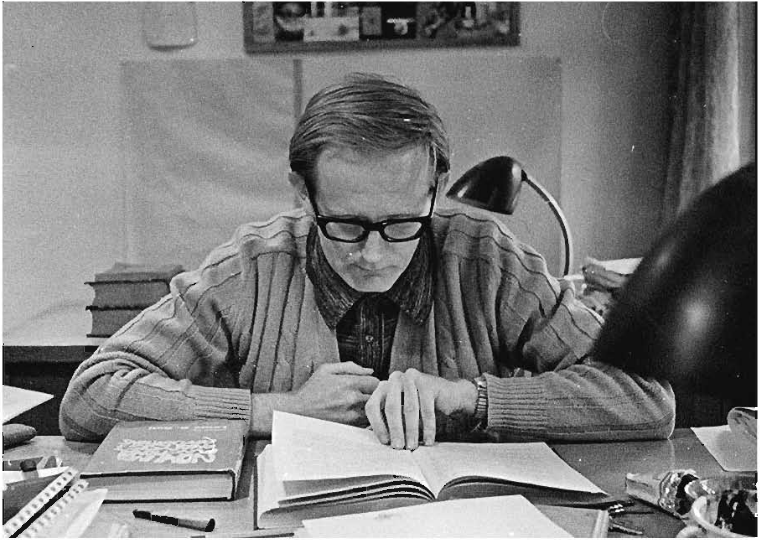
- On the Linear Control Problem x'= Ax+Bu. *Časopis pro pěstování matematiky*, 93, 141–144 (1968).

- *I*ntegral transforms (Lecture Notes) (in Czech) (with Jan Kučera), SPN 1969. 120 pp.



Generalized Differential Equations

- Über ein Differentialgleichungssystem mit unstetigen Lösungen endlicher Variation. *Zeitschrift für Angewandte Mathematik und Mechanik*, 48, T31–T32 (1968).
- Stetige Abhängigkeit von einem Parameter und invariante Mannifaltigkeiten für verallgemeinerte Differentialgleichungen. *Czechoslovak Mathematical Journal*, 19, 398–427 (1969).
- Veralgemeinerte gewöhnliche Differentialgleichungen; Systeme mit Impulsen auf Flächen I. *Czechoslovak Mathematical Journal*, 20, 468–490 (1970).
- Veralgemeinerte gewöhnliche Differentialgleichungen; Systeme mit Impulsen auf Flächen II. *Czechoslovak Mathematical Journal*, 21, 172–197 (1971).
- Bemerkungen zu Stabilitätsfragen für verallgemeinerte Differentialgleichungen. *Časopis pro pěstování matematiky*, 96, 57–66 (1971).
- Verallgemeinerte lineare Differentialgleichungssysteme. *Časopis pro pěstování matematiky*, 96, 183–211 (1971).
- Stetige Abhängigkeit von einem Parameter für ein Differentialgleichungssystem mit Impulsen. *Czechoslovak Mathematical Journal*, 21, 198–212 (1971).

Generalized Linear Differential Equations

$$x(t) = \widetilde{x} + \int_a^t d[A(s)]\, x(s) + f(t) - f(a) \qquad \left(\frac{dx}{d\tau} = \mathrm{D}[A(t)\, x + f(t)] \right)$$

