Model dravec – kořist se vzájemnou interferencí dravce

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EVROPSKÁ UNIE
MINISTERSTVO ŠKOLSTVÍ, MLÁDEŽE A TĚLOVÝCHOVY
OP Vzdělávání pro konkurenceschopnost
AMathNet síť pro transfer znalostí v aplikované matematice
INVESTICE DO ROZVOJE VZDĚLÁVÁNÍ

Model dravec – kořist

$$\begin{aligned}\frac{\mathrm{d}N}{\mathrm{d}t} &= rN\left(1-\frac{N}{K}\right) - Pf(N,P),\\ \frac{\mathrm{d}P}{\mathrm{d}t} &= ePf(N,P) - mP,\end{aligned}$$

N, P	...	populace kořisti a dravce
$f(N,P)$	...	funkční odpověď predátora
r, K, e, m	...	parametry

Dynamika modelu

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Lotkův – Volterrův model: $K \to \infty$, $f(N,P) = aN$

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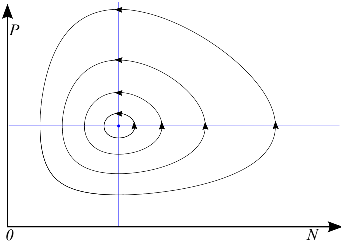
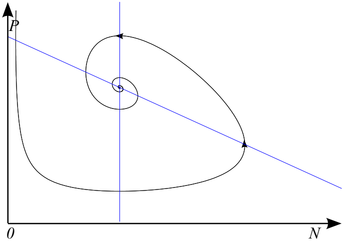


Dynamika modelu

Model s logistickým růstem kořisti: $K < \infty$, $f(N,P) = aN$

$$\begin{aligned} \frac{\mathrm{d}N}{\mathrm{d}t} &= rN\left(1-\frac{N}{K}\right) - Pf(N,P), \\ \frac{\mathrm{d}P}{\mathrm{d}t} &= ePf(N,P) - mP, \end{aligned}$$



Rosenzweigův – MacArthurův model

$$
\begin{aligned}
\frac{\mathrm{d}N}{\mathrm{d}t} &= rN\left(1-\frac{N}{K}\right)-Pf(N,P),\\
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\end{aligned}
$$

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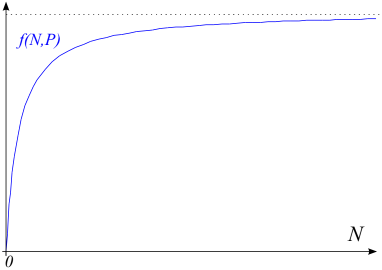
$$f(N,P) = \frac{\lambda N}{1+h\lambda N},$$



Rosenzweigův – MacArthurův model

$$\frac{\mathrm{d}N}{\mathrm{d}t} = rN\left(1-\frac{N}{K}\right) - Pf(N,P),$$
$$\frac{\mathrm{d}P}{\mathrm{d}t} = ePf(N,P) - mP,$$

$$f(N,P) = \frac{\lambda N}{1+h\lambda N},$$

λ … efektivita, s jakou dravec loví kořist
h … čas, který je třeba na ulovení a zpracování kořisti.

Dynamika Rosenzweigova – MacArthurova modelu

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\end{aligned}
$$

Rovnovážné body:

$[0,0]$	sedlo
$[K,0]$	stabilní rovnováha pro $e \leq hm$
	stabilní rovnováha pro $e > hm$ a $K < \frac{m}{\lambda(e-mh)}$
	sedlo pro $e > hm$ a $K > \frac{m}{\lambda(e-mh)}$
$[N^*, P^*]$	další rovnováha pro $e > hm$

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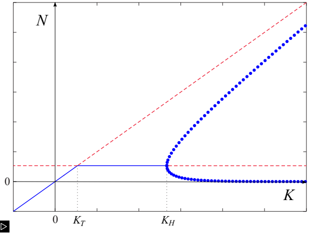

