Oscillation properties of the half-linear differential equations

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We will study the half-linear second order differential equation

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \quad \Phi(x) := |x|^{p-2}x, \ p > 1, \tag{1}$$

where r, c are continuous functions, $r(t) > 0$. We present a new oscillation criterion for (1). This equation is viewed as a perturbation of the nonoscillatory equation of the same form

$$(r(t)\Phi(x'))' + \tilde{c}(t)\Phi(x) = 0, p > 1, \tag{2}$$

and oscillation criterion for (1) is formulated in terms of the asymptotic behavior of the integral

$$\int^t [c(s) - \tilde{c}(s)]h^p(s)\, ds,$$

where h is a function which is "close" to the so-called nonprincipal solution of (2).

The main result is prersent in the following theorem.

Theorem 1. *Let $\tilde{x}$ be the positive principal solution of* (2) *such that*

$$\liminf_{t\to\infty} |G(t)| > 0, \quad G(t) := r(t)\tilde{x}(t)\Phi(\tilde{x}'(t)), \tag{3}$$

and

$$\int^\infty \frac{dt}{R(t)} = \infty, \quad R(t) := r(t)\tilde{x}^2(t)|\tilde{x}'(t)|^{p-2} \tag{4}$$

hold. Further suppose that

$$c(t) \ge \tilde{c}(t) + \frac{1}{2q\tilde{x}^p(t)R(t)(\int_{T_0}^t R^{-1}(s)\, ds)^2} \tag{5}$$

for some $T_0 \in \mathbb{R}$ and large t, and that

$$\int^\infty r^{1-q}(t)\, dt = \infty, \tag{6}$$

the below given integral in (7) *is convergent, and*

$$\int_t^\infty \left[\tilde{c}(s) + \frac{1}{2q\tilde{x}^p(s)R(s)(\int_T^s R^{-1}(\tau)\, d\tau)^2}\right] ds > 0 \tag{7}$$

for some $T \in \mathbb{R}$ and large t. If

$$\liminf_{t\to\infty} \frac{1}{\int_{T_0}^t R^{-1}(s)\, ds} \int_T^t [c(s) - \tilde{c}(s)]\tilde{x}^p(s) \left(\int_T^s R^{-1}(\tau)\, d\tau\right)^2 ds > \frac{1}{2q} \tag{8}$$

for T sufficiently large, then equation (1) *is oscillatory.*

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