- existence and uniqueness, variation-of-constants formula,
- boundary value problems and duality theory,
- Floquet theory,
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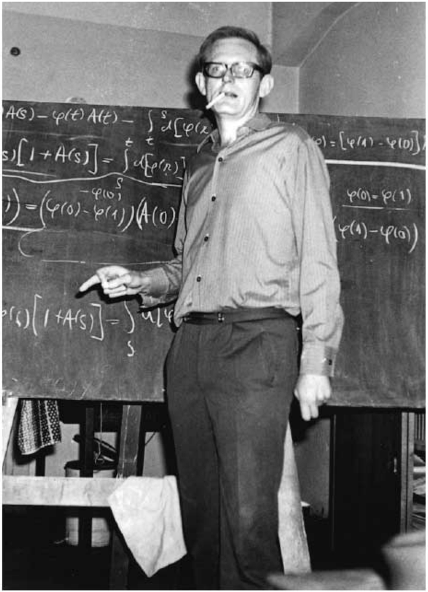
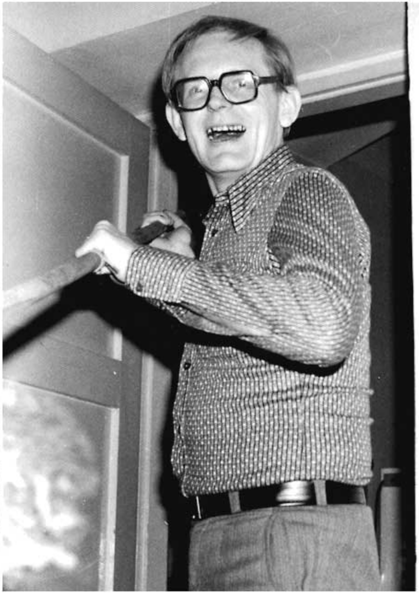
- boundary value problems and duality theory,
- Floquet theory,
- continuous dependence of solutions on a parameter,
- Perron-Stieltjes (=Kurzweil-Stieltjes) integration of BV functions with respect to BV functions,
- duality theory in the BV space,
- extensions to Volterra-Stieltjes or Fredholm-Stieltjes integral equations.





Differential and Integral Equations

Boundary Value Problems and Adjoints

ŠTEFAN SCHWABIK, MILAN TVRDÝ,
OTTO VEJVODA

ACADEMIA
PRAHA 1979

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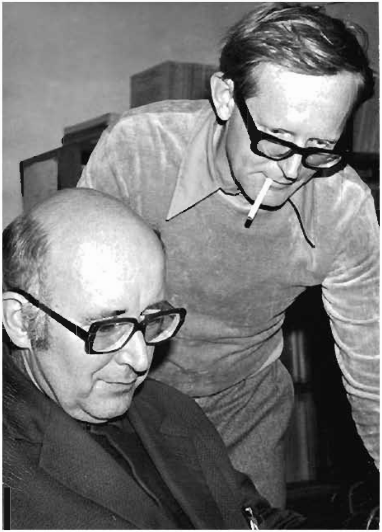


- Differential equations with interface conditions. *Časopis pro pěstování matematiky*, 105, 391–408 (1980).
- Generalized Volterra integral equations. *Czechoslovak Mathematical Journal*, 32, 245–270 (1982).

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- Generalized Volterra integral equations. *Czechoslovak Mathematical Journal*, 32, 245–270 (1982).
- Variational stability for generalized ordinary differential equations. *Časopis pro pěstování matematiky*, 109, 389–420 (1984).

- Differential equations with interface conditions. *Časopis pro pěstování matematiky*, 105, 391–408 (1980).
- Generalized Volterra integral equations. *Czechoslovak Mathematical Journal*, 32, 245–270 (1982).
- Variational stability for generalized ordinary differential equations. *Časopis pro pěstování matematiky*, 109, 389–420 (1984).

Inspired by *integral stability* introduced by I. Vrkoč for ODE's, Štefan for GDE's

$$(1) \qquad x(t) = \widetilde{x} + \int_a^t \mathrm{D}F(x(\tau), t)$$

defined:

The zero solution of (1) is variationally stable if: for every $\varepsilon > 0$ there is $\delta > 0$ such that

$$\left(\|\widetilde{x}\| < \delta \ \text{ and } \ \operatorname{var}_{t_0}^T \Big(x(s) - \int_{t_0}^s \mathrm{D}F(x(\tau), t)\Big) < \delta \right) \implies \|x(t)\| < \varepsilon \quad \text{for } t \in [t_0, T]$$

holds for each solution x of (1) on $[t_0, T]$ which has a bounded variation on $[t_0, T]$.

For this type of stability he proved some Lyapunov type results including converse Lyapunov theorems.

- Generalized Sturm-Liouvile equations. *Archivum mathematicum*, 23, 95–107 (1987).
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- A survey of some new results for regulated functions. In: *28. seminário brasileiro de análise* (Trabajos apresentados). - Sao Paolo, 201–209 (1988).
- Linear Operators in the Space of Regulated Functions. *Mathematica Bohemica*, 117, 79-92 (1992).
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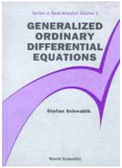
- *Generalized differential equations. Special results.* Rozpravy Československé akademie věd. Matematické a přírodní vědy, 99 [3] 3–79 (1989).
- ***Generalized Ordinary Differential Equations***. Singapore, World Scientific (Real Analysis Series 5), 382pp (1992).