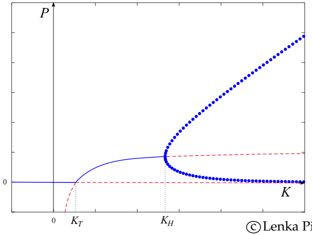

Model se vzájemnou interferencí dravce

$$\begin{aligned} \frac{dN}{dt} &= rN\left(1 - \frac{N}{K}\right) - Pf(N,P), \\ \frac{dP}{dt} &= ePf(N,P) - mP, \end{aligned}$$

$$f(N,P) = \frac{\lambda(P)N}{1 + h\lambda(P)N}, \quad \text{kde } \lambda'(P) < 0.$$

Model se vzájemnou interferencí dravce

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$$f(N,P) = \frac{\lambda(P)N}{1+h\lambda(P)N}, \quad \text{kde } \lambda'(P) < 0.$$

Speciální tvar funkční odpovědi:

$$\lambda(P) = \frac{\lambda_0}{(b+P)^w}, \quad \text{kde } w > 0$$

Pro $w = 0$ jde o Rosenzweigův – MacArthurův model.

Rovnovážné body

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$[0, 0]$ sedlo

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Rovnovážné body

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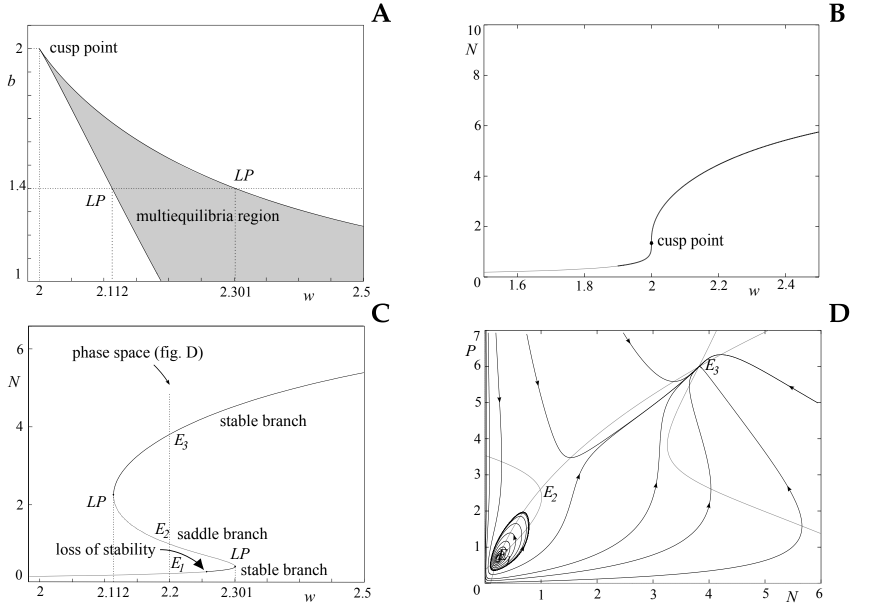
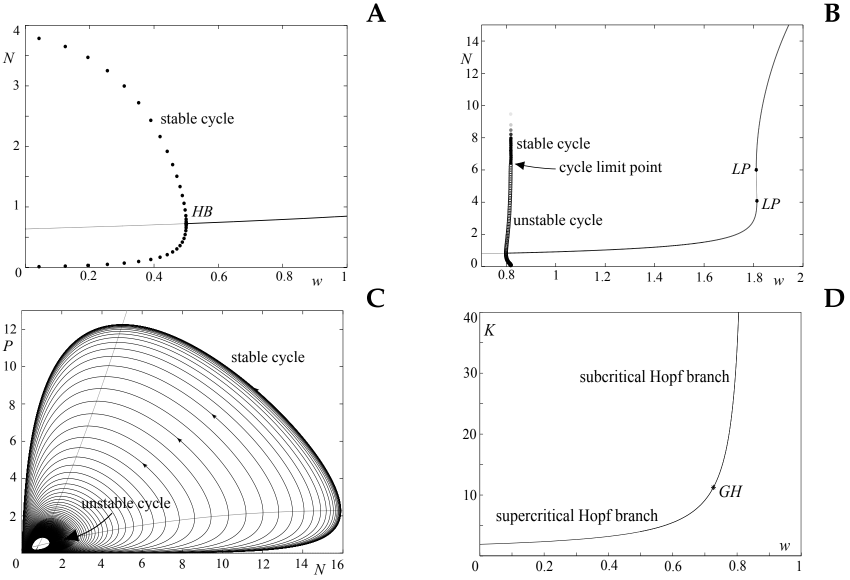
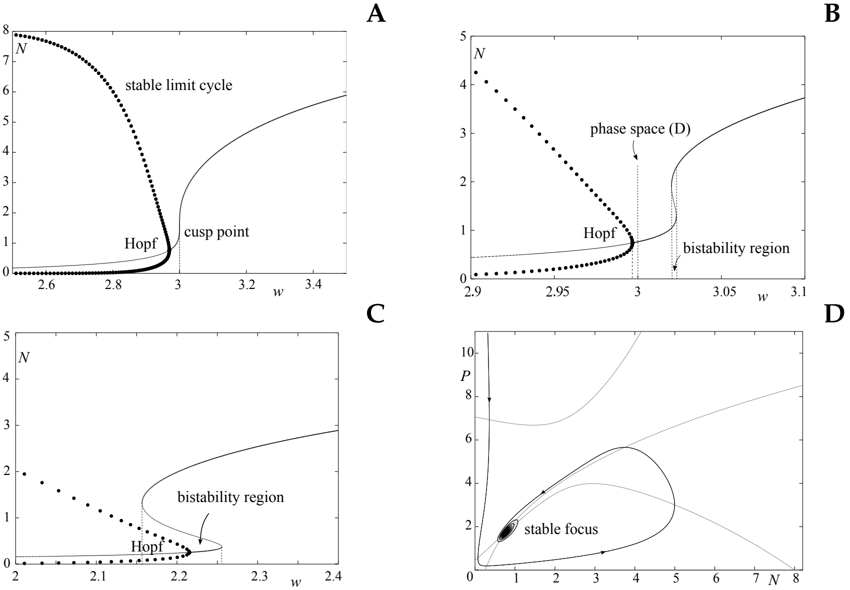
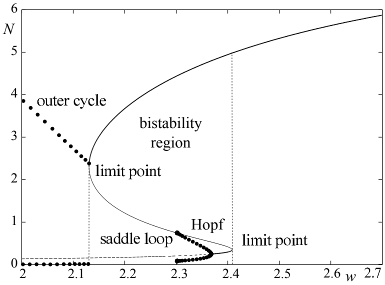
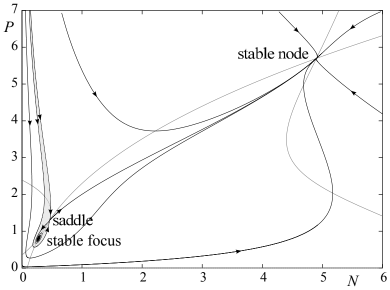
$[N^*, P^*]$	další rovnováha(**y**) pro $e > hm$


