Perturbation principle and Riccati technique in oscillation criteria for half-linear differential equations

Simona Fišnarová

Brno, Czech Republic

We consider a pair of half-linear second order differential equations

$$(r(t)\Phi(x'))' + c(t)\Phi(x) = 0, \tag{1}$$

$$\big[\big(r(t) + \tilde{r}(t)\big)\Phi(x')\big]' + \big(c(t) + \tilde{c}(t)\big)\Phi(x) = 0, \tag{2}$$

where $\Phi(x) = |x|^{p-2}x, p > 1$, and $r, c, \tilde{r}, \tilde{c}$ are continuous functions such that $r(t) > 0$, $r(t) + \tilde{r}(t) > 0$ for large t.

It is known (see [4]) that basic facts of the oscillation theory for linear Sturm-Liouville differential equations (the special case $p = 2$ in (1)) extend almost verbatim to (1). Consequently, similar to the linear case, any solution of (1) has either infinitely many or a finite number of zeros in a neighborhood of infinity, and hence (1) can be classified as oscillatory or nonoscillatory.

On the poster we present the results of the joint paper with O. Došlý [1]. We suppose that (1) is nonoscillatory and we examine the influence of the perturbation terms $\tilde{r}$, $\tilde{c}$ on the oscillatory nature of (2).

Our results are based on the Riccati technique, which relates nonoscillation of (2) to the solvability of the generalized Riccati equation

$$w' + c(t) + \tilde{c}(t) + (p-1)(r(t) + \tilde{r}(t))^{1-q}|w|^q = 0, \quad q = \frac{p}{p-1}, \tag{3}$$

in a neighborhood of infinity. The main idea we use is based on comparing the generalized Riccati equation (3) (which is related to (2)) with the Riccati equation associated to a certain linear equation. This connection between linear and half-linear equations enables to use results of linear oscillation theory when investigating (2).

Note that a similar problem was studied in several papers, where the case $\tilde{r} = 0$ was investigated, i.e., the perturbation was considered only in the c-term, see, e.g., [2, 3, 5] and other references of the paper [1]. The motivation for investigating the perturbations in the term involving derivative comes from the paper [6] dealing with linear Sturm-Liouville differential equations.

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