Series in Real Analysis - Vol. 5

GENERALIZED ORDINARY DIFFERENTIAL EQUATIONS



by **Š Schwabik** *(Czechoslovak Acad. Sci.)*

The contemporary approach of J Kurzweil and R Henstock to the Perron integral is applied to the theory of ordinary differential equations in this book. It focuses mainly on the problems of continuous dependence on parameters for ordinary differential equations. For this purpose, a generalized form of the integral based on integral sums is defined. The theory of generalized differential equations based on this integral is then used, for example, to cover differential equations with impulses or measure differential equations. Solutions of generalized differential equations are found to be functions of bounded variations.

The book may be used for a special undergraduate course in mathematics or as a postgraduate text. As there are currently no other special research monographs or textbooks on this topic in English, this book is an invaluable reference text for those interested in this field.

Contents:

- The Generalized Perron Integral
- Ordinary Differential Equations and the Perron Integral
- Generalized Ordinary Differential Equations
- Existence and Uniqueness of Solutions of Generalized Differential Equations
- Generalized Differential Equations and Other Concepts of Differential Systems
- Generalized Linear Differential Equations
- Product Integration and Generalized Linear Differential Equations
- Continuous Dependence on Parameters for Generalized Ordinary Differential Equations
- Emphatic Convergence for Generalized Ordinary Differential Equations
- Variational Stability for Generalized Ordinary Differential Equations

Readership: Mathematicians, undergraduate and postgraduate students.

"... the book is highly recommended to researchers ... and to readers who wish to learn an important chapter in the philosophy and thinking of differential equations."

Zvi Artstein
Mathematical Reviews

Integration theory in $\mathbb{R}^n$

- On Mawhin's Approach to Multiple Nonabsolutely Convergent Integral (with Jarník, J. and Kurzweil, J.). *Časopis pro pěstování matematiky*, 108, 356–380 (1983).
- Ordinary differential equations the solutions of which are ACG_*-functions (with Kurzweil, J.). *Archivum Mathematicum*, 26, 129–136 (1990).
- Convergence Theorems for the Perron Integral and Sklyarenko's Condition. *Commentationes Mathematicae Universitatis Carolinae*, 33 (2) 237–244 (1992).
- Henstock's condition for convergence theorems and equiintegrability. *Real Analysis Exchange*, 18 (1) 190–205 (1992).
- The Perron Integral in ordinary differential equations. *Differential and Integral Equations*, 6 (4) 836–882 (1993).
- On Kurzweil-Henstock equiintegrable Sequences (with Vrkoč, I.). *Mathematica Bohemica*, 121 (2) 189–207 (1996).
- On non-absolutely convergent integrals. *Mathematica Bohemica*, 121 (4) 369–383 (1996).
- McShane equiintegrability and Vitali's convergence theorem (with Kurzweil J.). *Mathematica Bohemica*, 129 (2), 141–157 (2004).

Equiintegrability

- Set $D=\{\alpha_0, \alpha_1, \ldots, \alpha_m\} \subset [a, b]$ is a *division* of $[a, b]$ if $a=\alpha_0<\alpha_1<\ldots<\alpha_m=b$.
- Couple (D, ξ) where $D = \{\alpha_0, \alpha_1, \ldots, \alpha_m\}$ is a division of $[a, b]$ $\xi = (\xi_1, \ldots, \xi_m) \in [a, b]^m$ and $\alpha_{j-1} \leq \xi_j \leq \alpha_j$, is a *partition* of $[a, b]$.
- Functions $\delta : [a, b] \to (0, 1)$ are called *gauges*.

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System $\mathcal{F}$ of functions $f\colon[a,b]\to\mathbb{R}$ is *equiintegrable* if

- each function $f\in\mathcal{F}$ is integrable,
- for each $\varepsilon>0$ there is a gauge δ such that

$$\left|\sum_j f(\xi_j)\,(\alpha_j-\alpha_{j-1})-\int_a^b f\right|<\varepsilon$$

hold for each δ-fine partition (D,ξ) and each $f\in\mathcal{F}$.

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hold for each δ-fine partition (D,ξ) and each $f \in \mathcal{F}$.

- If $\{f_n\}$ is equiintegrable and $f_n \to f$ pointwise, then f is integrable and $\int_a^b f_n \to \int_a^b f$.
- For McShane integral the equiintegrability convergence result is equivalent to the Vitali's convergence theorem.

