Contents

Chapter 1

Infinite Continued Fractions

Continued fractions have many applications, from the abstract to the concrete. They give good rational approximations to irrational numbers, and the have been used to understand why you can't tune a piano perfectly.[1]

There are many places to read about continued fractions, including Chapter X of Hardy and Wright's *Intro. to the Theory of Numbers*, §13.3 of Burton's *Elementary Number Theory*, Chapter IV of Davenport, and Khintchine's *Continued Fractions.* The notes

$$\begin{aligned} |x - c_n| &= \frac{1}{q_n\left(\frac{1}{t_n}q_n + q_{n-1}\right)} \\ &< \frac{1}{q_n(a_{n+1}q_n + q_{n-1})} \\ &= \frac{1}{q_n \cdot q_{n+1}} \leq \frac{1}{n(n+1)} \to 0. \end{aligned}$$

Example. *We use PARI to illustrate the convergence of the theorem for* $x = \pi$*.*

```
? a = contfrac(Pi)
? [c[2]*1.0,c[4]*1.0,c[6]*1.0]   \\ even ones swoop down on pi.
```

[1]See `http://www.research.att.com/~njas/sequences/DUNNE/TEMPERAMENT.HTML`

Chapter 2

Binary quadratic forms

Definition. The modular group $\mathrm{SL}_2(\mathbb{Z})$ is the group of all 2×2 integer matrices with determinant $+1$.

If $g = \left(\begin{smallmatrix} p & q \\ r & s \end{smallmatrix}\right) \in \mathrm{SL}_2(\mathbb{Z})$ and $f(x, y) = ax^2 + bxy + cy^2$ is a quadratic form, let

$$f|_g(x, y) = f(px + qy, rx + sy) = f\left(\begin{pmatrix} p & q \\ r & s \end{pmatrix} \begin{bmatrix} x \\ y \end{bmatrix}\right),$$

where for simplicity we will sometimes write $f\left(\begin{bmatrix} x \\ y \end{bmatrix}\right)$ for $f(x, y)$.

$$\mathcal{F} = \left\{\tau \in \mathfrak{h} : \mathrm{Re}(\tau) \in \left[-\frac{1}{2}, \frac{1}{2}\right), |\tau| > 1 \text{ or } |\tau| = 1 \text{ and } \mathrm{Re}(\tau) \leq 0\right\}.$$

Chapter 3

Cyclotomic units

In the Chapter 2 we have defined quadratic forms.

Let p, q and r be different primes such that $p, q, r \equiv 1 \pmod 4$. Let h denote the class number of $\mathbb{Q}(\sqrt{p}, \sqrt{q}, \sqrt{r})$.

By σ_l, where $l \in S$, we denote the automorphism determined by the equation $\mathrm{Gal}\left(K_S/K_{S\setminus\{l\}}\right) = \{1, \sigma_l\}$.

Let us further define

$$\epsilon_{n_S} = \begin{cases} 1 & \text{if } S = \emptyset, \\ \frac{1}{\sqrt{l}} \mathrm{N}_{\mathbb{Q}^S/K_S}(1-\zeta_S) & \text{if } S = \{l\}, \\ \mathrm{N}_{\mathbb{Q}^S/K_S}(1-\zeta_S) & \text{if } \#S > 1. \end{cases}$$

3.1 Finite sums

Below the usual way of calculating the sum of geometric series is shown:

$$\begin{array}{llll} 1+q+q^2+\cdots+q^n & & = S, & \\ q+q^2+\cdots+q^n & +q^{n+1} & = qS & / \cdot (-1) \\ \hline 1 & -q^{n+1} & = (1-q)\cdot S & \end{array}$$

Chapter 4

Study plans

1. rok studia

kód	název	kred.	rozsah		učitel
Podzimní semestr					
Povinné předměty					
M6531	Teorie množin	2+2	2/0	zk	Fuchs, E.
M7521	Pravděpodobnost a statistika 1	4+2	2/2	zk	Budíková, M.
Povinně volitelné předměty					
M7531	Diplomová práce	4	0/0	z	Šišma, P.
Jarní semestr					
Povinné předměty					
M4520	Seminář ze středoškolské matematiky 2	2	0/2	k	Kučera, R.
M7511	Historie matematiky 1	2+1	2/0	kz	Fuchs, E.
M8501	Didaktika matematiky 1	3	2/2	k	Šimša, J.
Povinně volitelné předměty					
M8532	Diplomová práce	4	0/0	z	Šišma, P.

Index