A
cusp point
b
LP
LP
multiequilibria region
w
B
N
cusp point
w
C
phase space (fig. D)
N
stable branch
E_3
LP
E_2
saddle branch
loss of stability
E_1
LP
stable branch
w
D
P
E_3
E_2
N

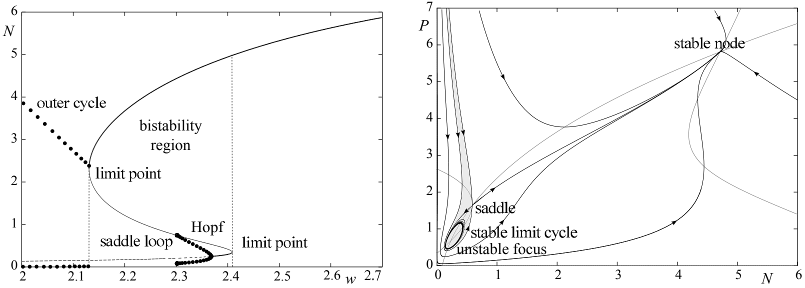

A
B
C
D
N
w
stable cycle
HB
cycle limit point
unstable cycle
LP
LP
P
K
subcritical Hopf branch
supercritical Hopf branch
GH


A
N
stable limit cycle
Hopf
cusp point
w
B
N
phase space (D)
Hopf
bistability region
w
C
N
bistability region
Hopf
w
D
P
stable focus
N

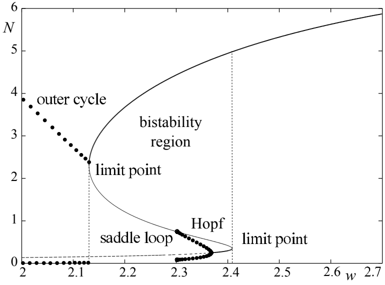

6
N
5
4
outer cycle
3
bistability
region
limit point
2
1
Hopf
saddle loop
limit point
0
2
2.1
2.2
2.3
2.4
2.5
2.6
w
2.7

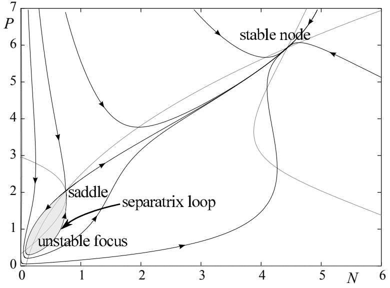

7
P
6
stable node
5
4
3
2
saddle
1
stable focus
0
0
1
2
3
4
5
N
6

$w = 2.39$

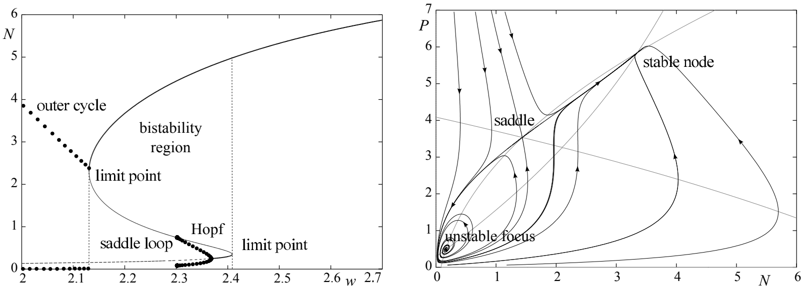

6
N
5
4
outer cycle
3
bistability
region
limit point
2
1
Hopf
saddle loop
limit point
0
2
2.1
2.2
2.3
2.4
2.5
2.6
w
2.7
7
P
6
stable node
5
4
3
2
saddle
1
stable limit cycle
unstable focus
0
0
1
2
3
4
5
N
6

$$w = 2.35$$


N
w
outer cycle
bistability
region
limit point
saddle loop
Hopf
limit point

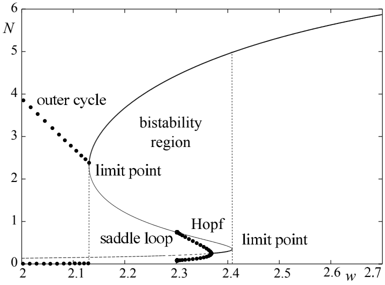

P
N
stable node
saddle
separatrix loop
unstable focus

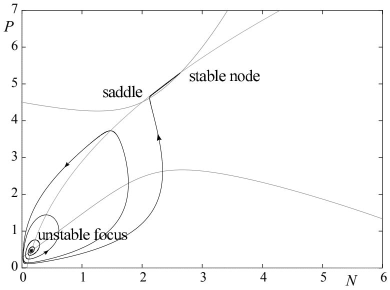
$w = 2.3$



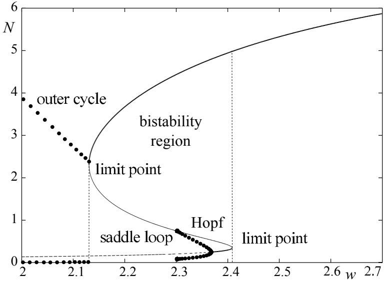
N
outer cycle
bistability
region
limit point
saddle loop
Hopf
limit point
w
P
stable node
saddle
unstable focus
N