- *Integration in $\mathbb{R}$ (Kurzweil's theory)* (in Czech). Praha, Karolinum. 326 pp (1999).

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- *A short guide to the history of integral* (in Czech) (with Šarmanová, P.). Praha, Prometheus, 95 pp (1996).

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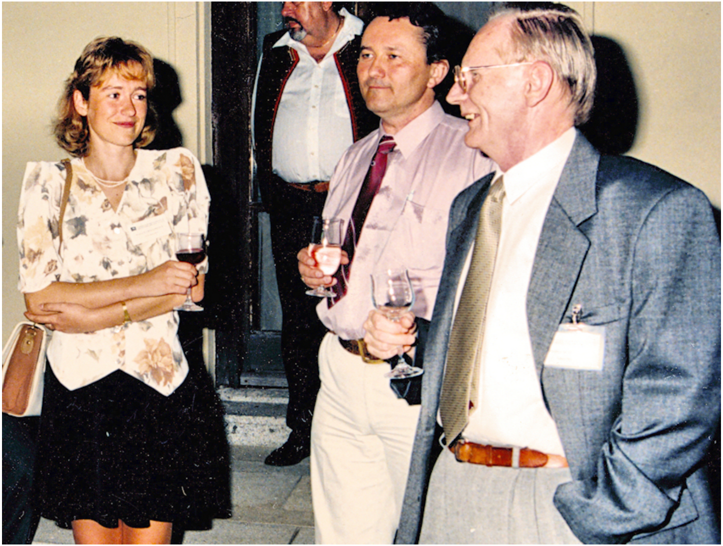

Linear Stieltjes integral equations in Banach spaces

- Abstract Perron-Stieltjes Integral. *Mathematica Bohemica*, 121 (4) 425–447 (1996).
- Linear Stieltjes integral equations in Banach spaces. *Mathematica Bohemica*, 124 (4) 433-457 (1999).
- Linear Stieltjes integral equations in Banach spaces II: Operator valued solutions. *Mathematica Bohemica*, 125 (4), 431-454 (2000).
- A note on integration by parts for abstract Perron-Stieltjes integrals. *Mathematica Bohemica*, 126 (3), 613–629 (2001).
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Integration in Banach space

- On the strong McShane integral of functions with values in a Banach space (with Ye Guoju). Czechoslovak Math.Journal 51 (126) (4), 819–828 (2001).
- The McShane and the Pettis integral of Banach space-valued functions defined on $\mathbb{R}^m$ (with Ye Guoju). *Illinois Journal of Math.* 46, 1125–1144 (2002).
- The McShane and the weak McShane integral of Banach space-valued functions defined on R^m (with Ye Guoju). *Mathematical Notes, Miskolc*, 2 (2), 127–136 (2001).
- On McShane integrability of Banach space-valued functions (with Jaroslav Kurzweil). *Real Analysis Exchange* 29 (2), 763–780 (2003/2004).
- A negative answer to a problem of Fremlin and Mendoza (with Ye Guoju). Acta Mathematica Scientia Series B, 27B (4), 813–820, (2007).

Linear Stieltjes integral equations in Banach spaces

- Abstract Perron-Stieltjes Integral. *Mathematica Bohemica*, 121 (4) 425–447 (1996).
- Linear Stieltjes integral equations in Banach spaces. *Mathematica Bohemica*, 124 (4) 433-457 (1999).
- Linear Stieltjes integral equations in Banach spaces II: Operator valued solutions. *Mathematica Bohemica*, 125 (4), 431-454 (2000).
- A note on integration by parts for abstract Perron-Stieltjes integrals. *Mathematica Bohemica*, 126 (3), 613–629 (2001).
- Operator - valued functions of bounded semivariation and convolutions. *Mathematica Bohemica*, 126 (4), 745–777 (2001).

Integration in Banach space

- On the strong McShane integral of functions with values in a Banach space (with Ye Guoju). Czechoslovak Math.Journal 51 (126) (4), 819–828 (2001).
- The McShane and the Pettis integral of Banach space-valued functions defined on $\mathbb{R}^m$ (with Ye Guoju). *Illinois Journal of Math.* 46, 1125–1144 (2002).
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- A negative answer to a problem of Fremlin and Mendoza (with Ye Guoju). Acta Mathematica Scientia Series B, 27B (4), 813–820, (2007).
- Bochner product integration. *Mathematica Bohemica*, 119 (3) 305–335 (1994).
- Henstock-Kurzweil and McShane product integration; Descriptive definitions (with Antonín Slavík). *Czechoslovak Mathematical Journal* 58 (133), 241-–269 (2008).

