

KAN EXTENSIONS AS PARTIAL COLIMITS

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One way of interpreting a left Kan extension is as taking a kind of “partial colimit”, where one replaces parts of a diagram by their colimits. We make this intuition precise by means of the “partial evaluations” sitting in the so-called bar construction of monads. The (pseudo)monads of interest for forming colimits are the monad of diagrams and the monad of small presheaves, both on the category \mathbf{CAT} of locally small categories. We also define a morphism of monads between them, which we call “image”, and which takes the “free colimit” of a diagram. This morphism allows us in particular to generalize the idea of “cofinal functors”, i.e. of functors which leave colimits invariant in an absolute way. This generalization includes the concept of absolute colimit as a special case. The main result of this work says that a pointwise left Kan extension of a diagram corresponds precisely to a partial evaluation of its colimit. This categorical result is analogous to what happens in the case of probability monads, where a conditional expectation of a random variable corresponds to a partial evaluation of its center of mass.

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