Dynamic equations $x^{(\Delta m)}(t) = f(t, x(t))$ on time scales

Aneta Sikorska – Nowak
Faculty of Mathematics and Computer Science,
Adam Mickiewicz University,
Poznań, Poland
e-mail: *anetas@amu.edu.pl*

Abstract

We prove the existence of solutions and Carathéodory's type solutions of the dynamic Cauchy problem

$$\begin{aligned}
&x^{(\Delta m)}(t) = f(t, x(t)), \quad t \in T,\\
&x(0) = 0,\\
&x^{\Delta}(0) = \eta_1, ..., x^{(\Delta(m-1))}(0) = \eta_{m-1}, \quad \eta_1, ...\eta_{m-1} \in E,
\end{aligned}$$

where $x^{(\Delta m)}$ denotes a $m - th$ order Δ - derivative, T denotes an unbounded time scale (nonempty closed subset of R such that there exists a sequence (a_n) in T and $a_n \to \infty$), E - a Banach space and f is a continuous function or satisfies Carathéodory's conditions and some conditions expressed in terms of measures of noncompactness.

The Sadovskii fixed point theorem and Ambrosetti's lemma are used to prove the main result.

As dynamic equations are an unification of differential and difference equations our result is also valid for differential and difference equations. The presented results are new not only for Banach valued functions but also for real valued functions.