Series in Real Analysis - Vol. 10

TOPICS IN BANACH SPACE INTEGRATION

by **Štefan Schwabik** *(Czech Academy of Sciences, Czech Republic)* & **Ye Guoju** *(Hohai University, China)*

The relatively new concepts of the Henstock–Kurzweil and McShane integrals based on Riemann type sums are an interesting challenge in the study of integration of Banach space-valued functions. This timely book presents an overview of the concepts developed and results achieved during the past 15 years. The Henstock–Kurzweil and McShane integrals play the central role in the book. Various forms of the integration are introduced and compared from the viewpoint of their generality. Functional analysis is the main tool for presenting the theory of summation gauge integrals.

Contents:

- Bochner Integral
- Dunford and Pettis Integrals
- McShane and Henstock–Kurzweil Integrals
- More on the McShane Integral
- Comparison of the Bochner and McShane Integrals
- Comparison of the Pettis and McShane Integrals
- Primitive of the McShane and Henstock–Kurzweil Integrals
- Generalizations of Some Integrals

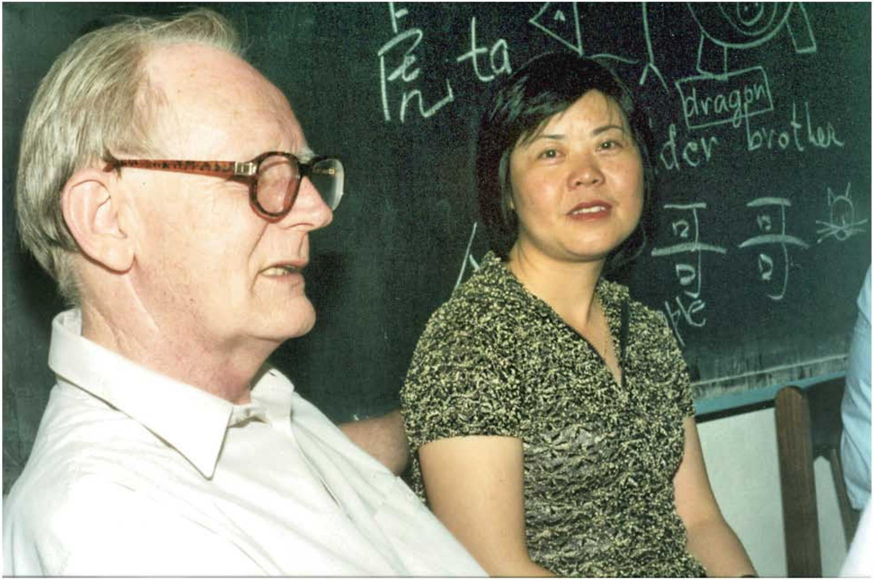
Readership: Graduate students and lecturers in mathematics.

“I can recommend this book for those seeking an overview of the concepts and results achieved during the past 15 years.”

Mathematical Reviews

“This book is carefully written and should be accessible to anyone with a basic knowledge of classical integration theory and elementary functional analysis. The book contains an extensive bibliography and should be useful to those with interests in Banach space integration.”

Zentralblatt MATH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Generalized ODE approach to functional differential equations

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- Generalized ODE approach to impulsive retarded functional differential equations (with Márcia Federson). *Differential and Integral Equations* 19 (11), 1201–1234 (2006).
- Stability for retarded functional differential equations (with Márcia Federson). *Ukrainian Mathematical Journal* 60 (1), 107–126 (2008).
- A new approach to impulsive retarded differential equations: stability results (with Márcia Federson). *Functional Differential Equations* 16 (4), 583–607 (2009).
- Discontinuous local semiflows for Kurzweil equations leading to LaSalle's Invariance Principle for non-autonomous systems with impulses (with Everaldo M. Bonotto and Márcia Federson), in preparation.

