The gregarious model structure for double categories

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In this talk, I will introduce a new model structure on the category of double categories and double functors, which I will argue is the most natural analogue for double categories of Lack's model structure for 2-categories. This "gregarious" model structure is completely characterised by the following two of its properties: every double category is fibrant, and a double functor is a trivial fibration iff it is surjective on objects, full on horizontal morphisms, full on vertical morphisms, and fully faithful on squares. Note that this model structure is preserved by all eight auto-equivalences of the category of double categories.

This model structure shares many of the excellent features of Lack's model structure for 2-categories. For instance, it is proper, it is monoidal with respect to Böhm's Gray tensor product for double categories, a double category is cofibrant iff its underlying categories of horizontal and vertical morphisms are free, and the double pseudofunctor classifier comonad is a cofibrant replacement comonad. Moreover, Lack's model structure for 2-categories is created by the (homotopically fully faithful) "double category of squares" functor from the gregarious model structure for double categories.

I will also introduce the notion of *double quasi-category*, defined as the fibrant objects of a Cisinski model structure on the category of bisimplicial sets, which I will argue presents the correct ∞-categorical generalisation of the notion of double category. I will prove that the gregarious model structure for double categories is right-induced by Watson's bisimplicial nerve functor from the model structure for double quasi-categories, and that this nerve functor is homotopically fully faithful.