HIGHER-ORDER ALGEBRAIC THEORIES AND RELATIVE MONADS

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There have traditionally been two ways to reason about universal algebraic structure categorically: via algebraic theories, and via monads. It is well known that the two are tightly related: in particular, there is a correspondence between algebraic theories and a class of monads on the category of sets.

Motivated by the study of simple type theories, Fiore and Mahmoud introduced second-order algebraic theories, which extend classical (first-order) algebraic theories by variable-binding operators, such as the existential quantifier $\exists x$ of first-order logic; the differential operators $\frac{d}{dx}$ of analysis; and the λ-abstraction operator of the unityped λ-calculus. Fiore and Mahmoud established a correspondence between second-order algebraic theories and a second-order equational logic, but did not pursue a general understanding of the categorical structure of second-order algebraic theories. In particular, the possibility of a monad–theory correspondence for second-order algebraic theories was left as an open question.

In this talk, I will present a generalisation of algebraic theories to higher-order structure, in particular subsuming the second-order algebraic theories of Fiore and Mahmoud, and describe a universal property of the category of n^{th}-order algebraic theories. The central result is a correspondence between $(n+1)^{\text{th}}$-order algebraic theories and a class of relative monads on the category of n^{th}-order algebraic theories, which extends to a monad correspondence subsuming that of the classical setting. Finally, I will discuss how the perspective lent by higher-order algebraic theories sheds new light on the classical monad–theory correspondence.

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