(1) $$\dot{y} = f(y_t, t), \quad y_{t_0} = \phi$$

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Imaz & Vorel

ASSUME:

- $f : C([-r,0],\mathbb{R}^n) \times [t_0,T] \to \mathbb{R}^n$
- there is $M \in L^1[t_0,T]$ such that

$$\left| \int_{t_1}^{t_2} f(x_s,s)\, ds \right| \le \int_{t_1}^{t_2} M(s)\, ds \quad \text{for } x \in C[-r,0],\ t_1,t_2 \in [t_0,T],$$

- there is $M \in L^1[t_0,T]$ such that

$$\left| \int_{t_1}^{t_2} [f(x_s,s) - f(y_s,s)]\, ds \right| \le \int_{t_1}^{t_2} L(s)\, \|x_s - y_s\|\, ds \quad \text{for } x,y \in C[-r,0],\ t_1,t_2 \in [t_0,T].$$

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DEFINE: $X = C([t_0 - r, T], \mathbb{R}^n)\}$ and

$$F(y,t)(\vartheta) = \begin{cases} 0 & \text{if } t_0 - r \le \vartheta \le t_0 \text{ or } t_0 - r \le t \le t_0, \\ \int_{t_0}^{\vartheta} f(y_s, s)\, ds & \text{if } t_0 \le \vartheta \le t \le T, \\ \int_{t_0}^{t} f(y_s, s)\, ds & \text{if } t_0 \le t \le \vartheta \le T, \end{cases} \qquad \tilde{x}(\vartheta) = \begin{cases} \varphi(\vartheta - t_0) & \text{if } t_0 - r \le \vartheta \le t_0, \\ \varphi(0) & \text{if } t_0 \le \vartheta \le T. \end{cases}$$

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THEN: (1) is equivalent with $\quad x(t) = \widetilde{x} + \displaystyle\int_a^t \mathrm{D}F(x(\tau), t).$

Variational measures and extensions of integrals

- Variational measures and extensions of the integral. *Real Anal. Exch.* 33, Suppl. 31st Summer Symp. Conf. Rep., 167–171 (2008).
- Variational measures and the Kurzweil-Henstock integral. *Mathematica Slovaca* 59 (6), 1–22 (2009).
- General integration and extensions. *Czechoslovak Mathematical Journal* 60 (135) (4) (2010), to appear.
- General integration and extensions II. *Czechoslovak Mathematical Journal* 60 (135) (4) (2010), to appear.

The Saks class $\mathfrak{S}$ of integrals

- $-\infty < a < b < \infty$, $\mathrm{Sub}([a,b])$ are compact subintervals in $[a,b]$,
- **Functionals** are mappings from the set of real valued functions defined on $[a,b]$ into $\mathbb{R}$,
- If S is an additive functional, then $F\colon [a,b]\to\mathbb{R}$ is S-**primitive** to $f \in \mathrm{Dom}(S)$ if

$$S(f,I) := S(f\cdot\chi_I) = F[I] = F(d) - F(c)$$

holds for all $I \in \mathrm{Sub}([a,b])$ with boundary points $c < d$.

- S is **integral** on $[a,b]$ if each S-primitive function to $f \in \mathrm{Dom}(S)$ is **continuous** on $[a,b]$. Denote $\mathfrak{S}$ **the set of all integrals** in $[a,b]$.
- Let $T, S \in \mathfrak{S}$, then T **contains** S ($S \sqsubset T$) if: $\mathrm{Dom}(S) \subset \mathrm{Dom}(T)$ and $T(f,I) = S(f,I)$ for all $f \in \mathrm{Dom}(S)$, $I \in \mathrm{Sub}([a,b])$.
- The relation $\sqsubset$ is a partial ordering in $\mathfrak{S}$.

Contribution by Štefan Schwabik:

- He presented a general approach to extensions of integrals, like the Cauchy and Harnack extensions. His results give, as a specimen, the Kurzweil-Henstock integration in the form of the extension of the Lebesgue integral.
- He introduced and studied 2 new general extensions in properly chosen class $\mathfrak{I}$ of integrals containing all the classical integrals like Newton, Riemann, Lebesgue, Perron, Kurzweil-Henstock.

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- He introduced and studied 2 new general extensions in properly chosen class $\mathfrak{I}$ of integrals containing all the classical integrals like Newton, Riemann, Lebesgue, Perron, Kurzweil-Henstock.

- These new extensions lead to approximate Nakanishi like description of the Kurzweil-Henstock integral based on the Lebesgue integral.


James Clavell
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