$$w = 2.1716$$

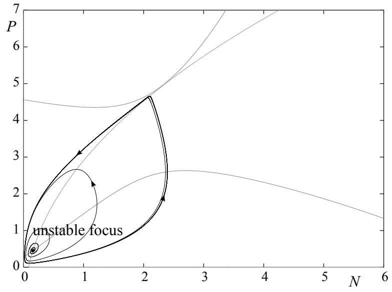

N
w
outer cycle
bistability
region
limit point
Hopf
saddle loop
limit point


P
N
saddle
stable node
unstable focus

$w = 2.135$

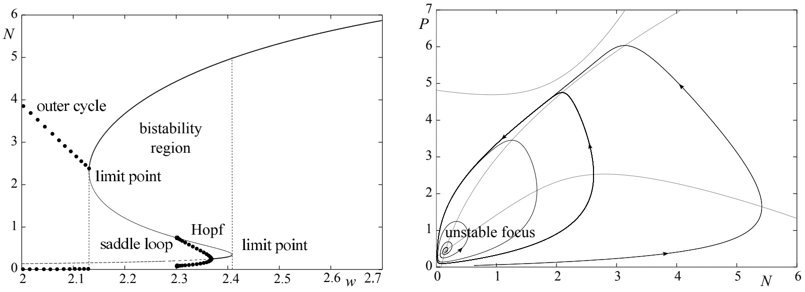


N
outer cycle
bistability
region
limit point
Hopf
saddle loop
limit point
w



P
unstable focus
N

$w = 2.13$



N
w
outer cycle
bistability
region
limit point
Hopf
saddle loop
limit point
P
N
unstable focus

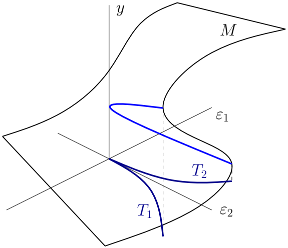
$w = 2.11$

Fold a cusp bifurkace

Nutná a postačující podmínka fold bifurkace rovnovážného bodu $[N^*, P^*]$ je:

$$\lambda^3(P^*) - C_1(2C_2 - \lambda(P^*))\lambda'(P^*) = 0\,,$$

kde $C_1 = \dfrac{er}{e-hm}$, $C_2 = \dfrac{m}{K(e-hm)}$.



$$\dot{y} = \varepsilon_1 + \varepsilon_2 y \pm y^3$$

Fold a cusp bifurkace

Pro

$$\lambda(P) = \frac{\lambda_0}{(b+P)^w}\,, \quad \text{kde } w > 0$$

podmínka přejde na

$$b\Lambda^2 + C_1(1-w)\Lambda + C_1 C_2(2w-1) = 0\,,$$

kde $\Lambda = \lambda(P^*),\ C_1 = \dfrac{er}{e-hm},\ C_2 = \dfrac{m}{K(e-hm)}.$

Fold a cusp bifurkace

Pro

$$\lambda(P) = \frac{\lambda_0}{(b+P)^w}\,, \quad \text{kde } w > 0$$

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$$b\Lambda^2 + C_1(1-w)\Lambda + C_1C_2(2w-1) = 0\,,$$

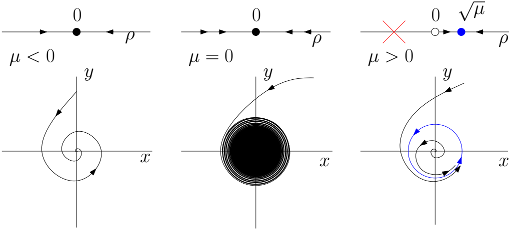
kde $\Lambda = \lambda(P^*)$, $C_1 = \dfrac{er}{e-hm}$, $C_2 = \dfrac{m}{K(e-hm)}$.
Odtud fold, resp. hysterese jen pro $w > 1$.
Kritická hodnota $w = w_c$ cusp bifurkace splňuje podmínku

$$(1-w_c)^2 = \frac{4bC_2}{C_1}(2w_c-1)\,.$$

Hopfova bifurkace

Nutnou podmínkou vzniku Hopfovy bifurkace rovnovážného bodu $[N^*, P^*]$ je:

$$\lambda'(P^*) = \frac{(e + hm - \lambda(P^*)Kh(e - hm))\lambda^2(P^*)}{e(\lambda(P^*)K(e - hm) - m)},$$



$$\dot{\rho} = \rho(\mu - \rho^2)$$

Hopfova bifurkace

Pro

$$\lambda(P) = \frac{\lambda_0}{(b+P)^w}, \quad \text{kde } w > 0$$

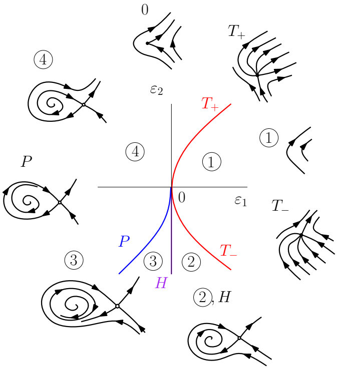
podmínka přejde na

$$A\Lambda^3 + B\Lambda^2 + C\Lambda + D = 0\,,$$

kde

$$\begin{aligned}
\Lambda &= \lambda(P^*)\,,\\
A &= K^2hb(e-hm)^3 > 0\,,\\
B &= K(e-hm)^2(hKer - b(e+hm) - wKe(e-hm))\,,\\
C &= Ke(e-hm)(wm(e-hm) - r(2hm+e))\,,\\
D &= emr(e+hm) > 0\,.
\end{aligned}$$

Bogdanova-Takensova bifurkace



$$\begin{aligned} \dot{y}_1 &= y_2 \\ \dot{y}_2 &= \varepsilon_1 + \varepsilon_2 y_1 + y_1^2 + s y_1 y_2 \end{aligned}$$

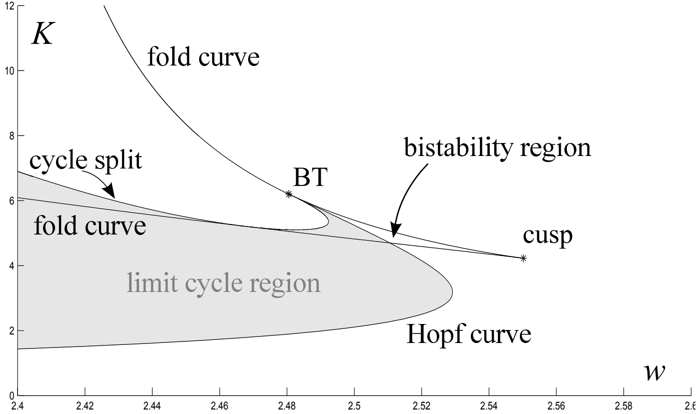
Bogdanova-Takensova bifurkace

$$\begin{aligned} b\Lambda^2 + C_1(1-w)\Lambda + C_1C_2(2w-1) &= 0\,, \\ A\Lambda^2 + B\Lambda + C &= 0\,, \end{aligned}$$

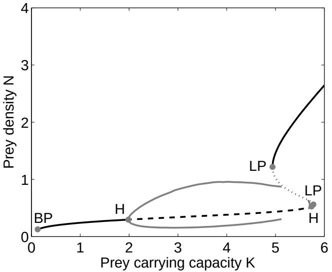
kde

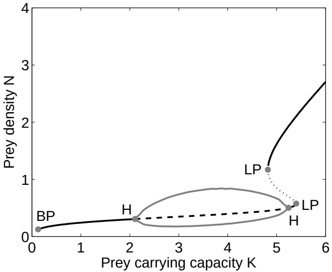
$$\begin{aligned} \Lambda &= \lambda(P^*)\,, \\ C_1 &= \frac{er}{e-hm}\,, \\ C_2 &= \frac{m}{K(e-hm)}\,, \\ A &= K^2(e-hm)^2(rh-(e-hm))\,, \\ B &= K(e-hm)(em-er-hm^2-3mhr)\,, \\ C &= 2mr(e+hm) > 0\,. \end{aligned}$$

Bogdanova-Takensova bifurkace



Bogdanova-Takensova bifurkace





DĚKUJI ZA POZORNOST.